$[\frac{1}{\sqrt{2}}(y+z)-t]$-type and $[t\sqrt{2}/(y+z)]$-type five dimensional plane wave solutions of Einstein's field equations in general relativity

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Abstract

In the present paper, we have studied $[\frac{1}{\sqrt{2}}(y+z)-t]$- type and $[t\sqrt{2}/(y+z)]$-type plane wave solutions of Einstein's field equations in general theory of relativity in the case where the zero mass scalar field coupled with gravitational field and zero mass scalar field coupled with gravitational & electromagnetic field and established the existence of these two types of plane wave solutions in V_5.
Furthermore we have considered the case of massive scalar field and shown that the non-existence of these two types of plane wave solutions in GR theory.

§1. Introduction

H.Takeno (1961) has obtained the non-flat plane wave solutions g_{ij} of the field equations $R_{ij}=0$ and established the existence of $(z-t)$-type and (t/z)-type plane waves in purely gravitational case in four-dimensional empty region of space-time. On the lines of Takeno (1961), Ladke and Thengane (2002) have also obtained the plane wave solutions g_{ij} of the field equations $R_{ij}=0$ in purely gravitational case by reformulating Takeno's (1961) definition of plane wave as follows:
Definition A plane wave g_{ij} is a non-flat solution of field equations

$$R_{ij}=0, \qquad [i, j=1,2,3,4,5] \tag{1.1}$$

in an empty region of space-time such that

$$g_{ij}=g_{ij}(Z), \quad Z=Z(x^i), \qquad \text{where } x^i=u,x,y,z,t \tag{1.2}$$

in some suitable co-ordinate system such that

$$g^{ij}Z,_i Z,_j=0, \qquad Z,_i=\frac{\partial Z}{\partial x^i}, \tag{1.3}$$

$$Z = Z(y,z,t), \qquad Z,_3 \neq 0, \qquad Z,_4 \neq 0, \qquad Z,_5 \neq 0 . \tag{1.4}$$

In this definition, the signature convention adopted is

$$g_{rr} < 0, \quad r = 1,2,3,4$$

$$\begin{vmatrix} g_{rr} & g_{rs} \\ g_{sr} & g_{ss} \end{vmatrix} > 0, \quad \begin{vmatrix} g_{rr} & g_{rs} & g_{rt} \\ g_{sr} & g_{ss} & g_{st} \\ g_{tr} & g_{ts} & g_{tt} \end{vmatrix} < 0 \quad , \qquad g_{55} > 0 \tag{1.5}$$

[not summed for *r*,*s* =1,2,3,4] and accordingly $g = \det(g_{ij}) > 0$ (1.6)

and in the paper refer it to [2], we have established the existence of $[\frac{1}{\sqrt{2}}(y+z)-t]$-type and $[t\sqrt{2}/(y+z)]$-type plane waves in V_5.

Furthermore , in the paper [3] by investigating the plane symmetric line elements

$$ds^2 = -Adu^2 - Adx^2 - B[dy^2 + dz^2 - dt^2] \tag{1.7}$$

and

$$ds^2 = -Adu^2 - Adx^2 - Z^2Bdy^2 - Z^2Bdz^2 + Bdt^2 \tag{1.8}$$

we have obtained the concrete form of g_{ij} as $P = 0$

where $P = 2R_{33} = 2R_{44} = R_{55} = 2R_{34} = -\sqrt{2}R_{35} = -\sqrt{2}R_{45}$, (1.9)

and $$P = \frac{(y+z)^2}{Z^2}R_{33} = \frac{(y+z)^2}{Z^2}R_{44} = \frac{(y+z)^2}{2}R_{55} = \frac{(y+z)^2}{Z^2}R_{34}$$

$$= -\frac{(y+z)^2}{Z\sqrt{2}}R_{35} = -\frac{(y+z)^2}{Z\sqrt{2}}R_{45} \tag{1.10}$$

for $[\frac{1}{\sqrt{2}}(y+z)-t]$-type and $[t\sqrt{2}/(y+z)]$-type plane waves respectively.

