C_{5n}*K_1- Minimal 10-Equitablel;for n-even

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Abstract

Every labeling of the vertices of a graph with distinct natural numbers induces a natural labeling of its edges: the label of an edge uv is the absolute value of the difference of the labels of u and v. A labeling of the vertices of a graph of order p is minimally k-equitable if the vertices are labeled with 1,2……,p and in the induced labeling of its edges every label either occurs exactly k times or does not occur at all. Here it is proven that the corona graphs C_{5n}*K_1 are minimally 10-equitable when n is even.

KEYWORDS: Corona graphs, Labeling of graphs, Minimal k-equitable graph.

INTRODUCTION

A labeling of the vertices of a graph G is an assignment of distinct natural numbers to the vertices of G. Every labeling induces a natural labeling of the edges: the label of an edge uv is the absolute value of the difference of the labels of u and v. Bloom[2]defined a labeling of the vertices of a graph to be k-equitable if in the induced labeling of its edges, every label occurs exactly k-times, if at all, furthermore, a k-equitable labeling of a graph of order p is said to be minimal if the vertices are labeled with 1,2,……p. A graph is minimally k-equitable if it has a minimal k-equitable labeling.

Bloom posed the following question: is the condition that k is a proper divisor of p sufficient for the cycle C_p to have a minimal k-equitable labeling? Wojciechowski[4]gave a positive answer to this question. Barrientos, Dejter and Hevia [1] have shown that forests of even size are 2-equitable. They also prove that for k=3 or k=4 a forest F of size kw is k-equitable if and only if the maximum degree F is at most 2w and that if 3 divides the size of the double star $S_{m,n}(1 \le m \le n)$, then $S_{m,n}$ is 3-equitable if and only if $q/3 \le m \le [(q-1)/2]$. Here $S_{m,n}$ is k_2 with m pendant edges attached at one end and n pendant edges attached at the other end. They discuss the k-equitability of forests for $k \ge 5$ and characterize all caterpillars of diameter 2 that are k-equitabile for all possible values of k.

The corona G_1*G_2 of two graphs G_1 and G_2 was defined by Frucht and Harary[3]as the graph G obtained by taking one copy of G_1 with p_1 vertices and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

Here, it is proven that the corona graphs C_{5n}*K_1 are minimally 10-equitable when n is even.

MAIN RESULT

THEOREM : The graphs C_{5n}*K_1 are minimally 10-equitable,when n is even.

Proof : Here $p(C_{5n}*K_1) = 10n = q(C_{5n}*K_1)$. It is proven that this graph is minimally 10-equitable with edge-weight set W = {1,2,…..,n} for even values of n.

Let $V(C_{5n}*K_1) = \{u_1, u_2, \ldots u_{5n}; v_1, v_2, \ldots v_{5n}\}$ where $u_1, u_2, \ldots u_{5n}$ is the cycle C_{5n} and v_i is the pendant vertex adjacent to u_i, $1 \le i \le 5n$.

Here the labeling function f is defined for $n \ge 12$. For the cases n= 2,4,6,8,10 a labeling function is obtained differently. Those are mentioned separately at the end.

A labeling function is a bijective function f: V(G) → {1,2,3,…….,10n} . Define f by dividing graph into 10-parts,viz,Part I for u_i, v_i, $1 \le i \le \frac{n}{2}$, Part II for u_i, v_i, $\frac{n}{2}+1 \le i \le n$,

Part III for u_i, v_i, $n+1 \le i \le (\frac{3n}{2})$ and so on…..Part X for u_i, v_i, $\frac{19n}{2}+1 \le i \le 10n$.

The labeling function for Part I is :

$f(u_{2i-1}) = n+1-2(i-1)$; $1 \le i \le \frac{n}{4}$ when $n \equiv 0 \pmod 4$ and $1 \le i \le [\frac{n}{4}]+1$ when $n \equiv 2 \pmod 4$

$f(v_{2i-1}) = 2i-1$; $1 \le i \le \frac{n}{4}$ when $n \equiv 0 \pmod 4$ and $1 \le i \le [\frac{n}{4}]+1$ when $n \equiv 2 \pmod 4$

$f(u_{2i}) = 2i$; $1 \le i \le \frac{n}{4}$ and $f(v_{2i}) = n-2(i-1)$; $1 \le i \le \frac{n}{4}$

From Part II to Part VI labeling function is defined by considering Two cases corresponding to $n \equiv 0,2 \pmod 4$.

