Computation of L_3 in 2+2 Body Problem When Perturbation Effects Act in Coriolis and Centrifugal Forces, Small Primary is a Radiating Body and Bigger Primary is a Triaxial Rigid Body

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Abstract

Collinear equilibrium point L_3 has been computed in 2+2 body problem when smaller primary is a radiating body, bigger primary is a triaxial rigid body and effect of perturbations in coriolis and centrifugal forces are taken in account to the system. Here other two bodies are so small that gravitational attractions due to these bodies on the two bigger bodies are neglected but two smaller bodies attract each other according to the Newton's law of gravitation.

KEYWORDS: Collinear equilibrium points, Photo-gravitation, Triaxial rigid bodies, Coriolis force.

1. INTRODUCTION

Whipple (1984) studied 2+2 body problem in which M_1 and M_2 ($M_2 \leq M_1$) be two finite bodies. They move in circular Keplerian orbits about their centre of mass. Two minor bodies (m_1 and m_2) $\ll M_2$, move in the gravitational field of two primaries (M_1 and M_2) and they attract each other but do not perturb the primaries. He found fourteen equilibrium points in which six are collinear and eight are triangular.

Sharma, Taqvi and Bhatnagar (2001) studied the existence and stability of libration points in the restricted three body problem when the bigger primary is a triaxial rigid body and a source of radiation.

Garain and Chakraborty (2007) studied Robe restricted three body problem when one of the primary is a triaxial rigid body. They considered perturbations in coriolis and centrifugal forces in the configuration.

2. EQUATION OF MOTION

Whipple's (1984) equation of motion of restricted problem of 2 + 2 bodies in synodic system be

$$\ddot{x}_i - 2\dot{y}_i = \frac{\partial T}{\partial x_i} \tag{1}$$

$$\ddot{y}_i + 2\dot{x}_i = \frac{\partial T}{\partial y_i} \tag{2}$$

$$\ddot{z}_i = \frac{\partial T}{\partial z_i}, \qquad \text{(i= 1, 2)} \qquad (3)$$

$$T = \sum_{i=1}^{2} \mu_i \left[\frac{(x_i^2 + y_i^2)}{2} + \frac{(1-\mu)}{r_{1i}} + \frac{\mu}{r_{2i}} + \frac{\mu_{3-i}}{2r} \right]$$

$$\mu = \frac{M_2}{M_1 + M_2}, \; \mu_i = \frac{m_i}{M_1 + M_2}, (i = 1, 2)$$

$$r_{1i}^2 = (x_i - \mu)^2 + y_i^2 + z_i^2, \; r_{2i}^2 = (x_i - \mu + 1)^2 + y_i^2 + z_i^2$$

$$r^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

Here, we consider M_2, being the radiating one. Also, coriolis and centrifugal forces are taken in to account and M_1 to be a triaxial rigid body. Then using the idea of Garain and Chakraborty (2007) and Sharma, Taqvi and Bhatnagar (2001), we replace T by U as follows:

$$U = \sum_{i=1}^{2} \mu_i \left[\frac{\beta(x_i^2 + y_i^2)}{2} + \frac{(1-\mu)}{r_{1i}} + \frac{q\mu}{r_{2i}} + \frac{\mu_{3-i}}{2r} + \frac{(1-\mu)(2\sigma_1 - \sigma_2)}{2r_{1i}^3} - \frac{3(1-\mu)(\sigma_1 - \sigma_2)y_i^2}{2r_{1i}^5} \right] \qquad (4)$$

Where q =1- ∈, β = 1- ∈′ and i = 1, 2.