In this paper, we investigate whether these two types of plane wave solutions exist in the case where zero mass scalar field coupled with gravitational field and the zero mass scalar field coupled with gravitational & electromagnetic field. Furthermore we consider the coupling of massive scalar field with gravitational field and the massive scalar field with gravitational & electromagnetic field in V_5 to investigate the existence of these two types of plane wave solutions of Einstein's field equation

$$R_{ij} = (-8\pi)[T_{ij} - \frac{1}{2}g_{ij}T], \quad [i,j = 1,2,3,4,5] \tag{1.11}$$

where R_{ij} is the Ricci tensor of the space-time ,

T_{ij} is the energy momentum tensor ,

g_{ij} is the fundamental tensor of the space-time

and $T = T_i^i = g^{ij} T_{ij}$.

We study $[\frac{1}{\sqrt{2}}(y+z)-t]$-type and $[t\sqrt{2}/(y+z)]$-type plane wave solutions separately.

Case I $[\frac{1}{\sqrt{2}}(y+z)-t]$-type plane wave solutions in V_5

§ 2. Zero Mass scalar field coupled with gravitational field

The energy momentum tensor of zero mass scalar field is given by

$$T_{ij} = \frac{1}{4\pi}[V_i V_j - \frac{1}{2} g_{ij} V_k V^k], \qquad [k = 1,2,3,4,5] \tag{2.1}$$

where V is scalar function of Z and $V_j = \frac{\partial V}{\partial x^j}$,

$(x^j = x^1, x^2, x^3, x^4, x^5$ i.e. $x^j = u, x, y, z, t)$.

Thus $$\sqrt{2}V_3 = \sqrt{2}V_4 = -V_5 = \overline{V} \qquad \because V_1 = V_2 = 0 \tag{2.2}$$

where (-) bar denotes partial derivative w. r. to Z

From line element (1.7) we have

$$g_{33} = -B, \qquad g_{44} = -B, \qquad g_{55} = B, \tag{2.3}$$

$$g^{33} = -\frac{1}{B}, \qquad g^{44} = -\frac{1}{B}, \qquad g^{55} = \frac{1}{B} \tag{2.4}$$

$$\Rightarrow V_k V^k = 0 \ . \tag{2.5}$$

Therefore equation (2.1) becomes

$$T_{ij} = \frac{1}{4\pi}[V_i V_j] \ . \tag{2.6}$$

And from equation (2.6), in zero mass scalar field

$$T = T_i^i = g^{ij} T_{ij} = 0 \tag{2.7}$$

Then using equations (2.6) and (2.7), Einstein's field equation (1.11) becomes

$$R_{ij} = -2[V_i V_j] \tag{2.8}$$

which further gives

$$R_{33} = -\overline{V}_{,}^{2} \quad R_{44} = -\overline{V}_{,}^{2} \quad R_{55} = -2\overline{V}_{,}^{2} \quad R_{34} = -\overline{V}_{,}^{2}$$

$$R_{35} = \sqrt{2}\,\overline{V}_{,}^{2} \quad R_{45} = \sqrt{2}\overline{V}^{2}. \tag{2.9}$$

Thus non-vanishing components of Ricci tensor are related as

$$2R_{33} = 2R_{44} = R_{55} = 2R_{34} = -\sqrt{2}R_{35} = -\sqrt{2}R_{45}. \tag{2.10}$$

It is observed that the equation (2.10) is compatible with the equation (1.9) which is obtained in the case of purely gravitational field. Hence we have

Conclusion $[\frac{1}{\sqrt{2}}(y+z)-t]$ *-type plane wave solutions exist in the case where zero mass scalar field is coupled with the gravitational field.*

