N-Dimensional Plane Gravitational Waves in Bimetric Relativity (IV)

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Abstract

The plane gravitational wave solutions of the field equations $N_i^j = 0$ in *n*-dimensional space-time V_n for BR theory of Rosen (1973,74) are given by g_{ij} which satisfied

$$Q\rho_i^j + P\sigma_i^j = 0$$

which further breaks in

$$\overline{\overline{w}}_{n-5}\rho_i^j + \overline{w}_{n-5}\sigma_i^j = 0 = \overline{\overline{\phi}}_{n-5}\rho_i^j + \overline{\phi}_{n-5}\sigma_i^j, \quad \overline{\overline{w}}_{n-4}\rho_i^j + \overline{w}_{n-4}\sigma_i^j = 0 = \overline{\overline{\phi}}_{n-4}\rho_i^j + \overline{\phi}_{n-4}\sigma_i^j,$$

$$\overline{\overline{w}}_{n-3}\rho_i^j + \overline{w}_{n-3}\sigma_i^j = 0 = \overline{\overline{\phi}}_{n-3}\rho_i^j + \overline{\phi}_{n-3}\sigma_i^j, \quad \overline{\overline{w}}_{n-2}\rho_i^j + \overline{w}_{n-2}\sigma_i^j = 0 = \overline{\overline{\phi}}_{n-2}\rho_i^j + \overline{\phi}_{n-2}\sigma_i^j,$$

$$\overline{\overline{w}}_{n-1}\rho_i^j + \overline{w}_{n-1}\sigma_i^j = 0 = \overline{\overline{\phi}}_{n-1}\rho_i^j + \overline{\phi}_{n-1}\sigma_i^j,$$

where $\rho_i^j = [(\phi_{n-5}^2 + \phi_{n-4}^2 + \phi_{n-3}^2 + \phi_{n-2}^2 + \phi_{n-1}^2) - 1]g^{hj}\overline{g}_{hi}$

$$\sigma_i^j = \frac{d}{dZ}\{[1 - (\phi_{n-5}^2 + \phi_{n-4}^2 + \phi_{n-3}^2 + \phi_{n-2}^2 + \phi_{n-1}^2)]g^{hj}\overline{g}_{hi}\}$$

$$\phi_{n-5} = \frac{Z,_{n-5}}{Z,_n}, \phi_{n-4} = \frac{Z,_{n-4}}{Z,_n}, \phi_{n-3} = \frac{Z,_{n-3}}{Z,_n}, \phi_{n-2} = \frac{Z,_{n-2}}{Z,_n}, \phi_{n-1} = \frac{Z,_{n-1}}{Z,_n}.$$

$$w_{n-5} = t + \phi_{n-5}x^{n-5}, w_{n-4} = t + \phi_{n-4}x^{n-4}, w_{n-3} = t + \phi_{n-3}x^{n-3}, w_{n-2} = t + \phi_{n-2}x^{n-2}, w_{n-1} = t + \phi_{n-1}x^{n-1}$$

If Z is independent of the variables $x^{n-5}, x^{n-4}, x^{n-3}, x^{n-2}$ respectively ,the corresponding work given in the papers [1],[2],[3] and [4] regarding the plane wave solutions in *n*-dimensional space-time V_n for bimetric theory of relativity proposed by Rosen (1973,74) can be brought out

§1. Introduction In the previous paper refer it to [1],we have obtained the plane wave solutions g_{ij} of the field equations $N_i^j = 0$ in n-dimensional space-time V_n for BR theory of Rosen (1973,74) by reformulating Karade's (1994) definition of plane wave as follows:

