Effectiveness of Polya's Heuristic Approach to Problem Solving In Acquisition of Mathematical Concepts among Secondary School Students

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Abstract

The present investigation was undertaken with an aim to study the effectiveness of Polya's heuristic approach to problem solving in acquisition of mathematical concepts among secondary school students.. For this purpose 320 secondary school students from Moga district of Punjab state were taken as a sample. The tool used was Mathematical Concept Acquisition Test (MCAT) on selected topics of Mathematics of class IX (developed by the investigator). Instructional material (Modules) based on Problem Solving Instructional Strategy on selected topics of Mathematics for Class IX was also prepared by the investigator. It was validated by the subject experts and later on was implemented to the experimental group. The present study employed an experimental method with "pre-test post-test"design. Results showed that students taught through problem solving instructional strategy were found to achieve significantly high on the acquisition of mathematical concepts than taught through traditional teaching method.

INTRODUCTION

Today, qualitative improvement of education is of great importance and it can be achieved only by improving the quality of instruction. Even though great advancement in science as well as educational technology was made in our country, the methods of teaching prevalent are not significant to meet the requirements of the students at all levels. Several studies on classroom practices reveal that even though the student's characteristics and societal expectations have changed, our educators still employ those traditional methods and mode of instruction. Hence, it is necessary to refine and to improve the teaching methods and instructional techniques to realize the fullest potentialities of individual student learning needs and to achieve acceptable levels of individual student mastery, proficiency and expertise.

Mathematics Teaching-Learning

An information and technology based society requires individuals to think critically about complex issues, analyze and adapt to new situations, solve problems of various kinds, and communicate their thinking effectively. The study of mathematics equips students with knowledge, skills and habits of mind that are essential for successful and rewarding participation in such a society. To learn mathematics in a way that will serve them well throughout their lives, students need classroom experiences that develop mathematical understanding, learn important facts, skills and procedures, develop the ability to apply the processes of mathematics, and acquire a positive attitude towards mathematics. Learning mathematics results in more than a mastery of basic skills. Students who truly understand or make sense of mathematical concepts are not just manipulating symbols or

following rules invented by others to solve problems. They are applying rules and exploring solutions by using logical thinking and reasonableness of solutions. Mathematical structures, operations, processes and language provide students with a framework and tools for reasoning, justifying conclusions and expressing ideas clearly through mathematical activities that are practical and relevant to their lives and, eventually in the workplace.

Need of Appropriate Instructional Strategy

To satisfy the needs of the learners new experiments, creative innovations and appropriate strategies are being developed and tried out to improve mathematics education at all levels. Teachers, as facilitators of a classroom environment where students learn to communicate mathematically and as an agent to improve quality of mathematics lessons, need to employ multiple instructional strategies/techniques such as mathematics laboratory, brainstorming, group discussion, inductive-deductive method, concept mapping, problem solving etc. A strategy that would transform the position of the teacher from being active speaker to facilitator, a strategy that makes the teaching systematic and a strategy that would help the students to master the mathematical concepts easily and may contribute towards the upliftment of mathematics instruction and education is the need of the hour.

Problem Solving as an Instructional Method

The last several decades in our nation have witnessed growing concerns that our educational system is not adequately preparing all of our citizens to develop the skills and knowledge they need to live and work successfully in today's complex society. Calls for change have centred on the need for schools to provide opportunities for students to engage in activities that promote in-depth understanding, better acquisition of concepts, critical thinking, creative problem solving and the ability to use knowledge in real-life settings.

In response to these concerns school improvement efforts have proliferated, aimed at all aspects of the educational system present a conception of teaching that focuses on developing thinking and reasoning among students and further on learning skills of problem formulation and solving. This conception is termed as " Problem Solving Instruction " because it sees learning as a dynamic internal process in which learners actively participate by connecting new information to what they already know, rather than as a process in which learners are passive recipients of information transferred to them from external sources. This newer conception's emphasis is on developing students' capacities for analysis and problem-solving, rather than on having them "cover the curriculum" in the most efficient manner possible.

Meaning of Skill of Problem Solving

Problem solving is a mental process and is a part of the larger problem process that includes problem finding and problem shaping. Problem solving occurs when an organism needs to move from a given state to a desired future goal state.

Ackoff (1978) defined problem solving as the skill that involves the selection of one or more courses of action (means) in the pursuit of one or more courses of action (ends).

Kirkley (2003) described problem solving as higher order thinking which includes skills such as visualization, comprehension, manipulation, reasoning, analysis, synthesis and generalization, each needing to be managed and coordinated.