Equation of motion changes to

$$\ddot{x}_i - 2\dot{y}_i = \frac{\partial U}{\partial x_i} \qquad (5)$$

$$\ddot{y}_i + 2\dot{x}_i = \frac{\partial U}{\partial y_i} \qquad (6)$$

$$\ddot{z}_i = \frac{\partial U}{\partial z_i}, \qquad \text{(i= 1, 2)} \qquad (7)$$

3. COLLINEAR EQUILIBRIUM POINT L_3

The equilibrium points are those points where

$$\dot{x}_i = \dot{y}_i = \dot{z}_i = \frac{\partial U}{\partial x_i} = \frac{\partial U}{\partial y_i} = \frac{\partial U}{\partial z_i} = 0, \quad (i = 1, 2)$$

Now differentiating both sides of equation (4) with respect to x_1, y_1, z_1, x_2, y_2 and z_2 respectively, we get

$$\beta x_1 - \frac{(1-\mu)(x_1-\mu)}{r_{11}^3} - \frac{q\mu(x_1-\mu+1)}{r_{21}^3} - \frac{\mu_2(x_1-x_2)}{r^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)(x_1-\mu)}{2r_{11}^5}$$
$$+\frac{15(1-\mu)(\sigma_1-\sigma_2)(x_1-\mu)y_1^2}{2r_{11}^7} = 0 \tag{8}$$

$$\beta y_1 - \frac{(1-\mu)y_1}{r_{11}^3} - \frac{q\mu y_1}{r_{21}^3} - \frac{\mu_2(y_1-y_2)}{r^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)y_1}{2r_{11}^5}$$
$$-\frac{3(1-\mu)(\sigma_1-\sigma_2)}{2}\left\{\frac{2y_1}{r_{11}^5} - \frac{5y_1^3}{r_{11}^7}\right\} = 0 \tag{9}$$

$$-\frac{(1-\mu)z_1}{r_{11}^3} - \frac{q\mu z_1}{r_{21}^3} - \frac{\mu_2(z_1-z_2)}{r^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)z_1}{2r_{11}^5} + \frac{15(1-\mu)(\sigma_1-\sigma_2)y_1^2 z_1}{2r_{11}^7} = 0 \tag{10}$$

$$\beta x_2 - \frac{(1-\mu)(x_2-\mu)}{r_{12}^3} - \frac{q\mu(x_2-\mu+1)}{r_{22}^3} - \frac{\mu_1(x_2-x_1)}{r^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)(x_2-\mu)}{2r_{12}^5}$$
$$+\frac{15(1-\mu)(\sigma_1-\sigma_2)(x_2-\mu)y_2^2}{2r_{12}^7} = 0 \tag{11}$$

$$\beta y_2 - \frac{(1-\mu)y_2}{r_{12}^3} - \frac{q\mu y_2}{r_{22}^3} - \frac{\mu_1(y_2-y_1)}{r^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)y_2}{2r_{12}^5}$$
$$-\frac{3(1-\mu)(\sigma_1-\sigma_2)}{2}\left\{\frac{2y_2}{r_{12}^5} - \frac{5y_2^3}{r_{12}^7}\right\} = 0 \tag{12}$$

$$-\frac{(1-\mu)z_2}{r_{12}^3} - \frac{q\mu z_2}{r_{22}^3} - \frac{\mu_1(z_2-z_1)}{r^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)z_2}{2r_{12}^5} + \frac{15(1-\mu)(\sigma_1-\sigma_2)y_2^2 z_2}{2r_{12}^7} = 0 \tag{13}$$

From equations (10) and (13), we get $z_1 = 0$ and this value in (13) yields $z_2 = 0$. By inspection, it can be seen that equations (9) and (12) are satisfied when $y_1 = y_2 = 0$. Now we have to determine x_1 and x_2 such that the following simplified forms of equations (8) and (11) are satisfied.