Definition : A plane wave g_{ij} is a non–flat solution of the field equations

$$N_i^j = 0, (i, j = 1,2...n) \tag{1.1}$$

in an empty region of the space-time such that

$$g_{ij} = g_{ij}(Z) \ , \ Z = Z(x^i), \ \ x^i = x^1, x^2, ...x^{n-3}, x^{n-2}, x^{n-1}, t \tag{1.2}$$

in some suitable co-ordinate system such that

$$g^{ij} Z,_i \ Z,_j = 0 \ , \qquad Z,_i = \partial Z / \partial x^i \tag{1.3}$$

$$Z = Z(x^{n-4}, x^{n-3}, x^{n-2}, x^{n-1}, t) \ , \ \ Z,_{n-4} \neq 0, Z,_{n-3} \neq 0, Z,_{n-2} \neq 0, Z,_{n-1} \neq 0, Z,_n \neq 0 \tag{1.4}$$

where $N_i^j = \frac{1}{2} f^{\alpha\beta}(g^{hj} g_{hi|\alpha})_{|\beta}$, $N = N_i^i$,...$k = \sqrt{g/f}$,...$g = \det(g_{ij})$,...$f = \det(f_{ij})$

and the bar (|) stands for f-covariant differentiation .In this definition , the signature convention adopted is

$$g_{aa} < 0, \quad a = 1,2,...(n-1)$$

$$\begin{vmatrix} g_{aa} & g_{ab} \\ g_{ba} & g_{bb} \end{vmatrix} > 0, \qquad \begin{vmatrix} g_{aa} & g_{ab} & g_{ac} \\ g_{ba} & g_{bb} & g_{bc} \\ g_{ca} & g_{cb} & g_{cc} \end{vmatrix} < 0$$

[not summed for $a, b, c = 1,2...(n-1)$]

$$\begin{vmatrix} g_{11} & g_{12} & g_{13} & \cdots & g_{(n-1)} \\ g_{21} & g_{22} & g_{23} & \cdots & g_{(n-1)} \\ \vdots & & & & \\ g_{(n-1)1} & g_{(n-1)2} & g_{(n-1)3} & \cdots & g_{(n-1)(n-1)} \end{vmatrix} < 0 \text{ ,when } n \text{ is even and}$$

$$\begin{vmatrix} g_{11} & g_{12} & g_{13} & \cdots & g_{(n\text{-}1)} \\ g_{21} & g_{22} & g_{23} & \cdots & g_{(n\text{-}1)} \\ \vdots & & & & \\ g_{(n\text{-}1)1} & g_{(n\text{-}1)2} & g_{(n\text{-}1)3} & \cdots & g_{(n\text{-}1)(n\text{-}1)} \end{vmatrix} < 0 \text{ , when } n \text{ is odd } \quad g_{nn} > 0\,. \qquad (1.5)$$

and accordingly $g < 0$ when n is even , $g > 0$ when n is odd.

The field equations $N_i^j = 0$ then yield

$$Q\rho_i^j + P\sigma_i^j = 0$$

which further breaks into

$$\overline{\overline{w}}_{n-4}\rho_i^j + \overline{w}_{n-4}\sigma_i^j = 0 = \overline{\overline{\phi}}_{n-4}\rho_i^j + \overline{\phi}_{n-4}\sigma_i^j\,,$$

$$\overline{\overline{w}}_{n-3}\rho_i^j + \overline{w}_{n-3}\sigma_i^j = 0 = \overline{\overline{\phi}}_{n-3}\rho_i^j + \overline{\phi}_{n-3}\sigma_i^j\,,$$

$$\overline{\overline{w}}_{n-2}\rho_i^j + \overline{w}_{n-2}\sigma_i^j = 0 = \overline{\overline{\phi}}_{n-2}\rho_i^j + \overline{\phi}_{n-2}\sigma_i^j\,,$$

$$\overline{\overline{w}}_{n-1}\rho_i^j + \overline{w}_{n-1}\sigma_i^j = 0 = \overline{\overline{\phi}}_{n-1}\rho_i^j + \overline{\phi}_{n-1}\sigma_i^j\,,$$

where $w_{n-4} = t + \phi_{n-4}x^{n-4}, w_{n-3} = t + \phi_{n-3}x^{n-3},\quad w_{n-2} = t + \phi_{n-2}x^{n-2}, w_{n-1} = t + \phi_{n-1}x^{n-1},$

