Five Dimensional Bianchi Type-I (Kasner form) Cosmological Model in $f(R)$ Theory of Gravitation

L.S.Ladke
Department of Mathematics, Sarvodaya Mahavidyalaya, Sindewahi (441222), India

Abstract

A five dimensional Bianchi type-I (Kasner form) cosmological model in *f*(*R*) theory of gravity has been studied. The general solution of the field equations of Bianchi type-I in Kasner form with five dimensions have been obtained by using special form of deceleration parameter. The physical aspects of the model are also discussed.
KEYWORDS: $f(R)$ theory of gravity, Bianchi type-I (Kasner form) in five dimensions, Special form of deceleration parameter.

1. Introduction:

The $f(R)$ theory of gravity provides a very natural unification of the early-time inflation and late-time acceleration as proved by Nojiri and Odintsov (2007,2008). Sharif & Shamir (2010) have been obtained non-vacuum solutions in Bianchi type-I & type-V using perfect fluid in $f(R)$ gravity. Bianchi type-III space time with anisotropic fluid in $f(R)$ gravity have been studied by Sharif & Kausar (2011). Aktas *et al.* (2012) have studied anisotropic models in $f(R)$ theory of gravity. Adhav (2012) discussed the Kantowski–Sachs string cosmological model in $f(R)$ gravity. Singh *et al.* (2013) studied functional form of $f(R)$ with power-law expansion in anisotropic model. Recently, Reddy *et al.* (2014) studied vacuum solution of Bianchi type-I and V models in $f(R)$ gravity with a special form of deceleration parameter.

Higher dimensional cosmological models play a vital role in many aspects of early stage of cosmological problems. The study of higher dimensional space-time provides an idea that our universe is much smaller at early stage of evolution as observed today. There is nothing in the equation of relativity which restricts them to four dimensions. Many researchers inspired to enter in to the field of higher dimension theory to explore the knowledge of the universe. Wesson (1983, 1984) and D R K Reddy (1999) have studied several aspects of five dimensional space-time in variable mass theory and bimetric theory of relativity. Lorentz and Petzold (1985), Ibanez and Verdaguer (1986), Reddy and Venkateswara (2001), Khadekar and Gaikwad (2001). Adhav *et. al.* (2007) have studied the multi dimensional cosmological model in general relativity and in other alternative theories of gravitation. Jaiswal *et al.* (2012) have studied the exact vacuum solutions of five-dimensional Bianchi type-I space-time in *f(R)*theory of gravity

Motivating with the above research work in $f(R)$ theory of gravity, Bianchi type-I (Kasner form) cosmological model with five dimensions is considered. The general solution of the field equations in Bianchi type-I space-time in Kasner form with five dimensions have been obtained using special form of deceleration parameter. The physical aspects of the model are also discussed.

2. $f(R)$ Theory of Gravity and Deceleration Parameter:

The $f(R)$ theory of gravity is the modification of general theory of relativity
The field equations of $f(R)$ theory of gravity in five dimensions are given by

$$F(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - \nabla_i \nabla_j \, F(R) + g_{ij} \Box F(R) = \kappa T_{ij}\,, \tag{2.1}$$

where $F(R) = {}^{df(R)}\!/_{dR}$, $\quad \Box \equiv \nabla^i \nabla_i$, (2.2)

with ∇_i the covariant derivative, T_{ij} is the standard matter energy-momentum tensor. These are the fourth order partial differential equations in the metric tensor. The fourth order is due to the last two terms on the left hand side of the equation. If we consider $f(R) \equiv R$, these equations of $f(R)$ theory of gravity reduce to the field equations of general relativity.
Now contraction of the field equations (2.1), we get

$$F(R)R - \frac{5}{2} f(R) + 4 \Box F(R) = \kappa T \tag{2.3}$$

In vacuum this field equation (4) reduced to

$$F(R)R - \frac{5}{2} f(R) + 4 \Box F(R) = 0\,, \tag{2.4}$$

From equation (2.4), we get (2.5)

$$f(R) = \frac{2}{5} F(R)R + \frac{8}{5} \Box F(R).$$

This gives an important relationship between $f(R)$ and $F(R)$.
Inserting this value of $f(R)$ in the vacuum field equations (2.1), we have

$$\frac{1}{5}\left[F(R)R - \Box F(R)\right] = \frac{F(R)R_{ij} - \nabla_i \nabla_j F(R)}{g_{ij}}. \tag{2.6}$$

