Mathematical Modelling and Topological Development in Secondary School: Advantages and Challenges

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ABSTRACT:

From the sciences which are the bases of technology and development are mathematics, and the most appropriate way to teach mathematics is using its application and also mathematical modelling and topological concept. In the present article we introduce mathematical modelling, topological development and their applications, review similarities and differences with problem solving, and consider how to introduce these in secondary levels.

We also state some advantages and challenges of introducing mathematical modelling and topological development to secondary school students. The main aim of the present article is to encourage the teachers to introduce mathematical modelling and topological development into their pedagogical system especially secondary schools. We believe that this could lead the nation to develop faster.

INTRODUCTION:

Rapid development of information and technology today changed society's expectations form people and education world. Today's world expects mathematics teachers to raise individuals who are able to create effective solutions in cases of real problems and use mathematics effectively in their daily lives. Thus, they will enjoy mathematics instated of being scared of it and comprehend and appreciate the importance and power of mathematics. This process of development and change caused new searches in our educational system and it became compulsory to try new approaches, methods and models in the educational realm. One of those new approaches in mathematics teaching is teaching by means of models.

MATHEMATICAL MODELLING:

Modelling is a cyclical process involving (1) the creation of a provisional model, which stems from (2) a series of interactive activities, which should be (3) continually tested and refined in order to improve or verify it (Kang & Noh, 2012).

M*athematical modelling* is the process of generating mathematical representations in attempting to solve real life problems (English, Fox & Watters, 2005).

Mathematical modelling is defined as the transformation of any problem situation into a mathematical model. However this concept started to be used commonly to define the process including all the steps of structuring mathematizing, mathematical working and interpretation/verification. Sometimes the problem situation that is given is nothing else than a pre-structured mathematical problem or a mathematical problem that is full of real life. This is the classic "word problem" situation that generally occurs in schools. Using mathematics to solve problems that are encountered in real life is called as application of mathematics. Sometimes the application concept is used for a relation that binds real life to mathematics. In the last ten years "application and modelling" concepts were used to explain any relations between real life and mathematics.

The main steps in mathematical modelling are:

Step 1: Understanding the problem

Step 2: Mathematical description and formulation

Step 3: Solving the mathematical problem

Step 4: Interpreting the solution

Step 5: Validating the model

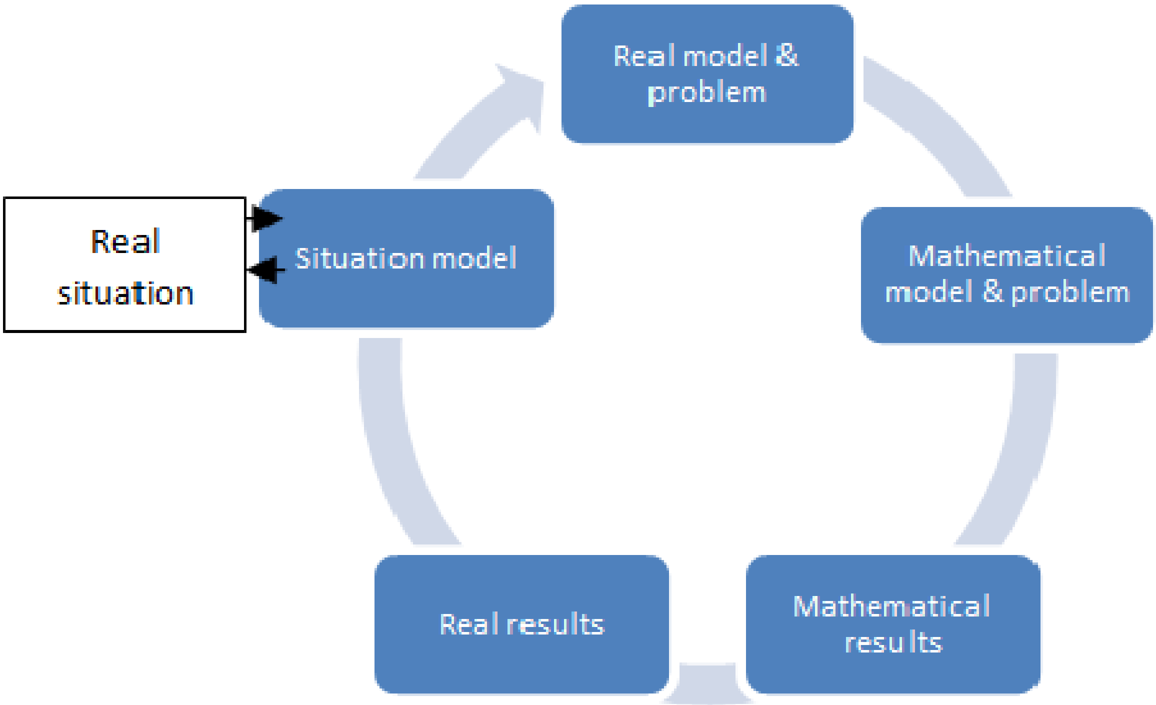


Figure 1: Modelling Cycle

PROBLEM SOLVING:

The problem solving method is one, which involves the use of process of problem solving or reflective thinking or reasoning. Problem solving method, as the name indicated, begins with the statement of a problem that challenges the students to find a solution.

Problem solving is a set of events in which human beings was rules to achieve some goals – **Gagne**

Problem solving involves concept formation and discovery learning –**Ausubel**

Steps in Problem Solving / Procedure for Problem solving

1. Identifying and defining the problem:

The student should be able to identify and clearly define the problem. The problem that has been identified should be interesting challenging and motivating for the students to participate in exploring.

2. Analysing the problem:

The problem should be carefully analysed as to what is given and what is to be find out. Given facts must be identified and expressed, if necessary in symbolic form.

3. Formulating tentative hypothesis

Formulating of hypothesis means preparation of a list of possible reasons of the occurrence of the problem. Formulating of hypothesis develops thinking and reasoning powers of the child. The focus at this stage is on hypothesizing – searching for the tentative solution to the problem.

4. Testing the hypothesis:

Appropriate methods should be selected to test the validity of the tentative hypothesis as a solution to the problem. If it is not proved to be the solution, the students are asked to formulate alternate hypothesis and proceed.

5. Verifying of the result or checking the result:

No conclusion should be accepted without being properly verified. At this step the students are asked to determine their results and substantiate the expected solution. The students should be able to make generalizations and apply it to their daily life.

TOPOLOGICAL DEVELOPMENT:

Topology developed as a field of study out of geometry and set theory, through analysis of concepts such as space, dimension, and transformation.

The term *topology* was introduced by Johann Benedict Listing in the 19th century, although it was not until the first decades of the 20th century that the idea of a topological space was developed. By the middle of the 20th century, topology had become a major branch of mathematics.

Topology has many subfields:

- **General topology** is the branch of topology dealing with the basic set-theoretic definitions and constructions used in topology.
- **Algebraic topology** is a branch of mathematics that uses tools from abstract algebra to study spaces. The basic goal is to find algebraic invariants that classify topological spaces up to homeomorphism, though usually most classify up to homotopy equivalence.
- **Differential topology** is the field dealing with differentiable functions on differentiable manifolds. It is closely related to differential geometry and together they make up the geometric theory of differentiable manifolds.
- **Geometric topology** is a branch of topology that primarily focuses on low-dimensional manifolds (i.e. dimensions 2, 3 and 4) and their interaction with geometry, but it also includes some higher-dimensional topology.

PEDAGOGICAL SYSTEM:

Pedagogy is the science and art of education, specifically instructional theory.

Here the syllabus of class ix and class x of W.B.B.S.E. and C.B.S.E. divide into braches and chapter wise and describe the origin of mathematical modelling and topological development.