$$Z,_{n-4} = \frac{\phi_{n-4}}{M_{n-4}},\ Z_{n-3} = \frac{\phi,_{n-3}}{M_{n-3}},\ Z_{n-2} = \frac{\phi_{n-2}}{M_{n-2}},\ Z_{n-1} = \frac{\phi_{n-1}}{M_{n-1}},\ Z_n = \frac{1}{P}.$$

$$\phi_{n-4} = \frac{Z,_{n-4}}{Z,_n},\ \phi_{n-3} = \frac{Z,_{n-3}}{Z,_n},\ \phi_{n-2} = \frac{Z,_{n-2}}{Z,_n},\ \phi_{n-1} = \frac{Z,_{n-1}}{Z,_n}.$$

$$M_{n-4} = \overline{w}_{n-4} - \overline{\phi}_{n-4}x^{n-4},\quad M_{n-3} = \overline{w}_{n-3} - \overline{\phi}_{n-3}x^{n-3},\quad M_{n-2} = \overline{w}_{n-2} - \overline{\phi}_{n-2}x^{n-2},$$

$$M_{n-1} = \overline{w}_{n-1} - \overline{\phi}_{n-1}x^{n-1},\quad N_{n-4} = \overline{\overline{w}}_{n-4} - \overline{\overline{\phi}}_{n-4}x^{n-4}\,,\quad N_{n-3} = \overline{\overline{w}}_{n-3} - \overline{\overline{\phi}}_{n-3}x^{n-3}\,,$$

$$N_{n-2} = \overline{\overline{w}}_{n-2} - \overline{\overline{\phi}}_{n-2}x^{n-2}\,,\quad N_{n-1} = \overline{\overline{w}}_{n-1} - \overline{\overline{\phi}}_{n-1}x^{n-1}\,,$$

$$\rho_i^j = [(\phi_{n-4}^2 + \phi_{n-3}^2 + \phi_{n-2}^2 + \phi_{n-1}^2) - 1]g^{hj}\overline{g}_{hi}\,,$$

$$\sigma_i^j = \frac{d}{dZ}\{[1-(\phi_{n-4}^2+\phi_{n-3}^2+\phi_{n-2}^2+\phi_{n-1}^2)]g^{hj}\overline{g}_{hi}\}.$$

In the present paper, we confine ourselves to the same space-time V_n but relax the condition (1.2),(1.3) and (1,5) with assuming

$$Z = Z(x^{n-5}, x^{n-4}, x^{n-3}, x^{n-2}, x^{n-1}, t),$$

$$Z,_{n-5} \neq 0, Z,_{n-4} \neq 0, Z,_{n-3} \neq 0, Z,_{n-2} \neq 0, Z,_{n-1} \neq 0, Z,_{n} \neq 0, \quad (1.6)$$

we get some interesting results for BR theory of Rosen (1973,74). If Z is independent of the variable x^{n-5} then earlier work regarding plane wave solutions in n-dimensional space-time given in the paper [1] for bimetrjc relativity can be deduced.

§ 2.Solutions of field equations

From the equations (1.3) and (1.5) we get

$$g^{(n-5)(n-5)}\phi_{n-5}^2 + 2g^{(n-5)(n-4)}\phi_{n-5}\phi_{n-4} + 2g^{(n-5)(n-3)}\phi_{n-5}\phi_{n-3} + 2g^{(n-5)(n-2)}\phi_{n-5}\phi_{n-2} +$$

$$2g^{(n-5)(n-1)}\phi_{n-5}\phi_{n-1} + 2g^{(n-5)n}\phi_{n-5} + g^{(n-4)(n-4)}\phi_{n-4}^2 + 2g^{(n-4)(n-3)}\phi_{n-4}\phi_{n-3} + 2g^{(n-4)(n-2)}\phi_{n-4}\phi_{n-2} +$$

$$2g^{(n-4)(n-1)}\phi_{n-4}\phi_{n-1} + 2g^{(n-4)n}\phi_{n-4} + g^{(n-3)(n-3)}\phi_{n-3}^2 + 2g^{(n-3)(n-2)}\phi_{n-3}\phi_{n-2} + 2g^{(n-3)(n-1)}\phi_{n-3}\phi_{n-1} +$$

