The comparative study of the Laplace Transform and New Integral Transform Elzaki Transform

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Abstract

In this paper, a new integral transform Elzaki transform is used to solve the linear ordinary differential equations. We compare the result of Elzaki transform with the well known integral transform the Laplace transforms.

KEYWORD: Integral Transform, Elzaki transform, linear differential equations, Laplace Transform.

1. Introduction:

The differential equation plays a central role in Applied Mathematics, Physics and in Engineering. Linear and Nonlinear Partial differential equations are generally difficult to be solved and their exact solution are difficult to be obtained. The exact solution and numerical solutions of this kind of equations play an important role in physical science and in engineering fields; therefore, there have been attempts to develop new techniques for obtaining analytical solutions which reasonably approximate the exact solutions.

In order to solve the differential equations, the integral transforms were extensively used. The importance of an integral transform is that they provide powerful operational methods for solving initial value problems and initial- boundary value problem for linear differential and integral equations. There are several methods like Laplace transform, Fourier transforms, Mellin Transforms, Series Methods, etc are available to solve linear differential equations [1].

The New Integral transform “Elzaki Transform” was first introduce by Tarig Elzaki in [2] and used heavily in the literature in [2-4]. The main objective is to introduce a comparative study to solve linear differential equations by using Laplace transform and Elzaki transform. The plan of the paper is as follows: In section 2, we introduce the basic idea of Elzaki transform, some properties in 3, then Application in 4 and conclusion in 5, respectively.

2. Elzaki Transform:

A new integral transform called the Elzaki transform defined for the function of exponential order, we consider functions in the set A defined by [2]:

$$A=\left\{f(t): \exists M, k_1, k_2>0, |f(t)|<Me^{\frac{|t|}{k_j}}, if\ t\in(-1)^j X[0,\infty)\right\} \quad (1)$$

For a given function in the set A, the constant M must be finite number, k_1, k_2 may be finite or infinite.

The Elzaki transform of the function $f(t)\in A$, denoted by $E[f(t)] or\ T(v)$, is defined by the integral equation

$$E[f(t)]=T(v)=v\int_0^{\infty} f(t)e^{\frac{-t}{v}}dt,\ t\geq 0, k_1\leq v\leq k_2 \quad (2)$$

Elzaki Transform of some functions:

For any function $f(t)$, we assume that the integral equation (2) exists. The Sufficient Conditions for the existence of Elzaki transform is that $f(t)$ for $t\ \geq\ 0$ be piecewise

Continuous and of exponential order, otherwise Elzaki transform may or may not exist.

The Elzaki transform of some simple functions are as follows:

i) If $f(t) = 1$ then $E(1) = v\int_0^\infty e^{\frac{-t}{v}}\,dt = v\left[-ve^{\frac{-t}{v}}\right]_0^\infty = v^2$

ii) If $f(t) = t$ then $E(t) = v\int_0^\infty te^{\frac{-t}{v}}\,dt = v^3$by integration by parts.
In general, $E(t^n) = n!\,v^{n+2}\ \forall\ n = 1,2,3,\ldots$

iii) If $f(t) = e^{at}$ then $E(t) = v\int_0^\infty e^{at}e^{\frac{-t}{v}}\,dt = \frac{v^2}{1-av}$ by integration by parts.

iv) Similarly $E(sinat) = \frac{av^3}{1+a^2v^2}$, $E(cosat) = \frac{v^2}{1+a^2v^2}$,
$E(sinhat) = \frac{av^3}{1-a^2v^2}$, $E(coshat) = \frac{v^2}{1-a^2v^2}$

3. Basic Properties of Elzaki Transform:

Theorem (3-1): If $E[f(t)] = T(v)$ then [3-5]

i. $E[f'(t)] = \frac{T(v)}{v} - vf(0)$

ii. $E[f''(t)] = \frac{T(v)}{v^2} - f(0) - vf'(0)$

iii. $E\left[f^{(n)}(t)\right] = \frac{T(v)}{v^n} - \sum_{k=0}^{n-1} v^{2-n+k} f^{(k)}(0)$