$$\beta x_1 - \frac{(1-\mu)(x_1-\mu)}{|x_1-\mu|^3} - \frac{q\mu(x_1-\mu+1)}{|x_1-\mu+1|^3} - \frac{\mu_2(x_1-x_2)}{|x_1-x_2|^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)(x_1-\mu)}{2|x_1-\mu|^5}$$
$$+\frac{15(1-\mu)(\sigma_1-\sigma_2)(x_1-\mu)y_1^2}{2|x_1-\mu|^7} = 0 \tag{14}$$

and

$$\beta x_2 - \frac{(1-\mu)(x_2-\mu)}{|x_2-\mu|^3} - \frac{q\mu(x_2-\mu+1)}{|x_2-\mu+1|^3} - \frac{\mu_2(x_2-x_1)}{|x_2-x_1|^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)(x_2-\mu)}{2|x_2-\mu|^5}$$
$$+\frac{15(1-\mu)(\sigma_1-\sigma_2)(x_2-\mu)y_2^2}{2|x_2-\mu|^7} = 0 \quad (15)$$

The solutions of equations (14) and (15) can be obtained with the help of power series.

Let $\in_i = \frac{\mu_i}{(\mu_1+\mu_2)^{\frac{2}{3}}}$, $(i=1,2)$

$$\therefore \mu_2 \in_1 = \frac{\mu_1\mu_2}{(\mu_1+\mu_2)^{\frac{2}{3}}} = \mu_1 \in_2$$

$$\therefore \mu_2 \in_1 = \mu_1 \in_2 = k \qquad (Let) \qquad (16)$$

Now, let $x_1 = L_3' + \sum_{j=1}^{n} a_{1j} \in_2^j$ (17)

Where L_3' be equilibrium points in photo-gravitational restricted problem of three bodies when bigger primary is a triaxial rigid body perturbed by coriolis and centrifugal forces and x_1 be the x - coordinate of first small body.

Similarly, let

$$x_2 = L_3' + \sum_{j=1}^{n} a_{2j} \in_1^j \qquad (18)$$

Similar to Whipple,

$$a_{11}\Omega^{\circ}_{xx} \in_2 - \frac{\mu_2(x_1-x_2)}{|x_1-x_2|^3} = 0 \qquad (19)$$

$$a_{21}\Omega^{\circ}_{xx} \in_1 - \frac{\mu_1(x_2-x_1)}{|x_2-x_1|^3} = 0 \qquad (20)$$

From equations (19) and (20), we get

$$a_{11} = -a_{21} \qquad (21)$$

From equations (19), (20) and (21), we have

$$a_{11} = (\pm 1)\frac{1}{(\Omega^{\circ}_{xx})^{\frac{1}{3}}} \qquad (22)$$

$$\Rightarrow\ x_1 = L_3' + \frac{(\pm 1)}{(\Omega^{\circ}_{xx})^{\frac{1}{3}}}\frac{\mu_2}{(\mu_1+\mu_2)^{\frac{2}{3}}} \qquad (23)$$

and

$$x_2 = L_3' - \frac{(\pm 1)}{(\Omega^{\circ}_{xx})^{\frac{1}{3}}}\frac{\mu_1}{(\mu_1+\mu_2)^{\frac{2}{3}}} \qquad (24)$$

Now, $\Omega^{\circ}_{xx} = A_3 + B_3 \in + C_3 \in' + 2D_3\sigma_1 - D_3\sigma_2$ (25)

Where, $A_3 = 1 + 2\left\{\frac{(1-\mu)}{a_3^3} + \frac{\mu}{(a_3+1)^3}\right\}$, $B_3 = \frac{6b_3(1-\mu)}{a_3^4} + \mu\left\{\frac{6b_3}{(a_3+1)^4} - \frac{2}{(a_3+1)^3}\right\}$

$$C_3 = 1 - 6\left\{\frac{(1-\mu)}{a_3^4} + \frac{\mu}{(a_3+1)^4}\right\}c_3,\quad D_3 = 6\left\{(1-\mu)\left(\frac{d_3}{a_3{}^4} + \frac{1}{a_3{}^5}\right) + \frac{d_3\mu}{(a_3+1)^4}\right\}$$

$$a_3 = 1 - \frac{7\mu}{12},\ b_3 = \frac{\mu}{12},\ c_3 = \frac{5\mu}{6}\ \text{ and }\ d_3 = \frac{19\mu}{24}$$