$$2g^{(n-3)n}\phi_{n-3} + g^{(n-2)(n-2)}\phi_{n-2}^2 + 2g^{(n-2)(n-1)}\phi_{n-2}\phi_{n-1} + 2g^{(n-2)n}\phi_{n-2} + g^{(n-1)(n-1)}\phi_{n-1}^2 + 2g^{(n-1)n}\phi_{n-1}$$

$$+ g^{nn} = 0. \quad (2.1)$$

where $$\phi_{n-5} = \frac{Z,_{n-5}}{Z,_n}, \phi_{n-4} = \frac{Z,_{n-4}}{Z,_n}, \phi_{n-3} = \frac{Z,_{n-3}}{Z,_n}, \phi_{n-2} = \frac{Z,_{n-2}}{Z,_n}, \phi_{n-1} = \frac{Z,_{n-1}}{Z,_n}. \quad (2.2)$$

which further yield

$$w_{n-5} = t + \phi_{n-5}x^{n-5}, \quad (2.3)$$

$$w_{n-4} = t + \phi_{n-4}x^{n-4}, \quad (2.4)$$

$$w_{n-3} = t + \phi_{n-3}x^{n-3}, \quad (2.5)$$

$$w_{n-2} = t + \phi_{n-2}x^{n-2}, \quad (2.6)$$

$$w_{n-1} = t + \phi_{n-1} x^{n-1} \tag{2.7}$$

where $w_{n-5}, w_{n-4}, w_{n-3}, w_{n-2}$ and w_{n-1} are arbtrary functions of Z.

Differentiating partially (2.3) with respect to x^{n-5}, t ,(2.4) wirth respect to x^{n-4}, t, (2.5) with respect to x^{n-3}, t, (2.6) with respect to x^{n-2}, t and (2.7) with respect to x^{n-1}, t we obtain

$$Z,_{n-5} = \frac{\phi_{n-5}}{M_{n-5}} \;, Z,_{n} = \frac{1}{M_{n-5}} \tag{2.8}$$

where $M_{n-5} = \overline{w}_{n-5} - \overline{\phi}_{n-5} x^{n-5}$ (2.9)

$$Z,_{n-4} = \frac{\phi_{n-4}}{M_{n-4}} \;, \quad Z,_{n} = \frac{1}{M_{n-4}} \tag{2.10}$$

where $M_{n-4} = \overline{w}_{n-4} - \overline{\phi}_{n-4} x^{n-4}$ (2.11)

$$Z,_{n-3} = \frac{\phi_{n-3}}{M_{n-3}} \;, \quad Z,_{n} = \frac{1}{M_{n-3}} \tag{2.12}$$

where $M_{n-3} = \overline{w}_{n-3} - \overline{\phi}_{n-3} x^{n-3}$ (2.13)

$$Z,_{n-2} = \frac{\phi_{n-2}}{M_{n-2}}, \; Z,_{n} = \frac{1}{M_{n-2}} \tag{2.14}$$

where $M_{n-2} = \overline{w}_{n-2} - \overline{\phi}_{n-2} x^{n-2}$ (2.15)

$$Z,_{n-1} = \frac{\phi_{n-1}}{M_{n-1}} \;, Z,_{n} = \frac{1}{M_{n-1}} \tag{2.16}$$

where $M_{n-1} = \overline{w}_{n-1} - \overline{\phi}_{n-1} x^{n-1}$ (2.17)