Thus problem solving is an ability to analyze a situation and then form a workable solution. The problem solving is a process of overcoming difficulties that appear to interfere with the attainment of a goal. It is a systematic way to move from a set of given principles or circumstances to the desired result by using higher cognitive skills like logic, associative reasoning, creative thinking and deductive reasoning.

Polya's Heuristic Approach to Problem Solving

George Polya (1887-1985) was a Hungarian mathematician who immigrated to the United States in 1940. His major contribution is for his work in problem solving. He was the first to introduce the concept of problem-solving model. He is popularly known as the "*Father of Problem Solving"*.

In 1945, Polya published the book "*How To Solve It"* which quickly became his most prized publication. It sold over one million copies and has been translated into 17 languages. In this he identifies four basic principles that uses the 3R's of problem solving i.e. request-response-result and a verification of the result. These are **:**

(a) Understand the problem

(b) Devise a plan

(c) Carry out the plan

(d) Look back.

Each of these steps are considered as separate skills and each step is categorized into sub skills. These separate skills or sub skills are also termed as heuristics or strategies. This four-step process forms the basis of any serious attempt at problem solving and is popularly termed as ***"Polya's Heuristic Approach To Problem Solving".***

Mathematical Concept Acquisition

Acquisition of mathematical concepts means the same as achievement in mathematics. It refers to the attainment of certain abilities and development of varied skills during the learning process. Achievement is regarded as the end product of all educational endeavors (Balasubramanayan,1997). It is considered as the sole criteria to evaluate the successful accomplishment of performance in a particular subject, area and course, usually visualized through skills, hard work and interest, typically summarized in various types of grades, marks, scores or descriptive commentary (Hawes & Hawes, 1982). *Here achievement in mathematics and acquisition of mathematical concepts are interchangeable terms and were considered as mean gain scores obtained by the students on mathematical concept acquisition test.*

OBJECTIVE

- To investigate the significance of difference in acquisition of mathematical concepts of the groups taught through Problem Solving Instructional Strategy and Traditional Instructional Strategy.

HYPOTHESIS

- There exists no significant difference in the mean scores on the acquisition of mathematical concepts of the groups taught through Problem Solving Instructional Strategy and Traditional Instructional Strategy.

SAMPLE

The sample in the present study was drawn at two levels such as school sample and the student sample. A sample of 320 students of 9^{th} class was selected through the multi-stage sampling technique.

EXPERIMENTAL DESIGN

The present study aimed at studying the effectiveness of PSIS on acquisition of mathematical concepts among group of students. For this purpose, " Pre-test-Post-test Design" was used. The variable of instructional strategy was studied at two levels as - Problem Solving Instructional Strategy (PSIS) and Traditional Instructional Strategy (TIS).

TOOLS USED

1. Mathematical Concept Acquisition Test (MCAT) on selected topics of Mathematics of class IX (developed by the investigator).
2. Modules based on Problem Solving Instructional Strategy (PSIS) on selected topics of Mathematics for Class IX (prepared by the investigator).
3. Lesson Plans based on Traditional Instructional Strategy (TIS) on selected topics of Mathematics for Class IX (prepared by the investigator).

PROCEDURE

The procedure for carrying out the present investigation involved two stages :

- Sample selection and allocation of students into two groups for instructional strategies
- Conducting the experiment

Sample Selection

The sample in the present study was drawn at two levels-the school sample and the student sample.

Conducting the Experiment

The experiment was conducted in three phases. The schematic representation of the treatment procedure is given in the table 1.

Table 1: Schematic Representation of the Treatment Procedure

Phase	Group - A_1	Group - A_2
Phase-I (Pre-test)	1. Test of Mathematical Concept Acquisition as a Pre-test. 2. Learning Style Inventory	1. Test of Mathematical Concept Acquisition as a Pre-test. 2. Learning Style Inventory
Phase-II (Treatment)	Exposure through Problem Solving Instructional Strategy.	Exposure through Traditional Instructional Strategy.
Phase-III (Post-test)	Test of Mathematical Concept Acquisition as a Post-test.	Test of Mathematical Concept Acquisition as a Post-test.

Table1 depicts teaching strategy designated as A and its two strategies exposure through problem solving instructional strategy and traditional instructional strategy as A_1 and A_2 respectively..

STATISTICAL TECHNIQUE EMPLOYED

t-test was employed to find out the significance of difference between means related to different groups and variables.