Putting the values of Ω°_{xx} and L_3' in (23) and (24) we get values of x_1 and x_2 as follow:

$$x_1 = a_{31} - b_{31} \in + c_{31} \in' - 2d_{31}\sigma_1 + d_{31}\sigma_2$$

Where, $a_{31} = \mu + a_3 + \frac{(\pm 1)\mu_2}{A_3^{\frac{1}{3}}(\mu_1+\mu_2)^{\frac{2}{3}}}$, $b_{31} = b_3 + \frac{(\pm 1)\mu_2 B_3}{3A_3^{\frac{4}{3}}(\mu_1+\mu_2)^{\frac{2}{3}}}$

$c_{31} = c_3 - \frac{(\pm 1)\mu_2 C_3}{3A_3^{\frac{4}{3}}(\mu_1+\mu_2)^{\frac{2}{3}}}$ and $d_{31} = d_3 + \frac{(\pm 1)\mu_2 D_3}{3A_3^{\frac{4}{3}}(\mu_1+\mu_2)^{\frac{2}{3}}}$.

$$x_2 = a_{32} - b_{32} \in + c_{32} \in' - 2d_{32}\sigma_1 + d_{32}\sigma_2$$

Where, $a_{32} = \mu + a_3 - \frac{1}{A_3^{\frac{1}{3}}}\frac{(\pm 1)\mu_1}{(\mu_1+\mu_2)^{\frac{2}{3}}}$, $b_{32} = b_3 - \frac{1}{3A_3^{\frac{4}{3}}}\frac{(\pm 1)\mu_1 B_3}{(\mu_1+\mu_2)^{\frac{2}{3}}}$

$c_{32} = c_3 + \frac{1}{3A_3^{\frac{4}{3}}} \frac{(\pm 1)\mu_1 C_3}{(\mu_1 + \mu_2)^{\frac{2}{3}}}$ and $d_{32} = d_3 - \frac{1}{3A_3^{\frac{4}{3}}} \frac{(\pm 1)\mu_1 D_3}{(\mu_1 + \mu_2)^{\frac{2}{3}}}$. The particular values of x_1 and x_2 may be shown by the following tables.

Table 1

For Collinear Equilibrium Solutions L_3

($\mu_1=\mu_2=10^{-10}$, $x_1 = a_{31} - b_{31}\epsilon + c_{31}\epsilon' - 2d_{31}\sigma_1 + d_{31}\sigma_2$)