TABLE 1: **Syllabus of class ix of W.B.B.S.E. and C.B.S.E.:**

Branch	Topics/Chapters W.B.B.S.E.	Topics/Chapters C.B.S.E.	Mathematical Modelling	Topological Development
Arithmetic	Real number, Distance measurement, Profit and Loss	Real number	Real problems solve by arithmetic techniques.	----------------
Algebra	Polynomial, Surd, Factorisation, Pair of Linear Equation, Logarithm	Polynomial, Linear Equation of two variable	Real problems solve by algebraic equations.	Basic concept of Algebraic Topology (variable)
Geometry	Graph plotting, Triangle(theorem), Properties of Parallelogram, Transversal and Midpoint Theorem, Area based theorems (Rectangle, Triangle), Co-ordinate Geometry, Construction	Introduction to Euclid's Geometry, Lines and Angles, Triangles(theorem), Quadrilaterals(theorem), Area theorems (Rectangle, Triangle), Circle(theorem), Co-ordinate Geometry, Construction	Real problems solve by different geometrical concept, properties and theorems.	Geometric Topology of two dimensional concept
Mensuration	Perimeter and Area of Triangle, Circumference and Area of Circle	Area(Triangle, Quadrilaterals), Surface Areas and Volumes(Cubes, Cuboids, Spheres, Right Circular Cylinders/Cones)	Real problems solve by the formula of area, volume.	Geometric Topology of two and three dimensional concepts

Branch	Topics/ Chapters W.B.B.S.E.	Topics/ Chapters C.B.S.E.	Mathematical Modelling	Topological Development
Statistics & Probability	Tabular Form, Bar Graph, Histogram, Frequency Polygon	Tabular Form, Bar Graph, Histogram, Frequency Polygon, Mean, Median, Mode. Basic concept of Probability.	Real problems solve by basic concept of statistics and probability.	
Arithmetic	Simple Interest, Compound Interest, Partnership Business	Real number,	Real problems solve by arithmetic techniques.	------------------ ---------------------
Algebra	Quadratic Equations with one variable, Ratio and Proportion, Quadratic Surd, Variation	Polynomial, Pair of Linear Equation of two variable, Quadratic Equation, Arithmetic Progressions	Real problems solve by algebraic equations.	Basic concept of abstract algebra (variable)
Geometry	Theorems related to: i) Circle, ii) Angles in a Circle, iii)Cyclic Quadrilaterals, iv)Tangent to a Circle, Pythagoras Theorem, Construction	Theorems related to Triangles, Theorems related to Tangent to a Circle, Construction	Real problems solve by different geometrical concept, properties and theorems.	Geometric Topology of two dimensional concept
Mensuration	Surface Areas and Volumes of i)Cuboids, Cubes, ii)Right Circular Cylinders, iii)Spheres, iv)Cones.	Areas related to Circles, Surface Areas and Volumes Cubes, Cuboids, Right Circular Cylinder, Cone, Spheres.	Real problems solve by the formula of area, volume.	Geometric Topology of two and three dimensional concepts
Trigonometry	Trigonometric Ratios and Identity, Trigonometric Ratios of Complementary angle, Heights and Distances	Introduction, Trigonometric Identities, Heights and Distances	Real problems solve by trigonometric ratios and identity.	Basic concepts of Algebraic Topology (Trigonometric Function) and Geometric Topology (Triangle)

Statistics & Probability	Mean, Median, Mode, Ogive	Mean, Median, Mode, Cumulative Frequency graph. Classical Definition of Probability, Simple Problems on Single Events.	Real problems solve by basic concept of statistics and probability.	---------------------

TABLE 2: **Syllabus of class x of W.B.B.S.E. and C.B.S.E.:**

Example 1: (Arithmetic)

Problem: Suppose Sudhir has invested Rs 15,000 at 8% simple interest per year. With the return from the investment, he wants to buy a washing machine that casts Rs 19,000. For what period should he invest Rs 15,000 so that he has enough money to buy a washing machine?

Solution: Step 1: Formulation of the problem:

We know the principal and the rate of interest. The interest amount Sudhir needs in addition to 15,000 to buy the washing machine. We have to find the number of years.