§ 3. Zero Mass scalar field coupled with gravitational & electromagnetic field

In this case the energy momentum tensor is given by

$$T_{ij} = \frac{1}{4\pi}[V_i V_j - \frac{1}{2} g_{ij} V_k V^k] + E_{ij} \tag{3.1}$$

where E_{ij} denotes electromagnetic energy momentum tensor.
But as in the previous case for zero mass scalar field

$$V_k V^k = 0 \text{ and } T = T_i^i = g^{ij} T_{ij} = 0. \tag{3.2}$$

Therefore equation (3.1) becomes

$$T_{ij} = \frac{1}{4\pi}[V_i V_j] + E_{ij}. \tag{3.3}$$

Hence Einstein's field equation (1.11) becomes

$$R_{ij} = -2[V_i V_j + 4\pi E_{ij}]. \tag{3.4}$$

Then equation (3.4) yields

$$R_{33}=-2[\frac{\overline{V}^2}{2}+4\pi E_{33}],\quad R_{44}=-2[\frac{\overline{V}^2}{2}+4\pi E_{44}],\quad R_{55}=-2[\overline{V}^2+4\pi E_{55}],$$

$$R_{34}=-2[\frac{\overline{V}^2}{2}+4\pi E_{34}],\quad R_{35}=-2[-\frac{\overline{V}^2}{\sqrt{2}}+4\pi E_{35}],\quad R_{45}=-2[-\frac{\overline{V}^2}{\sqrt{2}}+4\pi E_{45}]. \quad (3.5)$$

All above non-vanishing components of Ricci tensor are related as

$$2R_{33}=2R_{44}=R_{55}=2R_{34}=-\sqrt{2}R_{35}=-\sqrt{2}R_{45}. \quad (3.6)$$

It is observed that the equation (3.6) is compatible with equation (1.9) which is obtained in the case of purely gravitational field. Hence we have

Conclusion $[\frac{1}{\sqrt{2}}(y+z)-t]$*-type type plane wave solutions exists in the case where zero mass scalar field is coupled with gravitational & electromagnetic field.*

§ 4. Massive scalar field coupled with gravitational field

The energy momentum tensor of massive scalar field is given by

$$T_{ij}=\frac{1}{4\pi}[V_iV_j-\frac{1}{2}g_{ij}(V_kV^k-m^2V^2)],\ [k=1,2,3,4,5] \quad (4.1)$$

where V is scalar function of Z and $V_j=\frac{\partial V}{\partial x^j}$, $\quad (x^j=u,x,y,z,t)$,

m is mass associated with the massive scalar field.

Thus $V_3\sqrt{2}=V_4\sqrt{2}=-V_5=\overline{V} \qquad \because V_1=V_2=0 \quad (4.2)$

where (-) bar denotes partial derivative w. r. to *Z*.
From line element (1.7) we have

$$g_{33}=-B,\qquad g_{44}=-B\qquad g_{55}=B \quad (4.3)$$

$$g^{33}=-\frac{1}{B},\qquad g^{44}=-\frac{1}{B},\qquad g^{55}=\frac{1}{B} \quad (4.4)$$

$$\Rightarrow V_kV^k=0. \quad (4.5)$$

Therefore equation (4.1) implies

$$T_{ij}=\frac{1}{4\pi}[V_iV_j+\frac{1}{2}g_{ij}m^2V^2]. \quad (4.6)$$

Equation (4.6) yields

$$T = T_i^i = g^{ij}T_{ij} = \frac{1}{4\pi}[\frac{5}{2}m^2V^2]. \tag{4.7}$$

Using (4.6) and (4.7), Einstein's field equation (1.11) becomes

$$R_{ij} = -2[V_iV_j + (\frac{-3}{4})g_{ij}m^2V^2] \tag{4.8}$$

which further yields

$$R_{33} = -[\overline{V}^2 + \frac{3}{2}Bm^2V^2], \quad R_{44} = -[\overline{V}^2 + \frac{3}{2}Bm^2V^2], \quad R_{55} = -[2\overline{V}^2 - \frac{3}{2}Bm^2V^2],$$
$$R_{34} = -\overline{V}^2, \quad R_{35} = \overline{V}^2\sqrt{2}, \quad R_{45} = \overline{V}^2\sqrt{2}. \tag{4.9}$$

But from the line element (1.7) we have the relation of non-vanishing components of Ricci tensor as

$$2R_{33} = 2R_{44} = R_{55} = 2R_{34} = -\sqrt{2}R_{35} = -\sqrt{2}R_{45}. \tag{4.10}$$

It is to be noted that here the equation (4.9) is incompatible with the equation (4.10) which is obtained in the case of purely gravitational field. Hence we have

Conclusion *$[\frac{1}{\sqrt{2}}(y+z)-t]$-type plane wave solutions doesn't exist in the case where massive scalar field is coupled with gravitational field.*

Remark If $m^2 = 0$ then the equation (4.9) is compatible to (4.10) and we have a result as in the case where the zero mass scalar field is coupled with the gravitational field.