ANALYSIS AND INTERPRETATION

There exists no significant difference in the mean scores on the acquisition of mathematical concepts of the groups taught through Problem Solving Instructional Strategy and Traditional Instructional Strategy:

Table 2 : Significance of Difference Between Mean Gain Scores of Experimental and Control group

Group↓	Gain Scores				
	N	**M**	**SD**	**$S.E_D$**	**t-value**
Experimental Group	160	11.06	2.227	.234	22.08*
Control Group	160	5.89	1.952		

*significant at 0.01 level of significance

($t_{(table)}$ is 1.96 at 0.05 level and 2.58 at 0.01 level of significance, df=318)

The results presented in table 2 reveals that t-ratio for the difference between the mean gain scores on the variable of acquisition of mathematical concepts of the groups taught through PSIS and TIS came out to be 22.08 which in comparison to the table value was found to be highly significant at 0.01 level of significance as $p < 0.01$. Hence, the null hypotheses stating , "There exists no significant difference in the mean gain scores on the acquisition of mathematical concepts of the groups taught through Problem Solving Instructional Strategy and Traditional Instructional Strategy" stands rejected at specified level.

The means of the groups taught through PSIS and TIS on the variable of acquisition of mathematical concepts were found to be 11.06 and 5.89 respectively and the mean gain is in favour of group taught through PSIS meaning thereby that the students taught by PSIS achieved more as compared to those who taught through TIS. It means that both the groups were significantly different on the mean gain scores on the variable of acquisition of mathematical concepts.

The findings of the present study go in line with studies conducted by Russel and Chiapetta (1981); Greenbowe (1984); Dugger (1985); Dutt (1987) ; Conlon (1991); Dash (1996); Ayodhya (2007); Chavez (2007) ; Selcuk, Caliskan and Erol (2008); Aka, Guven and Mustafa (2010); Ali, Akhter and Khan (2010); Kousar (2010); Muhammad (2011); Michael, Salman and Ayinla (2012); Hoon, Kee and Singh (2013) and Novotna, Eisenmann, Pribyl, Ondrusova and Brehovsky (2014) who found that students taught through PSIS achieved significantly higher as compared to those taught through conventional mode.

FINDINGS AND CONCLUSIONS

The t-ratio for the difference between the mean gain scores on the variable of acquisition of mathematical concepts of the groups taught through PSIS and TIS came out to be 22.08 which is significant at 0.01 level of confidence. There was significant difference in the means of the two groups {M (PSIS)=11.06} and {M(TIS)=5.89} and the mean gain was in favour of group taught through PSIS. Thus it can be concluded that students taught through PSIS were found to achieve significantly high on the acquisition of mathematical concepts than taught through TIS. Thus PSIS proved to be a better instructional strategy over TIS.

IMPLICATIONS

On the basis of the findings of the study, it is strongly recommended that PSIS should be used to enhance quality of education at school level and even at college level. As we all know that the performance of the students in mathematics is not all satisfactory at every level of education. This is due to the reason that students lack conceptual understanding of the subject and they are also poor in higher order thinking skills. Much has been written about the ways of improving mathematical concepts of the students. But , efforts made in reality are far from satisfactory. The results of the present study showed that

PSIS modules enhanced the concept acquisition of students in mathematics. Polya's method centered around 'doing' mathematics with great emphasis on teaching students how to think mathematically. The use of different heuristics, principles and self-directed questions (suggested by Polya) arouses and sustain their interest in mathematics and helped them to explore their innate intellectual abilities. Moreover, the stage-wise problem solving helped the students to become familiar with the problem- solving process which thereby leads to better acquisition of mathematical concepts.

Therefore students should be taught mathematics with the help of this method and mathematics pupil teachers should be trained for using Polya's problem solving approach. The present investigation will help the students in the following ways :

- ✓ Explaining Polya's four principles of problem solving
- ✓ Enlisting the various problem solving strategies and heuristics for solving different problems and further selecting a particular strategy to tackle a problem
- ✓ Apply a variety of appropriate heuristic questions to solve problems
- ✓ Developing concept clarity and thereby leading to better understanding of the subject matter
- ✓ Enhancing interest in learning mathematics by active participation and at their own pace
- ✓ Boosting confidence by improving their conceptual clarity and using mathematics meaningfully
- ✓ Monitor and reflect on the process of mathematical problem solving
- ✓ Build new mathematical knowledge and concepts through problem solving and helping them in formulating generalisations
- ✓ Discouraging the concept of rote memorisation in learning of mathematics that will help in retaining the content for a longer period of time

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