Differentiating partially (2.9) with respect to x^{n-5}, t ,(2.11) with respect to x^{n-4}, t, (2.13) with respect to x^{n-3}, t, (2.15) with respect to x^{n-2}, t and (217) with respect to x^{n-1}, t we obtain

$$M_{(n-5),(n-5)} = \frac{N_{n-5}\phi_{n-5}}{M_{n-5}} - \bar{\phi}_{n-5}, \qquad M_{(n-5),n} = \frac{N_{n-5}}{M_{n-5}} \tag{2.18}$$

where $N_{n-5} = \bar{\bar{w}}_{n-5} - \bar{\bar{\phi}}_{n-5} x^{n-5}$ (2.19)

$$M_{(n-4),(n-4)} = \frac{N_{n-4}\phi_{n-4}}{M_{n-4}} - \bar{\phi}_{n-4}, \qquad M_{(n-4),n} = \frac{N_{n-4}}{M_{n-4}} \tag{2.20}$$

where $N_{n-4} = \bar{\bar{w}}_{n-4} - \bar{\bar{\phi}}_{n-4} x^{n-4}$ (2.21)

$$M_{(n-3),(n-3)} = \frac{N_{n-3}\phi_{n-3}}{M_{n-3}} - \bar{\phi}_{n-3}, \; M_{(n-3),n} = \frac{N_{n-3}}{M_{n-3}} \tag{2.22}$$

where $N_{n-3} = \bar{\bar{w}}_{n-3} - \bar{\bar{\phi}}_{n-3} x^{n-3}$ (2.23)

$$M_{(n-2),(n-2)} = \frac{N_{n-2}\phi_{n-2}}{M_{n-2}} - \bar{\phi}_{n-2}, \quad M_{(n-2),n} = \frac{N_{n-2}}{M_{n-2}} \tag{2.24}$$

where $N_{n-2} = \bar{\bar{w}}_{n-2} - \bar{\bar{\phi}}_{n-2} x^{n-2}$ (2.25)

$$M_{(n-1),(n-1)} = \frac{N_{n-1}\phi_{n-1}}{M_{n-1}} - \bar{\phi}_{n-1}, \; M_{(n-1),n} = \frac{N_{n-1}}{M_{n-1}} \tag{2.26}$$

where $N_{n-1} = \bar{\bar{w}}_{n-1} - \bar{\bar{\phi}}_{n-1} x^{n-1}$ (2.27)

The equations (2.8),(2.10),(2.12),(2.14),(2.16) and (2.18),(2.20), (2.22),(2.24),(2.26), reveal that

$$M_{n-5} = M_{n-4} = M_{n-3} = M_{n-2} = M_{n-1} = P \quad \text{(say) and}$$

$$N_{n-5} = N_{n-4} = N_{n-3} = N_{n-2} = N_{n-1} = Q \qquad \text{(say)} \tag{2.28}$$

Then the equations (2.8),(2.10),(2.12),(2.14),(2.16) and (2.18),(2.20), (2.22),(2.24),(2.26) can be rewritten as

$$Z,_{n-5} = \frac{\phi_{n-5}}{P}, Z,_{n-4} = \frac{\phi_{n-4}}{P}, Z_{n-3} = \frac{\phi,_{n-3}}{P}, Z_{n-2} = \frac{\phi_{n-2}}{P}, Z_{n-1} = \frac{\phi_{n-1}}{P}, Z_n = \frac{1}{P}. \tag{2.29}$$

and

$$P,_{n-5}=\frac{Q\phi_{n-5}}{P}-\bar{\phi}_{n-5}, P,_{n-4}=\frac{Q\phi_{n-4}}{P}-\bar{\phi}_{n-4}, P,_{n-3}=\frac{Q\phi_{n-3}}{P}-\bar{\phi}_{n-3}, P,_{n-2}=\frac{Q\phi_{n-2}}{P}-\bar{\phi}_{n-2},$$

$$P,_{n-1}=\frac{Q\phi_{n-1}}{P}-\bar{\phi}_{n-1}, \quad P,_{n}=\frac{Q}{P} \tag{2.30}$$

where a bar (-) over a letter means the derivative with respect to Z. It is to be noted that we have retained the format of the mathematical expressions derived by Karade (1994) here too.