§ 5. Massive scalar field coupled with gravitational & electromagnetic field

In this case the energy momentum tensor is given by

$$T_{ij} = \frac{1}{4\pi}[V_iV_j - \frac{1}{2}g_{ij}(V_kV^k - m^2V^2)] + E_{ij} \tag{5.1}$$

where E_{ij} denotes electromagnetic energy momentum tensor.

But as in the previous case for massive scalar field

$$V_kV^k = 0 \quad \text{and} \quad T = T_i^i = g^{ij}T_{ij} = \frac{1}{4\pi}[\frac{5}{2}m^2V^2]. \tag{5.2}$$

Therefore equation (5.1) becomes

$$T_{ij} = \frac{1}{4\pi}[V_iV_j + \frac{1}{2}g_{ij}m^2V^2] + E_{ij}. \tag{5.3}$$

Hence Einstein's field equation (1.11) becomes

$$R_{ij} = -2[V_i V_j - \frac{3}{4} g_{ij} m^2 V^2 + 4\pi E_{ij}]. \qquad (5.4)$$

Then from equation (5.4) we have

$$R_{33} = -[\overline{V}^2 + \frac{3}{2} B m^2 V^2 + 8\pi E_{33}], \qquad R_{44} = -[\overline{V}^2 + \frac{3}{2} B m^2 V^2 + 8\pi E_{44}],$$
$$R_{55} = -[2\overline{V}^2 - \frac{3}{2} B m^2 V^2 + 8\pi E_{55}], \qquad R_{34} = [\overline{V}^2 + 8\pi E_{34}],$$
$$R_{35} = -[-\sqrt{2}\overline{V}^2 + 8\pi E_{35}], \qquad R_{45} = -[-\sqrt{2}\overline{V}^2 + 8\pi E_{45}]. \qquad (5.5)$$

But line element (1.7) gives the relation of non-vanishing components of Ricci tensor as

$$2R_{33} = 2R_{44} = R_{55} = 2R_{34} = -R_{35}\sqrt{2} = -R_{45}\sqrt{2}\,. \qquad (5.6)$$

Here it has been observed that the equation (5.5) is incompatible with the equation (5.6) which is obtained in the case of purely gravitational field. Hence we have

Conclusion *$[\frac{1}{\sqrt{2}}(y+z)-t]$-type plane wave solutions of Einstein's field equation in general relativity doesn't exist in the case where massive scalar field is coupled with gravitational & electromagnetic field.*

Remark If $m^2 = 0$ then the equation (5.5) is compatible to (5.6) and we have a result as in the case where the zero mass scalar field is coupled with the gravitational & electromagnetic field .

Case II we consider the case of $[t\sqrt{2}/(y+z)]$-type plane wave solutions.

§ 6. Zero mass scalar field coupled with gravitational field

The energy momentum tensor of zero mass scalar field is given by

$$T_{ij} = \frac{1}{4\pi}[V_i V_j - \frac{1}{2} g_{ij} V_k V^k], \qquad [k = 1,2,3,4,5] \qquad (6.1)$$

where V is scalar function of Z and $V_j = \frac{\partial V}{\partial x^j}$, $(x^j = u, x, y, z, t)$.