Theorem (3-2): If $E[f(t)] = T(v)$ then [3-5]

i. $E[tf(t)] = v^2\frac{dT}{dv} - vT(v)$

ii. $E[t^2f(t)] = v^4\frac{d^2T}{dv^2}$

Theorem (3-3): If $E[f(t)] = T(v)$ then [3-5]

i. $E[tf'(t)] = v^2\frac{d}{dv}\left[\frac{T(v)}{v} - vf(0)\right] - v\left[\frac{T(v)}{v} - vf(0)\right]$

ii. $E[tf''(t)] = v^2\frac{d}{dv}\left[\frac{T(v)}{v^2} - f(0) - vf'(0)\right] - v\left[\frac{T(v)}{v^2} - f(0) - vf'(0)\right]$

iii. $E[t^2f''(t)] = v^4\frac{d^2}{dv^2}\left[\frac{T(v)}{v^2} - f(0) - vf'(0)\right]$

Theorem (3-4): If $E[f(t)] = T(v)$ then [3-5]

$$E[e^{at}f(t)] = \frac{1}{1-av}T\left[\frac{v}{1-av}\right]$$

4. Application:

As stated in introduction the Elzaki transform can be used as an effective tool for solving ordinary differential equations. The following examples illustrate the application of Elzaki Transform to solve ordinary differential equations with initial conditions and such a result is verified by well-known transform, the Laplace Transform.

4.1.Example 1: consider the first order ordinary differential equation $\frac{dy}{dt} - y = t^2, y(0) = 0$

4.1.1. Solution by Elzaki Transform:

Let $E[y(t)] = T(v)$, then apply Elzaki transform to given equation we get

$E\left[\frac{dy}{dt}\right] - E[y] = E(t^2)$,

$\frac{T(v)}{v} - vy(0) - T(v) = 2v^4$,

$T(v) = \frac{2v^5}{1-v}$,

$T(v) = 2\left[\frac{v^2}{1-v} - v^4 - v^3 - 2v^2\right]$,

Apply Inverse Elzaki Transform we get,

$y(t) = 2e^t - t^2 - 2t - 2.$

4.1.2. Solution by Laplace Transform:

Let $L[y(t)] = f(s)$, then apply Laplace transform to given equation we get

$L\left[\frac{dy}{dt}\right] - L[y] = L(t^2)$,

$sf(s) - y(0) - f(s) = \frac{2}{s^3}$

$f(s) = \frac{2}{s^3(s-1)}$, by partial fraction method we get

$f(s) = \frac{2}{s-1} - \frac{2}{s^3} - \frac{2}{s^2} - \frac{2}{s}$

Apply inverse Laplace Transform we get

$y(t) = 2e^t - t^2 - 2t - 2.$

4.2.Example 2: Consider the second order ordinary differential equation $y''(t) + y(t) = 6cos2t$ with $y(0) = 3, y''(0) = 1$

4.2.1. Solution by Elzaki Transform:

$\frac{T(v)}{v^2} - y(0) - vy'(0) + T(v) = \frac{6v^2}{1+4v^2}$

$T(v) = \frac{6v^4}{(1+4v^2)(1+v^2)} + \frac{3v^2+v^3}{1+v^2}$

$T(v) = v^2\left[\frac{6v^2}{(1+4v^2)(1+v^2)} + \frac{3+v}{1+v^2}\right]$

$T(v) = v^2\left[\frac{-2}{(1+4v^2)} + \frac{2}{1+v^2} + \frac{3+v}{1+v^2}\right]$

$T(v) = \frac{-2v^2}{(1+4v^2)} + \frac{5v^2}{1+v^2} + \frac{v^3}{1+v^2}$

Taking inverse transform we get,

$y(t) = 5cost + sint - 2cos2t$

4.2.2. Solution by Laplace Transform:

$s^2f(s) - sy(0) - y'(0) + f(s) = \frac{6s}{s^2+4}$

$f(s) = \frac{5s}{s^2+1} + \frac{1}{s^2+1} - \frac{2s}{s^2+4}$

$y(t) = 5cost + sint - 2cos2t$

4.3. Example 3: Consider the system of ordinary differential equations $\frac{dx}{dt} = 2x - 3y$, $\frac{dy}{dt} = y - 2x$, with $x(0) = 8, y(0) = 3$.