Step 2: Mathematical Description:

The formula for simple interest is I= $\frac{Pnr}{100}$

Where p = Principal,

n = Number of years,

r% = Rate of interest,

I = Interest earned

Here, the principal = 15,000

The money required by Sudhir for buying a washing machine = 19,000

So, the interest to be earned = Rs (19,000 – 15,000)

= Rs 4,000

Rate of interest, r% = 8%

The number of years, n to be found

The interest on Rs 15,000 for n years at the rate of 8% = I

Then, I = $\frac{15000 \times n \times 8}{100}$

So, I = 1200n (1)

We have to find the period in which the interest earned is Rs 4000.

Putting I = 4000 in (1), we have

$$4000 = 1200n \qquad (2)$$

Step 3: Solution of the problem:

Solving Equation (2), we get

$$n = \frac{4000}{1200} = 3\frac{1}{3}$$

Step 4: Interpretation:

Since $n = 3\frac{1}{3}$ and one third of a year is 4 months,

Sudhir can buy a washing machine after 3 years and 4 months.

Step 5: Validating the model:

We have to assume that the interest rate remains the same for the period for which we calculate the interest. Otherwise, the formula $I = \frac{Pnr}{100}$ will not be valid. We have also assumed that the price of the washing machine does not increase by the time Sudhir has gathered the money.

Example 2: (Algebra):

Problem: A motorboat goes upstream on a river and covers the distance between two towns on the riverbank in six hours. It covers this distance downstream in five hours. If the speed of the steam is 2 km/hr, find the speed of the boat in still water.

Solution: Step 1: Formulation of the problem:

We know the speed of the river and the time taken to cover the distance between two places. We have to find the speed of the boat in still water.

Step 2: Mathematical Description:

Let us write x for the speed of the boat, t for the time taken and y for the distance travelled.

Then $y = tx \qquad (1)$

Let d be the distance between the two places.

While going upstream, the actual speed of the boat = speed of the boat – speed of the river, because the boat is travelling against the flow of the river.

So, the speed of the boat upstream = $(x - 2)$ km/hr

It takes 6 hours to cover the distance between the towns upstream. So, from (1),

We get $d = 6(x - 2)$ (2)

When going downstream, the speed of the river has to be added to the speed of the boat.

So, the speed of the boat downstream = $(x + 2)$ km/hr

The boat takes 5 hours to cover the same distance downstream. So,

$$d = 5(x + 2) \quad (3)$$

From (2) and (3), we have

$$6(x - 2) = 5(x + 2) \quad (4)$$

Step 3: Solution of the problem:

Solving for x in Equation (4), we get

$$6(x - 2) = 5(x + 2)$$

Or, $6x - 12 = 5x + 10$

Or, $x = 22$

Step 4: Interpretation:

Since x = 22, therefore the speed of the motorboat in still water is 22 km/hr.

Step 5: Validating the model:

We have assumed that the speed of the river and the boat remains constant all time and the effect of friction between the boat and water and the friction due to air is negligible.

Example 3: (Geometry):

Problem: In a huge park, people are concentrated at three points:

A: where there are different slides and swings for children,

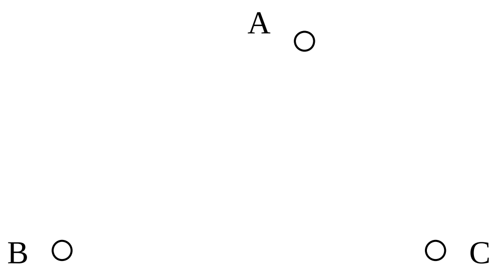
B: near which a man-made lake is situated,

C: which is near to a large parking and exit.

Where should an ice-cream parlour be set up so that maximum number of persons can approach it?

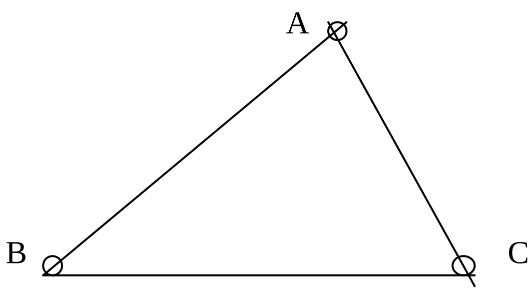
Solution: Step 1: Formulation of the problem:

Suppose the three points A, B, C are situated in the park as



Step 2: Mathematical Description:

Now we have to join the points A, B, C as AB, BC and CA.