Thus $V_1 = V_2 = 0$ and $V_3 = V_4 = -V_5 \frac{Z}{\sqrt{2}} = -\frac{\overline{V} Z}{(y+z)}$ (6.2)

here (-) bar denotes partial derivative w. r. to Z

From line element (1.8) we have

$$g_{33} = -Z^2 B \quad , \quad g_{44} = -Z^2 B \quad , \quad g_{55} = B \ , \tag{6.3}$$

$$g^{33} = -\frac{1}{Z^2 B}, \quad g^{44} = -\frac{1}{Z^2 B}, \quad g^{55} = \frac{1}{B} \tag{6.4}$$

$$\Rightarrow V_k V^k = 0 . \tag{6.5}$$

Therefore equation (6.1)

$$\Rightarrow T_{ij} = \frac{1}{4\pi}[V_i V_j] \tag{6.6}$$

Equation (6.6) yields

$$T = T_i^i = g^{ij} T_{ij} = 0 . \tag{6.7}$$

Then from (6.6) and (6.7), Einstein's field equation (1.11) becomes

$$R_{ij} = -2[V_i V_j] \tag{6.8}$$

which further gives

$$R_{33} = -2[\frac{\bar{V}^2 Z^2}{(y+z)^2}], \quad R_{44} = -2[\frac{\bar{V}^2 Z^2}{(y+z)^2}], \quad R_{55} = -2[\frac{2\bar{V}^2}{(y+z)^2}]$$

$$R_{34} = -2[\frac{\bar{V}^2 Z^2}{(y+z)^2}], \quad R_{35} = 2\sqrt{2}[\frac{\bar{V}^2 Z}{(y+z)^2}], \quad R_{45} = 2\sqrt{2}[\frac{\bar{V}^2 Z}{(y+z)^2}]. \tag{6.9}$$

Thus non-vanishing components of Ricci tensor are related as

$$\frac{(y+z)^2}{Z^2} R_{33} = \frac{(y+z)^2}{Z^2} R_{44} = \frac{(y+z)^2}{2} R_{55} = \frac{(y+z)^2}{Z^2} R_{34} = -\frac{(y+z)^2}{Z\sqrt{2}} R_{35} = -\frac{(y+z)^2}{Z\sqrt{2}} R_{45} . \tag{6.10}$$

It is observed that the equation (6.10) is compatible with equation (1.10) which is obtained in the case of purely gravitational field. Hence we have

Conclusion $[t\sqrt{2}/(y+z)]$-type plane *wave solutions exist in the case where zero mass scalar field is coupled with the gravitational field.*

§ 7. Zero Mass scalar field coupled with gravitational & electromagnetic field

In this case the energy momentum tensor is given by

$$T_{ij} = \frac{1}{4\pi}[V_i V_j - \frac{1}{2} g_{ij} V_k V^k] + E_{ij} \tag{7.1}$$

where E_{ij} denotes electromagnetic energy momentum tensor.

But as in the previous case for zero mass scalar field

$$V_k V^k = 0 \quad \text{and} \quad T = T_i^i = g^{ij} T_{ij} = 0\,. \tag{7.2}$$

Therefore equation (7.1) becomes

$$T_{ij} = \frac{1}{4\pi}[V_i V_j] + E_{ij}\,. \tag{7.3}$$

Hence Einstein's field equation (1.11) becomes

$$R_{ij} = -2[V_i V_j + 4\pi E_{ij}] \tag{7.4}$$

which further yields

$$R_{33} = -2[\frac{\overline{V}^2 Z^2}{(y+z)^2} + 4\pi E_{33}], \quad R_{44} = -2[\frac{\overline{V}^2 Z^2}{(y+z)^2} + 4\pi E_{44}], \quad R_{55} = -2[\frac{2\overline{V}^2}{(y+z)^2} + 4\pi E_{55}],$$

$$R_{34} = -2[\frac{\overline{V}^2 Z^2}{(y+z)^2} + 4\pi E_{34}], \quad R_{35} = -2[\frac{-\overline{V}^2 Z\sqrt{2}}{(y+z)^2} + 4\pi E_{35}],$$

$$R_{45} = -2[\frac{-\overline{V}^2 Z\sqrt{2}}{(y+z)^2} + 4\pi E_{45}]. \tag{7.5}$$

