Bianchi Type I Electromagnetic Massive String Cosmological Model in General Theory of Relativity

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Abstract

In this paper, spatially homogenous and anisotropic bianchi type I electromagnetic massive string cosmological model is studied in the context of general theory of relativity and found that there is no contribution of Maxwell's field in this theory. Further for massive string cosmological model we assume the condition that $B=A^n$where A and B are the metric coefficients and n is constant and hence we have discussed general solution, its physical and Kinematic properties.

KEYWORD: Anisotropic bianchi type I, Electromagnetic field, Massive string.
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Introduction

When we study bianchi type I models it is observed that the models contain isotropic special cases and they permit arbitrarily small isotropic level at some instant of cosmic times. Bianchi type models are important in the sense that there are homogenous and isotropic from which the process of isotropic is studied through the passage of time. Hence these models are to be known as suitable models of our universe, therefore study of bianchi type models create much more interest.

There are two types of models. (i) Orthogonal models: in which the matter moves orthogonally to the hyper surface of homogeneity and (ii) The tilted models: in which the fluid flow is not normal to the hyper surface of homogeneity. The non-tilted models have been studied by King and Ellis [1], the homogenous and anisotropic bianchi type I models have been studied by number of researchers in different aspects. Bali [2-5] has obtained bianchi type I, III, IX string cosmological models in general theory of relativity. Yadav [6] has studied some bianchi type I viscous fluid string cosmological model with magnetic field. Tripathi and Dubey [7] have studied LRS bianchi type I cosmological models with variable deceleration parameter. Bianchi type I universe is also studied with the various matters in the context of bimetric theory of relativity by Deo and singh [8, 9], Deo & Ronghe [10].

In this paper we have discussed an anisotropic bianchi type I cosmological model with the matter cosmic string coupled with electromagnetic field and observed some physical as well as kinematic aspects of general theory of relativity.

1. Metric & Field equation

We consider the spatially homogenous and anisotropic type I metric in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 \text{ -------------------------------- } (2.1)$$

where A, B, C are the metric functions of cosmic time t only.

The energy momentum tensor for a cloud of massive string coupled with electromagnetic field has the form

$$T_i^j = \rho v_i v^j - \lambda x_i x^j + E_i^j \text{ -------------------------------- } (2.2)$$

where, ρ is the rest energy density for a cloud of strings with particles attached along the extension

Thus $\rho = \rho_p + \lambda$ -------------------------------- (2.3)

where ρ_p is particle energy density and λ is the tension density of the string. v^i Are the four vectors representing the velocity of cloud of particles and x^i - the four vectors representing the direction of anisotropy,i.e. x-direction.

where v_i and x_i satisfy condition

$$v_i v^i = 1,\ x_i x^i = -1 \text{ and } v_i x^i = 0 \text{ -------------------------------- } (2.4)$$

v^i is the four velocity vector satisfying the conditions $g_{ij}v^i v^j = 1$

Electromagnetic field is defined as

$$E_i^j = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^j \text{ -------------------------------- } (2.5)$$

where E_i^j is electromagnetic energy tensor and F_i^j is the electromagnetic field tensor and also we have the Maxwell's equation

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \text{ -------------------------------- } (2.6)$$

In the space time equation (2.6) gives us the only non-vanishing components of F_{ij} are F_{12}, F_{13}, F_{24} and F_{34} such that $F_{12} = eF_{24}$ and $F_{13} = eF_{34}$ -------------------------------- (2.7)

where $e = \pm 1$, we choose e=1 in equation (2.6) so that it represents an outgoing wave

The Einstein field equation in the general relativity is given by

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi kT_i^j \text{ -------------------------------- } (2.8)$$

where R_i^j is known as Ricci tensor and $R = g^{ij}R_{ij}$ is the Ricci scalar and T_i^j is energy momentum tensor for matter.

The field equations (2.8) together with the line element (2.1) with equations (2.2) to (2.7) we get

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = 8\pi k\lambda \text{ -------------------------------- } (2.9)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = 8\pi k\left[\frac{(F_{13})^2}{A^2C^2} - \frac{(F_{12})^2}{A^2B^2}\right] \text{ -------------------------------- } (2.10)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi k\left[\frac{(F_{13})^2}{A^2C^2} - \frac{(F_{12})^2}{A^2B^2}\right] \text{ -------------------------------- } (2.11)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=8\pi k\rho \text{ --------------------------------- } (2.12)$$

$$0=\left[\frac{(F_{13})^2}{A^2C^2}+\frac{(F_{12})^2}{A^2B^2}\right] \text{ --------------------------------- } (2.13)$$

where $\dot{A}=\frac{\partial A}{\partial t}, \ddot{A}=\frac{\partial^2 A}{\partial t^2}$ etc.

The spatial volume for the model is given by

$$V^3 = ABC \text{ --------------------------------- } (2.14)$$

where $V=(ABC)^{\frac{1}{3}}$ as the average scale factor, so that the Hubble parameter in anisotropic models can be defined as

$$H=\frac{\dot{V}}{V}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \text{ --------------------------------- } (2.15)$$

Here we can define the generalized mean Hubble parameter as

$$H=\frac{1}{3}\left(H_1+H_2+H_3\right) \text{ --------------------------------- } (2.16)$$

where $H_1=\frac{\dot{A}}{A}, H_2=\frac{\dot{B}}{B}, H_3=\frac{\dot{C}}{C}$ are the direction Hubble's parameter in the x, y, z direction respectively.