μ	$\mu_2/(\mu_1+\mu_2)^{2/3}$	a_{31}	b_{31}	c_{31}	d_{31}
0.00001	0.000292401774	1.000206906339	0.000000833390	-0.000014192039	0.000143079320
0.00002	0.000292401774	1.000150061012	0.000001666779	-0.000005857404	0.000150998596
0.00003	0.000292401774	1.000154391658	0.000002500169	0.000002477231	0.000158917873
0.00004	0.000292401774	1.000158688304	0.000003333559	0.000010811866	0.000166837149
0.00005	0.000292401774	1.000162964347	0.000004166948	0.000019146500	0.000174756425
0.00006	0.000292401774	1.000167226334	0.000005000338	0.000027481135	0.000182675701
0.00007	0.000292401774	1.000171478013	0.000005833728	0.000035815770	0.000190594977
0.00008	0.000292401774	1.000175721753	0.000006667117	0.000044150405	0.000198514254
0.00009	0.000292401774	1.000179959156	0.000007500507	0.000052485040	0.000206433530
0.0001	0.000292401774	1.000184191365	0.000008333897	0.000060819675	0.000214352806
0.0002	0.000292401774	1.000226347700	0.000016667793	0.000144166026	0.000293545572
0.0003	0.000292401774	1.000268353769	0.000025001690	0.000227512378	0.000372738341
0.0004	0.000292401774	1.000310288289	0.000033335587	0.000310858731	0.000451931114
0.0005	0.000292401774	1.000352179562	0.000041669484	0.000394205086	0.000531123891
0.0006	0.000292401774	1.000394041392	0.000050003381	0.000477551442	0.000610316671
0.0007	0.000292401774	1.000435881666	0.000058337279	0.000560897799	0.000689509455
0.0008	0.000292401774	1.000477705364	0.000066671176	0.000644244158	0.000768702242
0.0009	0.000292401774	1.000519515852	0.000075005074	0.000727590518	0.000847895034
0.001	0.000292401774	1.000561315525	0.000083338971	0.000810936879	0.000927087829
0.002	0.000292401774	1.000978968285	0.000166677956	0.001644400564	0.001719015981
0.003	0.000292401774	1.001396310741	0.000250016953	0.002477864380	0.002510944502
0.004	0.000292401774	1.001813506339	0.000333355963	0.003311328329	0.003302873392
0.005	0.000292401774	1.002230613531	0.000416694986	0.004144792409	0.004094802653
0.006	0.000292401774	1.002647660713	0.000500034021	0.004978256622	0.004886732283
0.007	0.000292401774	1.003064664060	0.000583373070	0.005811720966	0.005678662285
0.008	0.000292401774	1.003481633755	0.000666712132	0.006645185443	0.006470592659
0.009	0.000292401774	1.003898576665	0.000750051206	0.007478650053	0.007262523406
0.01	0.000292401774	1.004315497665	0.000833390294	0.008312114795	0.008054454525
0.02	0.000292401774	1.008484011269	0.001666781886	0.016646769547	0.015973786390
0.03	0.000292401774	1.012651893929	0.002500174792	0.024981437722	0.023893156307
0.04	0.000292401774	1.016819472941	0.003333569028	0.033316119464	0.031812564958
0.05	0.000292401774	1.020986864421	0.004166964609	0.041650814919	0.039732013047
0.06	0.000292401774	1.025154124433	0.005000361551	0.049985524237	0.047651501294
0.07	0.000292401774	1.029321284726	0.005833759870	0.058320247571	0.055571030441
0.08	0.000292401774	1.033488365153	0.006667159582	0.066654985077	0.063490601248
0.09	0.000292401774	1.037655378996	0.007500560704	0.074989736913	0.071410214497
0.1	0.000292401774	1.041822335587	0.008333963252	0.083324503243	0.079329870991
0.2	0.000292401774	1.083489682901	0.016668070747	0.166673002214	0.158529008244
0.3	0.000292401774	1.125153939509	0.025002338883	0.250023160327	0.237733547910
0.4	0.000292401774	1.166815044860	0.033336782980	0.333375219656	0.316944802184
0.5	0.000292401774	1.208471824545	0.041671412410	0.416729489404	0.396164551866