Case : n≡ 0 (mod 4)

In Part II, a labeling function is defined by considering Three sub-parts, viz , S_1, S_2 and S_3

Sub-part S_1: It contains only two vertices $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}+1}$. The labeling function is

$$f(u_{\frac{n}{2}+1})=\frac{n}{2}+1 \text{ and } f(v_{\frac{n}{2}+1})=\frac{3n}{2}+1$$

Sub-part S_2 : The labeling function is

$f(u_{\frac{n}{2}+2i})=\frac{3n}{2}-2(i-1)$, $1 \le i \le \frac{n}{8}$ when $n \equiv 0 \pmod 8$ and $1 \le i \le [\frac{n}{8}]+1$ when $n \equiv 4 \pmod 8$

$f(v_{\frac{n}{2}+2i})=\frac{3n}{2}+2i$, $1 \le i \le \frac{n}{8}$ when $n \equiv 0 \pmod 8$ and $1 \le i \le [\frac{n}{8}]+1$ when $n \equiv 4 \pmod 8$

$f(u_{\frac{n}{2}+2i+1})=\frac{3n}{2}+2i+1$ and $f(v_{\frac{n}{2}+2i+1})=\frac{3n}{2}-(2i-1)$ $1 \le i \le [\frac{n}{8}]$.

Sub-part S_3 : Here the labeling function is defined by considering two sub-cases n≡ 0 (mod 8) and n≡ 4 (mod 8).

For n≡ 0 (mod 8) the function is:

$f(u_{\frac{3n}{4}+2i})= \frac{7n}{4} + 2i$ and $f(v_{\frac{3n}{4}+2i})= \frac{5n}{4} - 2(i-1)$;$1 \le i \le \frac{n}{8}$

$f(u_{\frac{3n}{4}+2i+1})= \frac{5n}{4} - (2i-1)$ and $f(v_{\frac{3n}{4}+2i+1})= \frac{7n}{4} + 2i + 1$;$1 \le i \le \frac{n}{8} - 1$

For n≡ 4 (mod 8) the function is:

$f(u_{\frac{3n}{4}+2i})= \frac{5n}{4} - 2(i-1)$, $f(v_{\frac{3n}{4}+2i})= \frac{7n}{4} + 2i$, $f(u_{\frac{3n}{4}+2i+1})= \frac{7n}{4} + 2i + 1$ and

$f(v_{\frac{3n}{4}+2i+1})= \frac{5n}{4} - (2i-1)$ for $1 \le i \le [\frac{n}{8}]$

Then there is parity in labeling function from Part II to Part V. Labeling function for Part III is obtained by adding 2n to respective labels of Part II, Labeling function for Part IV by adding 2n to respective labels of Part III and so on as shown in Figure 1.