Since the left side does not depend on the index i , so the field equation can be expressed as

$$K_i \equiv \frac{F(R)R_{ii} - \nabla_i \nabla_i F(R)}{g_{ii}} \tag{2.7}$$

is independent of the index i and hence $K_i - K_j = 0$ for all i and j.

Here K_i is just a notation for the traced quantity.

Many authors assume various physical or mathematical assumptions to obtain exact solution of the modified Einstein's field equations in $f(R)$ gravity. Some authors used condition on deceleration parameter. Berman in 1983 proposed a law of variation for Hubble parameter which yields constant deceleration parameter models of the universe. Akarsu and Kilinc (2010), Yadav *et al*. (2011) and Adhav (2011a, 2011b) have extended this law for Bianchi type-I, Bianchi type-V anisotropic cosmological models respectively. Cosmological models of the universe are either decelerating or accelerating. For a universe which was decelerating in the past and accelerating at present time, decelerating parameter must shows singnature flipping (Riess *et al*. 2001). Cunha & Lima (2008) favours recent acceleration and past deceleration by analyzing three SNe type Ia samples. In order to match this observation Singh and Debnath (2009) has defined a special form of deceleration parameter for FRW metric as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^{\alpha}}, \tag{2.8}$$

where $\alpha > 0$ is a constant and a is mean scale factor of the universe.
After solving equation (9) one can obtain the mean Hubble parameter H as

$$H = \frac{\dot{a}}{a} = k\left(1 + \frac{1}{a^{\alpha}}\right), \tag{2.9}$$

where k is a constant of integration.
Integrating equation (2.9), we obtain the mean scale factor as

$$a = (e^{k\alpha t} - 1)^{1/\alpha}. \tag{2.10}$$

The physical parameters that are of cosmological importance are

The mean anisotropy parameter: $$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2. \tag{2.11}$$

The shear scalar: $$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - 4H^2\right). \tag{2.12}$$

The expansion scalar : $$\theta = 4H. \tag{2.13}$$

3. Bianchi type-I (Kasner form) Cosmological Model:

The Bianchi type-I (Kasner form) metric in five dimension is

$$ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2 - t^{2p_4}du^2, \tag{3.1}$$

where p_1, p_2, p_3 and p_4 are constants.
The corresponding Ricci scalar becomes

$$R = -\left[S^2 - 2S + \theta\right]t^{-2}, \tag{3.2}$$

where $S = p_1 + p_2 + p_3 + p_4$ and $\phi = p_1^2 + p_2^2 + p_3^2 + p_4^2$.
Now we can obtain the corresponding field equations for the metric (3.1) with the help of equation (2.7) as

$$\left[(S-\phi) + p_1(S-1)\right]t^{-2} + \frac{p_1}{t}\frac{\dot{F}}{F} - \frac{\ddot{F}}{F} = 0, \tag{3.3}$$

$$\left[(S-\phi) + p_2(S-1)\right]t^{-2} + \frac{p_2}{t}\frac{\dot{F}}{F} - \frac{\ddot{F}}{F} = 0, \tag{3.4}$$

$$\left[(S-\phi)+p_3(S-1)\right]t^{-2}+\frac{p_3}{t}\frac{\dot{F}}{F}-\frac{\ddot{F}}{F}=0 \quad , \tag{3.5}$$

$$\left[(S-\phi)+p_4(S-1)\right]t^{-2}+\frac{p_4}{t}\frac{\dot{F}}{F}-\frac{\ddot{F}}{F}=0 \quad , \tag{3.6}$$