All above non-vanishing components of Ricci tensor are related as

$$\frac{(y+z)^2}{Z^2} R_{33} = \frac{(y+z)^2}{Z^2} R_{44} = \frac{(y+z)^2}{2} R_{55} = \frac{(y+z)^2}{Z^2} R_{34} = -\frac{(y+z)^2}{Z\sqrt{2}} R_{35} = -\frac{(y+z)^2}{Z\sqrt{2}} R_{45}\,. \tag{7.6}$$

It is observed that the equation (7.6) is compatible with equation (1.10) which is obtained in the case of purely gravitational field. Hence we have

Conclusion $[t\sqrt{2}/(y+z)]$-type plane *wave solutions exist in the case where zero mass scalar field is coupled with gravitational & electromagnetic field.*

§ 8. Massive scalar field coupled with gravitational field

The energy momentum tensor of massive scalar field is given by

$$T_{ij} = \frac{1}{4\pi}[V_i V_j - \frac{1}{2} g_{ij}(V_k V^k - m^2 V^2)], \quad [k = 1,2,3,4,5] \tag{8.1}$$

where V is scalar function of Z and $V_j = \frac{\partial V}{\partial x^j}$, $(x^j = u, x, y, z, t)$,

m is mass associated with the massive scalar field.

Thus $V_3 = V_4 = -V_5 \frac{Z}{\sqrt{2}} = -\frac{\bar{V} Z}{(y+z)}, \quad \because V_1 = V_2 = 0$ (8.2)

where (-) bar denotes partial derivative w. r. to *Z*.

From line element (1.8) we have

$$g_{33} = -Z^2 B \quad , \quad g_{44} = -Z^2 B \quad , \quad g_{55} = B \ , \tag{8.3}$$

$$g^{33} = -\frac{1}{Z^2 B}, \quad g^{44} = -\frac{1}{Z^2 B}, \quad g^{55} = \frac{1}{B} \tag{8.4}$$

$$\Rightarrow V_k V^k = 0 . \tag{8.5}$$

Therefore equation (8.1) becomes

$$T_{ij} = \frac{1}{4\pi}[V_i V_j + \frac{1}{2} g_{ij} m^2 V^2] . \tag{8.6}$$

From equation (8.6) we have

$$T = T_i^i = g^{ij} T_{ij} = \frac{1}{4\pi}[\frac{5}{2} m^2 V^2] . \tag{8.7}$$

Then from (8.6) and (8.7), Einstein's field equation (1.11) becomes

$$R_{ij} = -2[V_i V_j - \frac{3}{4} g_{ij} m^2 V^2] \tag{8.8}$$

which further yields

$$R_{33} = -2[\frac{\overline{V}^2 Z^2}{(y+z)^2} + \frac{3}{4} Z^2 B m^2 V^2], \qquad R_{44} = -2[\frac{\overline{V}^2 Z^2}{(y+z)^2} + \frac{Z^2 B m^2 V^2}{2}],$$

$$R_{55} = -2[\frac{2\overline{V}^2}{(y+z)^2} - \frac{3}{4} B m^2 V^2], \qquad R_{34} = -2[\frac{\overline{V}^2 Z^2}{(y+z)^2}],$$

$$R_{35} = -2[\frac{-\overline{V}^2 Z\sqrt{2}}{(y+z)^2}], \qquad R_{45} = -2[\frac{-\overline{V}^2 Z\sqrt{2}}{(y+z)^2}] \,. \tag{8.9}$$

But from the line element (1.8) we have the relation of non-vanishing components of Ricci tensor as

$$\frac{(y+z)^2}{Z^2} R_{33} = \frac{(y+z)^2}{Z^2} R_{44} = \frac{(y+z)^2}{2} R_{55} = \frac{(y+z)^2}{Z^2} R_{34}$$
$$= \frac{-(y+z)^2}{Z\sqrt{2}} R_{35} = \frac{-(y+z)^2}{Z\sqrt{2}} R_{45} \,. \tag{8.10}$$

Here we observed that the equation (8.9) is incompatible with the equation (8.10) which is obtained in the case of purely gravitational field. Hence we have

Conclusion $[t\sqrt{2}/(y+z)]$-type plane *wave solutions of Einstein's field equation in general theory of relativity doesn't exist in the case where massive scalar field is coupled with the gravitational field.*

Remark If $m^2 = 0$ then the equation (8.9) is compatible to (8.10) and we have a result as in the case where the zero mass scalar field is coupled with the gravitational field.