One of the important observational quantity-deceleration parameter q is defined as

$$q=-\frac{V\ddot{V}}{\dot{V}^2} \text{ --------------------------------- } (2.17)$$

The physical quantities of the expansion scalar θ and the average anisotropic parameter A_m are defined as

$$\theta=V^i{}_{;i}=3\frac{\dot{V}}{V}=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \text{ --------------------------------- } (2.18)$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}=\frac{1}{2}\left(\frac{\dot{A}^2}{A^2}+\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}\right)-\frac{\theta^2}{6} \text{ --------------------------------- } (2.19)$$

and $A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2$ --------------------------------- (2.20)

where $\Delta H_i = H_i - H$, i=1, 2, 3.

2. Solution of Field equations

From equation (2.13) we have $F_{12}=F_{13}=0$

i.e. $F_{12}=F_{24}=0$ and $F_{13}=F_{34}=0$ --------------------------------- (3.1)

Which gives us in this theory an isotropic bianchi type I cosmological model does not accommodate electromagnetic field.

Using equation (3.1) in (2.9) to (2.12) we get a new set of field equations which are

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=8\pi k\lambda \text{ -------------------------------- } (3.2)$$
$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=0 \text{ -------------------------------- } (3.3)$$
$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=0 \text{ -------------------------------- } (3.4)$$
$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=8\pi k\rho \text{ -------------------------------- } (3.5)$$

Therefore the system of four field equations (3.2) to (3.5) are in five unknown parameter A, B, C, λ and ρ. To obtain explicit solutions of the system we require additional constraints relating these parameters, so we assume that

$$B=A^{n} \text{ -------------------------------- } (3.6)$$

wheren is constant

From equation (3.4) and (3.6) give us

$$(n+1)\frac{\ddot{A}}{\dot{A}}+n^{2}\frac{\dot{A}}{A}=0 \text{ -------------------------------- } (3.7)$$

which yields $\dot{A}^{n+1}A^{n^2}=P_1$ -------------------------------- (3.8)

where P_1 is constant of integration

After solving equation (3.8) we have

$$A=\left(\frac{P_2t+d}{P_3}\right)^{P_3} \text{ -------------------------------- } (3.9)$$

where d is constant of integration, $P_2^{\,n+1}=P_1$ and $P_3=\dfrac{n+1}{n^2+n+1}$

From (3.6) and (3.9) we obtain

$$B=\left(\frac{P_2t+d}{P_3}\right)^{nP_3} \text{ -------------------------------- } (3.10)$$

Using equation (3.3) and (3.4)

$$\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB} \text{ -------------------------------- } (3.11)$$

which is on integration, reduces to

$$\left(\dot{B}C-C\dot{B}\right)A=P_4 \text{ -------------------------------- } (3.12)$$

where P_4 is a constant of integration. Again integrating (3.12) we obtain

$$\frac{C}{B}=P_4\int\frac{dt}{AB^2}+P_5 \text{ -------------------------------- } (3.13)$$

where P_5 is a constant of integration

Using equation (3.9) and (3.10) in (3.13) we get

$$C=\frac{P_4}{1-(n+1)P_3}\left(\frac{P_2t+d}{P_3}\right)^{1-(n+1)P_3}+P_5\left(\frac{P_2t+d}{P_3}\right)^{nP_3} \text{ -------------------------------- } (3.14)$$

Using values A, B, C from (3.9), (3.10) and (3.14) in (2.1) we get,

$$ds^2 = dt^2 - \left(\frac{P_2 t + d}{P_3}\right)^{2P_3} dx^2 - \left(\frac{P_2 t + d}{P_3}\right)^{2nP_3} dy^2 - \left[\frac{P_4}{1-(n+1)P_3}\left(\frac{P_2 t + d}{P_3}\right)^{1-(n+1)P_3} + P_5\left(\frac{P_2 t + d}{P_3}\right)^{nP_3}\right]^2 dz^2 \text{ -----------------------} \quad (3.15)$$

By using a suitable transformation of coordinate equation (3.15) becomes

$$ds^2 = \left(\frac{P_3}{P_2}\right)^2 dT^2 - T^{2P_3}dx^2 - T^{2nP_3}dy^2 - \left[\frac{P_4}{1-(n+1)P_3}T^{1-(n+1)P_3} + P_5 T^{nP_3}\right]^2 dz^2 \text{ ---------} \quad (3.16)$$

3. Particular model for $P_5 = 0$

When $P_5 = 0$, the line element equation (3.16) becomes

$$ds^2 = \left(\frac{P_3}{P_2}\right)^2 dT^2 - T^{2P_3}dx^2 - T^{2nP_3}dy^2 - \left[\frac{P_4}{1-(n+1)P_3}T^{1-(n+1)P_3}\right]^2 dz^2 \text{ -------------------} \quad (4.1)$$

The rest energy density ρ, the string tension λ and particle density ρ_p for equation (4.1) are given by

$$\rho = 0 \text{ ---------------------------------} \quad (4.2)$$

$$8\pi\lambda = -\left(\frac{n^2 - n - 1}{n+1}\right)\frac{P_2^2}{T^2} \text{ ---------------------------------} \quad (4.3)$$

$$8\pi\rho_p = \left(\frac{n^2 - n - 1}{n+1}\right)\frac{P_2^2}{T^2} \text{ ---------------------------------} \quad (4.4)$$

Using equation (4.2) to (4.4) the equation (2.3)
i.e, $\rho = \rho_p + \lambda$ is satisfied.

4. Conclusion

In this paper, spatially homogenous and anisotropic bianchi type I cosmological model representing electromagnetic massive strings is studied in general theory of relativity.
In the general, model is expanding, shearing and non- rotating. Here we have observed that in this model, electromagnetic field does not exists and some physical as well as kinematic properties are discussed and for particular model when $P_5 = 0$, by using (4.2) to (4.4) we observe that for cosmic cloud string $\rho = \rho_p + \lambda$ is satisfied.

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