Here dot (˙) denotes derivative with respect to time t.
The field equations (3.3)-(3.6) are a system of four non-linear differential equations with five unknowns p_1, p_2 , p_3, p_4 and F. In order to solve the system completely we impose a special form of deceleration parameter as given in equation (2.8) and then the corresponding mean Hubble parameter H and mean scale factor a is given in equations (2.9) and (2.10) respectively.
We define the spatial volume V for Bianchi type-I universe as

$$V=a^4=t^S . \tag{3.7}$$

The mean Hubble parameter H for Bianchi type-I universe is defined as

$$H=\frac{\dot{a}}{a}=\frac{1}{4}\left(H_1+H_2+H_3+H_4\right) , \tag{3.8}$$

where $H_1=\frac{(t^{p_1})^\bullet}{t^{p_1}}$, $H_2=\frac{(t^{p_2})^\bullet}{t^{p_2}}$, $H_3=\frac{(t^{p_3})^\bullet}{t^{p_3}}$, $H_4=\frac{(t^{p_4})^\bullet}{t^{p_4}}$ are the directional Hubble parameters in the directions of x , y , z and u respectively.
Subtracting Equations (3.4) from (3.3), (3.5) from (3.4), (3.6) from (3.5) and (3.6) from (3.3) respectively, we get

$$\left[(p_1-p_2)(S-1)\right]t^{-2}+\frac{\dot{F}}{F}\frac{(p_1-p_2)}{t}=0 \quad , \tag{3.9}$$

$$\left[(p_2-p_3)(S-1)\right]t^{-2}+\frac{\dot{F}}{F}\frac{(p_2-p_3)}{t}=0 \quad , \tag{3.10}$$

$$\left[(p_3-p_4)(S-1)\right]t^{-2}+\frac{\dot{F}}{F}\frac{(p_3-p_4)}{t}=0 \quad , \tag{3.11}$$

$$\left[(p_1-p_4)(S-1)\right]t^{-2}+\frac{\dot{F}}{F}\frac{(p_1-p_4)}{t}=0 \quad , \tag{3.12}$$

These equation imply that

$$t^{p_1}=t^{p_2}n_1\exp\left(m_1\int\frac{dt}{t^S F}\right) \quad , \tag{3.13}$$

$$t^{p_2}=t^{p_3}n_2\exp\left(m_2\int\frac{dt}{t^S F}\right) \quad , \tag{3.14}$$

$$t^{p_3} = t^{p_4} n_3 \exp\left(m_3 \int \frac{dt}{t^S F} \right) \quad , \tag{3.15}$$

$$t^{p_1} = t^{p_4} n_4 \exp\left(m_4 \int \frac{dt}{t^S F} \right) \quad , \tag{3.16}$$

where m_1 , m_2 , m_3, m_4 and n_1 , n_2 , n_3, n_4 are constants of integration which satisfy the relation

$$m_1 + m_2 + m_3 + m_4 = 0 \quad , \quad n_1 n_2 n_3 n_4 = 1. \tag{3.17}$$

Using Equations (3.13)-(3.16) we can write the metric functions explicitly as

$$t^{p_1} = t^{S/4} r_1 \exp\left(q_1 \int \frac{dt}{t^S F} \right) , \tag{3.18}$$

$$t^{p_2} = t^{S/4} r_2 \exp\left(q_2 \int \frac{dt}{t^S F} \right) , \tag{3.19}$$

$$t^{p_3} = t^{S/4} r_3 \exp\left(q_3 \int \frac{dt}{t^S F} \right) , \tag{3.20}$$

$$t^{p_4} = t^{S/4} r_4 \exp\left(q_4 \int \frac{dt}{t^S F} \right) , \tag{3.21}$$