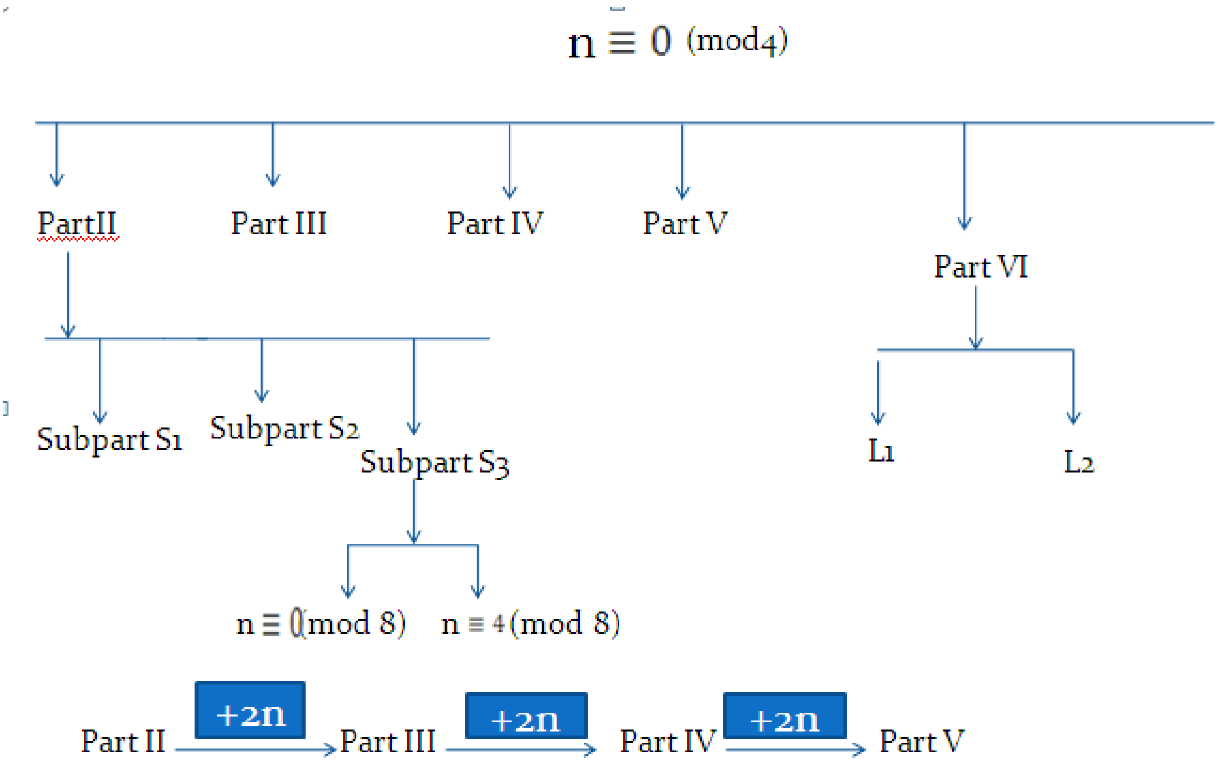

Figure 1

Part VI : It contains vertices u_i, v_i, $\frac{5n}{2} + 1 \le i \le 3n$ and labeling function is defined by considering Two sub-parts L_1 and L_2.

Sub-part L_1: It contains 2 vertices $u_{\frac{5n}{2}+1}$ and $v_{\frac{5n}{2}+1}$. The labeling function is

$f(u_{\frac{5n}{2}+1})=\frac{17n}{2}+1$, $f(v_{\frac{5n}{2}+1})=\frac{19n}{2}+1$

Sub-part L_2 :

$f(u_{\frac{5n}{2}+2i})=\frac{19n}{2}-2(i-1)$ and $f(v_{\frac{5n}{2}+2i})=\frac{19n}{2}+2i$; $1\leq i\leq\frac{n}{4}$

$f(u_{\frac{5n}{2}+2i+1})=\frac{19n}{2}+2i+1$ and $f(v_{\frac{5n}{2}+2i+1})=\frac{19n}{2}-(2i-1)$; $1\leq i\leq\frac{n}{4}-1$

As mentioned earlier from Part II to Part VI labeling function is defined by considering Two cases corresponding to $n\equiv 0,2 \pmod 4$. So here is the second case:

<u>Case : n≡ 2 (mod 4) :</u>

In Part II, a labeling function is defined by considering Four sub-parts, viz , S_1', S_2' , S_3' and S_4'

Sub-part S_1': It contains only two vertices $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}+1}$. The labeling function is

$$f(u_{\frac{n}{2}+1})=\frac{n}{2}+1 \text{ and } f(v_{\frac{n}{2}+1})=\frac{3n}{2}$$

Sub-part S_2' : This Sub-part contains six vertices u_i , v_i ; $\frac{n}{2}+2\leq i\leq\frac{n}{2}+4$ The labeling function is : $f(u_{\frac{n}{2}+2})=\frac{3n}{2}+1$, $f(v_{\frac{n}{2}+2})=\frac{3n}{2}+3$,

$$f(u_{\frac{n}{2}+3})=\frac{3n}{2}-2 \text{ , } f(v_{\frac{n}{2}+3})=\frac{3n}{2}+2,$$

$$f(u_{\frac{n}{2}+4})=\frac{3n}{2}+4 \text{ and } f(v_{\frac{n}{2}+4})=\frac{3n}{2}-1$$

Sub-part S_3' : The labeling function f is defined as :

$f(u_{\frac{n}{2}+3+2i})=\frac{3n}{2}-(2i+1)$, $1\leq i\leq\left[\frac{n}{8}\right]-1$ when $n\equiv 2 \pmod 8$ and

$$1\leq i\leq\left[\frac{n}{8}\right] \text{ when } n\equiv 6 \pmod 8$$

$f(v_{\frac{n}{2}+3+2i})=\frac{3n}{2}+3+2i$, $1\leq i\leq[\frac{n}{8}]-1$ when $n\equiv 2 \pmod 8$ and

$$1\leq i\leq[\frac{n}{8}] \text{ when } n\equiv 6 \pmod 8$$

$f(u_{\frac{n}{2}+4+2i})=\frac{3n}{2}+4+2i$ and $f(v_{\frac{n}{2}+4+2i})=\frac{3n}{2}-2(i+1)$, $1\leq i\leq[\frac{n}{8}]-1$.