4.3.1. Solution by Laplace Transform:

Let $L\{x(t)\} = X(s)$ *and* $L\{y(t)\} = Y(s)$ then taking Laplace transform we get,

$sX - 8 = 2X - 3Y$ & $sY - 3 = Y - 2X$

$(s-2)X + 3Y = 8$ & $2X + (s-1)Y = 3$

Solving these two equations we get

$X(s) = \frac{8s-17}{s^2-3s-4} = \frac{5}{s+1} + \frac{3}{s-4}$

$Y(s) = \frac{3s-22}{s^2-3s-4} = \frac{5}{s+1} - \frac{2}{s-4}$

Taking inverse Laplace transform we get,

$x(t) = 5e^{-t} + 3e^{4t}$& $y(t) = 5e^{-t} - 2e^{4t}$

4.3.2. Solution by Elzaki Transform:

Let $E[x(t)] = X(v)$ & $E[y(t)] = Y(v)$ then taking Elzaki transform we get

$\frac{X}{v} - vx(0) = 2X - 3Y$ & $\frac{Y}{v} - vy(0) = Y - 2X$

$$X\left(\frac{1-2v}{v}\right) - 3Y = 8v \;\&\; 2X + Y\left(\frac{1-v}{v}\right) = 3v$$

Solving these two equations simultaneously we get

$X(v) = \frac{-v^2(8+v)}{8v^2-3v+1}$ and

$Y(v) = \frac{-v^2(3-22v)}{8v^2-3v+1}$

Taking inverse Elzaki transform we can find values of $x(t)$ and $y(t)$, but it is much more complicated.

5. Conclusion:

The main goal of this paper is to conduct a comparative study between new Elzaki transform and well known Laplace transform. Elzaki transform gives a new tool to solve ordinary differential equations with initial condition.

An important conclusion can made here. The Elzaki transform method for solving ordinary differential equations as compare to Laplace transform requires more calculations and hence time consuming. Hence such a method is not useful for practical method.

References:

[1] Lokenath Debnath and D. Bhatta., Integral Transform and their Applications, 2nd edition, Chapman & Hall/CRC(2006)

[2] Tarig M. Elzaki, The New Integral Transform “Elzaki Transform”, Global Journal of Pure and Applied Mathematics, ISSN 0973-1768 Volume 7, Number 1(2011),pp 57-64.

[3] Tarig M. Elzaki, Salil M. Elzaki & E.A. Elnour, On the New Integral Transform “ Elzaki Transform” Fundamental Properties Investigations and Applications, Global Journal of Mathematical Sciences: Theory and Practical, ISSN 0974-3200 Volume 4, Number 1(2012),pp 1-13

[4] Tarig M. Elzaki & Salil M. Elzaki, On the connection between Laplace and Elzaki Transforms, Advances in Theoretical and Applied Mathematics, ISSN 0973-4554 volume 6, Number 1(2011),pp.1-10.

[5] Tarig M. Elzaki & Salil M. Elzaki, Application of New Integral Transform “Elzaki Transform” to Partial Differential Equations, Global Journal of Pure and Applied Mathematics, ISSN 0973-1768 Volume 7, Number 1(2011),pp 65-70.

[6] Badradeen A.A. Adam, A comparative study of Adomain decomposition method and the new integral transform “ Elzaki transform “, International Journal of Applied Mathematical Research, 4(1)(2015),pp.8-14.

[7] Murray R. Spiegel, Laplace Transforms, Schaum’s Outline Series(1965)