where $r_1 = \left(n_1^2 n_2 n_4\right)^{\frac{1}{4}}$, $r_2 = \left(n_1^{-2} n_2 n_4\right)^{\frac{1}{4}}$, $r_3 = \left(n_1^{-2} n_2^{-3} n_4\right)^{\frac{1}{4}}$, $r_4 = \left(n_1^2 n_4^{-3} n_2\right)^{\frac{1}{4}}$ (3.22)

and

$$q_1 = \frac{2m_1 + m_2 + m_4}{4} , \quad q_2 = \frac{m_2 + m_4 - 2m_1}{4} , \quad q_3 = \frac{m_4 - 2m_1 - 3m_2}{4} , \quad q_4 = \frac{2m_1 + m_2 - 3m_4}{4} . \tag{3.23}$$

Note that r_1 , r_2 , r_3 , r_4 and q_1 , q_2 , q_3 , q_4 also satisfy the relation

$$r_1 r_2 r_3 r_4 = 1 \text{ and } q_1 + q_2 + q_3 + q_4 = 0 . \tag{3.24}$$

Now, we use the power-law to solve the integral part in the above equations.

$$F \propto a^m ,$$

$$F = h a^m , \tag{3.25}$$

where h is the constant of proportionality and m is any integer [here $m = -2$].

Using equations (2.10) and (3.25) for $k = 1, \alpha = 2$ and $m = -2$ and noting equation (3.7), equations (3.18), (3.19), (3.20) and (3.21) gives scale factors as

$$t^{p_1} = r_1(e^{2t}-1)^{\frac{1}{2}} \exp\left[\frac{q_1}{h}\log\left(\frac{e^{2t}-1}{e^{2t}}\right)^{\frac{1}{2}}\right], \tag{3.26}$$

$$t^{p_2} = r_2(e^{2t}-1)^{\frac{1}{2}} \exp\left[\frac{q_2}{h}\log\left(\frac{e^{2t}-1}{e^{2t}}\right)^{\frac{1}{2}}\right], \tag{3.27}$$

$$t^{p_3} = r_3(e^{2t}-1)^{\frac{1}{2}} \exp\left[\frac{q_3}{h}\log\left(\frac{e^{2t}-1}{e^{2t}}\right)^{\frac{1}{2}}\right], \tag{3.28}$$

$$t^{p_4} = r_4(e^{2t}-1)^{\frac{1}{2}} \exp\left[\frac{q_4}{h}\log\left(\frac{e^{2t}-1}{e^{2t}}\right)^{\frac{1}{2}}\right], \tag{3.29}$$

Using equations (3.26)-(3.29) , the directional Hubble parameters in the directions of x, y, z and u -axes are

$$H_1 = \frac{q_1}{h\left(e^{2t}-1\right)} + \frac{e^{2t}}{(e^{2t}-1)}, \qquad H_2 = \frac{q_2}{h\left(e^{2t}-1\right)} + \frac{e^{2t}}{(e^{2t}-1)}$$

$$H_3 = \frac{q_3}{h\left(e^{2t}-1\right)} + \frac{e^{2t}}{(e^{2t}-1)} \qquad \text{and} \qquad H_4 = \frac{q_4}{h\left(e^{2t}-1\right)} + \frac{e^{2t}}{(e^{2t}-1)}. \tag{3.30}$$

The Mean Hubble parameter is given by

$$H = \frac{e^{2t}}{(e^{2t}-1)}. \tag{3.31}$$

The volume of the universe is given by

$$V = \left(e^{2t}-1\right)^2. \tag{3.32}$$

The expansion scalar $\theta = 4H$ is given by

$$\theta = \frac{4e^{2t}}{(e^{2t}-1)}. \tag{3.33}$$

The mean anisotropy parameter Δ is given by

$$\Delta = \frac{\left(q_1^2+q_2^2+q_3^2+q_4^2\right)}{4h^2e^{4t}}. \tag{3.34}$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{\left(q_1^2+q_2^2+q_3^2+q_4^2\right)}{2h^2}\frac{1}{(e^{2t}-1)^2}. \tag{3.35}$$