Presuming f_{ij} as Lorentz metric (-1,-1,-1,…-1,+1) ,the f-covariant derivative becomes the ordinary partial derivative and the field equations (1.1) assume the simple form

$$f^{\alpha\beta}[g^{hj}g_{hi,\alpha}],_{\beta}=0 \tag{2.31}$$

which in view of (1.6) becomes

$$f^{(n-5)(n-5)}[g^{hj}g_{hi,(n-5)}],_{(n-5)}+f^{(n-4)(n-4)}[g^{hj}g_{hi,(n-4)}],_{n-4}+f^{(n-3)(n-3)}[g^{hj}g_{hj,(n-3)}],_{(n-3)}+$$

$$f^{(n-2)(n-2)}[g^{hj}g_{hi,(n-2)}],_{(n-2)}+f^{(n-1)(n-1)}[g^{hj}g_{hi,(n-1)}],_{(n-1)}+f^{nn}[g^{hj}g_{hi,n}],_{n}=0$$

which further yield

$$Q\{[(\phi^2_{n-5}+\phi^2_{n-4}+\phi^2_{n-3}+\phi^2_{n-2}+\phi^2_{n-1})-1]g^{hj}\bar{g}_{hi}\}+P\{[1-(\phi^2_{n-5}+\phi^2_{n-4}+\phi^2_{n-3}+\phi^2_{n-2}+\phi^2_{n-1})]g^{hj}\bar{\bar{g}}_{hi}$$

$$+[1-(\phi^2_{n-5}+\phi^2_{n-4}+\phi^2_{n-3}+\phi^2_{n-2}+\phi^2_{n-1})]\overline{g}^{hj}\bar{g}_{hi}-[2(\phi_{n-5}\bar{\phi}_{n-5}+\phi_{n-4}\bar{\varepsilon}_{n-4}+\phi_{n-3}\bar{\phi}_{n-3}+\phi_{n-2}\bar{\phi}_{n-2}$$

$$+\phi_{n-1}\bar{\phi}_{n-1})]g^{hj}\bar{g}_{hi}\}=0 \tag{2.32}$$

Equation (2.32) can be put in the form analogous to that of Karade (1994) as

$$Q\rho^j_i+P\sigma^j_i=0 \tag{2.33}$$

where $\rho^j_i=[(\phi^2_{n-5}+\phi^2_{n-4}+\phi^2_{n-3}+\phi^2_{n-2}+\phi^2_{n-1})-1]g^{hj}\bar{g}_{hi}$

$$\sigma^j_i=\frac{d}{dZ}\{[1-(\phi^2_{n-5}+\phi^2_{n-4}+\phi^2_{n-3}+\phi^2_{n-2}+\phi^2_{n-1})]g^{hj}\bar{g}_{hi}\}$$

Substituting the values of P and Q the equations (2.33) reduced to

$$\bar{\bar{w}}_{n-5}\rho_i^j + \bar{w}_{n-5}\sigma_i^j = 0 = \bar{\bar{\phi}}_{n-5}\rho_i^j + \bar{\phi}_{n-5}\sigma_i^j,\ \bar{\bar{w}}_{n-4}\rho_i^j + \bar{w}_{n-4}\sigma_i^j = 0 = \bar{\bar{\phi}}_{n-4}\rho_i^j + \bar{\phi}_{n-4}\sigma_i^j,$$

$$\bar{\bar{w}}_{n-3}\rho_i^j + \bar{w}_{n-3}\sigma_i^j = 0 = \bar{\bar{\phi}}_{n-3}\rho_i^j + \bar{\phi}_{n-3}\sigma_i^j,\ \bar{\bar{w}}_{n-2}\rho_i^j + \bar{w}_{n-2}\sigma_i^j = 0 = \bar{\bar{\phi}}_{n-2}\rho_i^j + \bar{\phi}_{n-2}\sigma_i^j,$$

$$\bar{\bar{w}}_{n-1}\rho_i^j + \bar{w}_{n-1}\sigma_i^j = 0 = \bar{\bar{\phi}}_{n-1}\rho_i^j + \bar{\phi}_{n-1}\sigma_i^j, \qquad (2.34)$$

which are again in the format of Karade (1994).

Conclusion. *we conclude that the plane gravitational waves g_{ij} are given by the equations* (2.33) or (2.34).

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