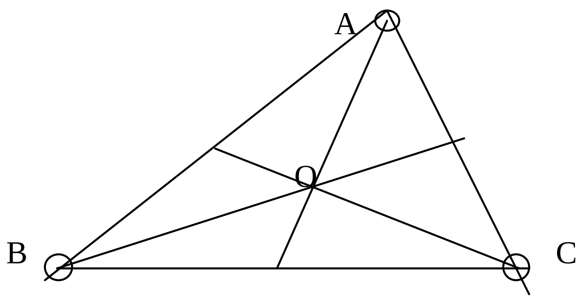


Then we get ABC as a triangle. In this triangle we find out a point where the ice-cream parlour is set up so that maximum number of persons can approach it.

Step 3: Solution of the problem:

If we put the point in ΔABC so that the distance between the point and the each point A, B, C are equidistance then maximum number of persons can approach the point i.e., the ice-cream parlour.

Now we draw three medians AD, BE and CF and the median meets at O.



Step 4: Interpretation: Thus the point O is the point where the ice-cream parlour should be set up so that maximum number of persons can approach it.

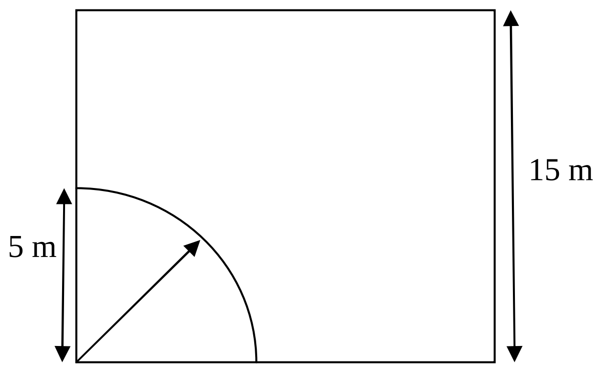
Step 5: Validating the model: A, B, C consider as a point where the people are concentrated.

Example 4: (Mensuration):

Problem: A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze.

Solution: Step 1: Formulation of the problem:

We draw a square shaped grass field of side 15 m and a 5 m long rope where the horse is tied to a peg.



Step 2: Mathematical Description:

Here area of the sector in which the horse can graze = $\frac{\theta}{360} \times \pi r^2$; where θ is the angle of corner of the square = 90° and r is the radius of arc of circle = 5 m.

Step 3: Solution of the problem:

Hence area of the sector = $\frac{90}{360} \times 3.14 \times 5^2$

$= \frac{1}{4} \times 3.14 \times 25$

= 19.625 sq. m.

Step 4: Interpretation:

The area of that part of the field in which the horse can graze is 19.625 sq. m.

Step 5: Validating the model: The grass field consider square shaped and the rope is 5 m long.

DISCUSSION AND CONCLUSIONS:

Mathematical models and modelling exist all around us; we especially encounter them in technological devices. More generally, mathematical modelling;

- Helps students to understand the world better,
- Supports mathematical learning (motivation, concept formation, giving meanings etc.)
- Ensures developing various mathematical qualifications and accurate attitudes.
- Provides sufficient support for the framework of mathematics.

Mathematics becomes more meaningful for the students with modelling. Development of mathematical modelling skills of students is underscored as one of the main objectives of mathematics teaching in the secondary school program.

Suggestion:

- Mathematical modelling is to be attached greater importance and used more in mathematics syllabus when developing constructivist mathematical course books and other materials.
- Teachers are to be given in service education and studies on teachers are to be carried out to determine to what extent they adopt mathematical modelling approach and develop activities and lesson plans based on modelling approach.
- Universities are to cooperate to carry out more comprehensive research and projects. In this context, joint studies with countries in which mathematics education is applied should be carried out.

We can say that using models in mathematics teaching is necessary to motivate the students, eliminate their fear and anxiety and allow them to develop a positive approach towards mathematics in addition to its many cognitive benefits such as realizing meaningful learning, establishing a relation between mathematics and daily life and developing problem solving skills.

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