§ 9. Massive scalar field coupled with gravitational & electromagnetic field

In this case the energy momentum tensor is given by

$$T_{ij} = \frac{1}{4\pi}[V_i V_j - \frac{1}{2} g_{ij}(V_k V^k - m^2 V^2)] + E_{ij} \tag{9.1}$$

where E_{ij} denotes electromagnetic energy momentum tensor.

But as in the previous case for massive scalar field

$$V_k V^k = 0 \quad \text{and} \quad T = T_i^i = g^{ij} T_{ij} = \frac{1}{4\pi}[\frac{5}{2} m^2 V^2]. \tag{9.2}$$

Therefore equation (9.1) becomes

$$T_{ij} = \frac{1}{4\pi}[V_i V_j + \frac{1}{2} g_{ij} m^2 V^2] + E_{ij} \,. \tag{9.3}$$

Hence Einstein's field equation (1.11) becomes

$$R_{ij} = -2[V_i V_j - \frac{3}{4} g_{ij} m^2 V^2 + 4\pi E_{ij}]. \tag{9.4}$$

And from equation (9.4) we have

$$R_{33} = -2[\frac{\bar{V}^2 Z^2}{(y+z)^2} + \frac{3}{4} Z^2 Bm^2 V^2 + 8\pi E_{33}], \quad R_{44} = -2[\frac{\bar{V}^2 Z^2}{(y+z)^2} + \frac{3}{4} Z^2 Bm^2 V^2 + 8\pi E_{44}],$$

$$R_{55} = -2[\frac{2\bar{V}^2}{(y+z)^2} - \frac{3}{4} Bm^2 V^2 + 8\pi E_{55}], \qquad R_{34} = -2[\frac{\bar{V}^2 Z^2}{(y+z)^2} + 8\pi E_{34}],$$

$$R_{35} = -2[\frac{-\bar{V}^2 Z\sqrt{2}}{(y+z)^2} + 8\pi E_{35}], \qquad R_{45} = -2[-\frac{\bar{V}^2 Z\sqrt{2}}{(y+z)^2} + 8\pi E_{45}]. \tag{9.5}$$

But line element (1.8) gives the relation of non-vanishing components of Ricci tensor as

$$\frac{(y+z)^2}{Z^2} R_{33} = \frac{(y+z)^2}{Z^2} R_{44} = \frac{(y+z)^2}{2} R_{55} = \frac{(y+z)^2}{Z^2} R_{34}$$
$$= \frac{-(y+z)^2}{Z\sqrt{2}} R_{35} = \frac{-(y+z)^2}{Z\sqrt{2}} R_{45}. \tag{9.6}$$

Here we observed that the equation (9.5) is incompatible with the equation (9.6) which is obtained in the case of purely gravitational field. Hence we have

Conclusion $[t\sqrt{2}/(y+z)]$-type plane *wave solutions of Einstein's field equation doesn't exist in the case where massive scalar field is coupled with gravitational & electromagnetic field.*

Remark If $m^2 = 0$ then the equation (9.5) is compatible to (9.6) and we have a result as in the case where the zero mass scalar field is coupled with the gravitational & electromagnetic field.

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References

[1] Ladke L S and Thengane K D (2002) : " Plane wave solutions of field equations $R_{ij} = 0$ in five dimensional space-time." *Proceeding of Einstein Foundation International Vol.*12,2002, *India.*

[2] Jumale J K and Thengane K D (2004) : " $[\frac{1}{\sqrt{2}}(y+z)-t]$-type and $[t\sqrt{2}/(y+z)]$-type plane waves in five dimensional space-time" *INSA New Delhi, India.*

[4] H Takeno(1961) : "The Mathematical theory of plane gravitational wave in general relativity". *Scientific report of the Research Institute to theoretical physics. Hiroshima, University,* Takchara, Hiroshima-ken, Japan-1961.