The deceleration parameter q is given by

$$q=\frac{2}{(e^{2t}-1)}-1. \tag{3.36}$$

Using equation (2.5), we obtain the function of Ricci scalar $f(R)$ as

$$f(R)=\frac{2}{5(e^{2t}-1)^3}\left\{16he^{2t}(1-e^{2t})+h\,(e^{2t}-1)^2R\right\}. \tag{3.37}$$

The Ricci scalar R is

$$R=\frac{1}{(e^{2t}-1)^2}\left\{\frac{-1}{h^2}\left(q_1^2+q_2^2+q_3^2+q_4^2+q_1q_2+q_2q_3+q_3q_4+q_1q_4\right)-8e^{2t}(e^{2t}-1)\right\}. \tag{3.38}$$

4. Conclusion:

i) Exact solution for Bianchi Type-I (Kasner form) cosmological model with five dimensions in $f(R)$ theory of gravity is obtained.

ii) From equation (3.32), it is observed that, the spatial volume V is vanishes at $t=0$. It expands exponentially as t increases and becomes infinitely large as $t\to\infty$.

iii) From equation (3.33), it is observed that, the expansion scalar θ start with infinite value at $t=0$ and then rapidly becomes constant after some finite time.

iv) All the results obtained here are similar to the results obtained by Reddy *et.al.*(2014).

References:

Adhav. K.S. *et al*.: Astrophysics and Space Science, **310**(3-4), 231, (2007).

Adhav. K.S. *et al*.: Astrophysics and Space Science, **332**(2), 497(2011a).

Adhav. K.S. *et al*.: Cent. Eur. J. Phys., **9**(4) , 919 (2011b).

Adhav. K.S. *et al*.: Eur.Phys. J. Plus, **126** , 122 (2011c).

Adhav. K.S. *et al*.: Int. Jou.Math.Arch., **2(11)** , 2149 (2011d).

Adhav, K.S.: Can.J.Phys.,**90**,119, (2012).

Akarsu O.and Kilinc C.B., Gen. Relat. Gravit. **42**, 763 (2010).

Berman, M.S.: Nuovo Cimento B, **74**, 2, 182, (1983)

Cunha J.V. and Lima J.A.S.: Mon. Not. R. Astron. Soc.: **390**(1),210 (2008).

Harko,T. : Phys. Lett. B, **669,** 376, (2008).

Ibanez, Z. and Verdaguer,E.: Phys. Rev. D, **34**, 1202, (1986).

Jaiswal *et al*.: Prespacetime Journal, **3**,7, pp. 677, (2012)

Khadekar, G.S., and Gaikwad,M.: Proc. Einstein Found Int. **11**, 95, (2001) Lorentz and Petzold : Gen. Relativ. Grav. **17**, 1189, (1985).

Nojiri S and Odintsov S D : J. Phys. Conf.Ser. **66**, 012005,(2007).

Nojiri S and Odintsov S D : Phys. Rev. **D, 78**, 046006, (2008).

Reddy, D.R.K. and Venkateswara Rao,N.: Astrophys Space Sci, **277**, 461, (2001).

Reddy, D.R.K. *et al.*: IJSAT, **4**, 3, (2014).

Riess A.G. *et al.*: Astrophys J., **560**, 49, (2001)

Sharif,M. and Shamir,M.F. : Gen. Relativ. Grav. **42**, 1557 (2010).

Sharif, M. and Kausar, H.R.: Phys. Lett. B, **697**, 1, (2011).

Singh,V . and Singh, C.P.: Astrophys Space Sci, **346**, 285, (2013).

Singh A.K. and Debnath U.: Int. J. Theor Phys., **48**(2),351 (2009).

Wesson, P.S. : Astro. and Astrophys. **119**, 145 (1983).

Wesson, P.S.: Gen. Relativ. Grav. **16**, 193 (1984).

Yadav,A.K. *et al.* : Int. J. Theor. Phys. **50**, 871-881, (2011)