Table 2					
For Collinear Equilibrium Solutions L_3					
($\mu_1=\mu_2=10^{-10}$, $x_2 = a_{32} - b_{32}\epsilon + c_{32}\epsilon' - 2d_{32}\sigma_1 + d_{32}\sigma_2$)					
μ	$\mu_2/(\mu_1+\mu_2)^{2/3}$	a_{32}	b_{32}	c_{32}	d_{32}
0.00001	0.000292401774	0.999801426994	0.000000833277	0.000030858706	-0.000127245987
0.00002	0.000292401774	0.999866605655	0.000001666554	0.000039190738	-0.000119331930
0.00003	0.000292401774	0.999870608342	0.000002499831	0.000047522769	-0.000111417873
0.00004	0.000292401774	0.999874645029	0.000003333108	0.000055854801	-0.000103503815
0.00005	0.000292401774	0.999878702319	0.000004166385	0.000064186833	-0.000095589758
0.00006	0.000292401774	0.999882773666	0.000004999662	0.000072518865	-0.000087675701
0.00007	0.000292401774	0.999886855320	0.000005832939	0.000080850896	-0.000079761644
0.00008	0.000292401774	0.999890944914	0.000006666216	0.000089182928	-0.000071847587
0.00009	0.000292401774	0.999895040844	0.000007499493	0.000097514960	-0.000063933530
0.0001	0.000292401774	0.999899141968	0.000008332770	0.000105846991	-0.000056019473
0.0002	0.000292401774	0.999940318966	0.000016665540	0.000189167307	0.000023121095
0.0003	0.000292401774	0.999981646231	0.000024998310	0.000272487622	0.000102261659
0.0004	0.000292401774	1.000023045045	0.000033331080	0.000355807935	0.000181402219
0.0005	0.000292401774	1.000064487105	0.000041663849	0.000439128247	0.000260542776
0.0006	0.000292401774	1.000105958608	0.000049996619	0.000522448558	0.000339683329
0.0007	0.000292401774	1.000147451667	0.000058329388	0.000605768867	0.000418823879
0.0008	0.000292401774	1.000188961302	0.000066662157	0.000689089175	0.000497964424
0.0009	0.000292401774	1.000230484148	0.000074994926	0.000772409482	0.000577104966
0.001	0.000292401774	1.000272017808	0.000083327695	0.000855729788	0.000656245505
0.002	0.000292401774	1.000687698382	0.000166655378	0.001688932769	0.001447650686
0.003	0.000292401774	1.001103689259	0.000249983047	0.002522135620	0.002239055498
0.004	0.000292401774	1.001519826995	0.000333310704	0.003355338338	0.003030459941
0.005	0.000292401774	1.001936053136	0.000416638348	0.004188540924	0.003821864014
0.006	0.000292401774	1.002352339287	0.000499965979	0.005021743378	0.004613267717
0.007	0.000292401774	1.002768669274	0.000583293596	0.005854945700	0.005404671048
0.008	0.000292401774	1.003185032912	0.000666621202	0.006688147890	0.006196074007
0.009	0.000292401774	1.003601423335	0.000749948794	0.007521349947	0.006987476594
0.01	0.000292401774	1.004017835668	0.000833276373	0.008354551871	0.007778878808
0.02	0.000292401774	1.008182655398	0.001666551447	0.016686563786	0.015692880276
0.03	0.000292401774	1.012348106071	0.002499825208	0.025018562278	0.023606843693
0.04	0.000292401774	1.016513860393	0.003333097639	0.033350547203	0.031520768375
0.05	0.000292401774	1.020679802246	0.004166368725	0.041682518414	0.039434653620
0.06	0.000292401774	1.024845875567	0.004999638449	0.050014475763	0.047348498706
0.07	0.000292401774	1.029012048607	0.005832906797	0.058346419095	0.055262302892
0.08	0.000292401774	1.033178301514	0.006666173751	0.066678348256	0.063176065419
0.09	0.000292401774	1.037344621004	0.007499439296	0.075010263087	0.071089785503
0.1	0.000292401774	1.041510997747	0.008332703415	0.083342163423	0.079003462342
0.2	0.000292401774	1.083176983766	0.016665262587	0.166660331119	0.158137658422
0.3	0.000292401774	1.124846060491	0.024997661117	0.249976839673	0.237266452090
0.4	0.000292401774	1.166518288473	0.033329883686	0.333291447010	0.316388531149
0.5	0.000292401774	1.208194842122	0.041661920923	0.416603843929	0.395502114800

3. CONCLUSION

We get the values of L_3 which are (x_1, 0, 0) and (x_2, 0, 0) respectively. For different values μ, μ_1 and μ_2 we may compute x_1 and x_2 for L_3.

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