Sub-part S_4' : In this sub-part the labeling function is different for $n\equiv 2 \pmod 8$ and n $\equiv 6 \pmod 8$

When n $\equiv 2 \pmod 8$ the labeling function is :

$f(u_{\frac{3n+2}{4}+2i})=\frac{7n+2}{4}+2i$ and $f(v_{\frac{3n+2}{4}+2i})=\frac{5n+2}{4}-(2i-1)$, $1\leq i\leq[\frac{n}{8}]$

$f(u_{\frac{3(n+2)}{4}+2i})=\frac{5n+2}{4}-2i$ and $f(v_{\frac{3(n+2)}{4}+2i})=\frac{7n+6}{4}+2i$, $1\leq i\leq[\frac{n}{8}]-1$

When n $\equiv 6 \pmod 8$ the labeling function is defined for $1\leq i\leq[\frac{n}{8}]$:

$f(u_{\frac{3n+2}{4}+2i})=\frac{5n+2}{4}-(2i-1)$, $f(v_{\frac{3n+2}{4}+2i})=\frac{7n+2}{4}+2i$

$f(u_{\frac{3(n+2)}{4}+2i})=\frac{7n+6}{4}+2i$, $f(v_{\frac{3(n+2)}{4}+2i})=\frac{5n+2}{4}-2i$

In Part III, a labeling function is defined by considering Three sub-parts, viz , T_1', T_2' and T_3'

Sub-part T_1': It contains only two vertices u_{n+1} and v_{n+1} . The labeling function is

$$f(u_{n+1})=\frac{5n}{2}+2 \text{ and } f(v_{n+1})=\frac{7n}{2}+1$$

Sub-part T_2' : The labeling function f is defined as :

$f(u_{n+2i})=\frac{7n}{2}+2i$ and $f(v_{n+2i})=\frac{7n}{2}-2(i-1)$, $1\leq i\leq[\frac{n}{8}]+1$

$f(u_{n+2i+1})=\frac{7n}{2}-(2i-1)$, $1\leq i\leq\left[\frac{n}{8}\right]$ when n $\equiv 2 \pmod 8$ and

$1\leq i\leq[\frac{n}{8}]+1$ when n$\equiv 6 \pmod 8$

$f(v_{n+2i+1})=\frac{7n}{2}+2i+1$, $1\leq i\leq[\frac{n}{8}]$ when n $\equiv 2 \pmod 8$ and

$1\leq i\leq[\frac{n}{8}]+1$ when n$\equiv 6 \pmod 8$

Sub-part T_3' : In this sub-part the labeling function is different for n $\equiv 2 \pmod 8$ and

n $\equiv 6 \pmod 8$

When n $\equiv 2 \pmod 8$ the labeling function is :

$f(u_{\frac{5n+2}{4}+2i})=\frac{15n+2}{4}+2i$ and $f(v_{\frac{5n+2}{4}+2i})=\frac{13n+6}{4}-2i$, $1\leq i\leq[\frac{n}{8}]$

$f(u_{\frac{5n+6}{4}+2i})=\frac{13n+2}{4}-2i$ and $f(v_{\frac{5n+6}{4}+2i})=\frac{15n+6}{4}+2i$, $1\leq i\leq[\frac{n}{8}]-1$

When n $\equiv 6 \pmod 8$ the labeling function is defined for $1\leq i\leq[\frac{n}{8}]$:

f($u_{\frac{5n+2}{4}+2i}$)= $\frac{13n+6}{4}-2i$, f($v_{\frac{5n+2}{4}+2i}$)= $\frac{15n+2}{4}+2i$

f($u_{\frac{5n+6}{4}+2i}$)= $\frac{15n+6}{4}+2i$, f($v_{\frac{5n+6}{4}+2i}$)= $\frac{13n+2}{4}-2i$

Labeling function for Part IV and Part V is obtained by adding 2n and 4n respectively to that of respective labels of Part II

Figure 2 briefs the pattern of labeling function in Case : n≡ 2 (mod 4)

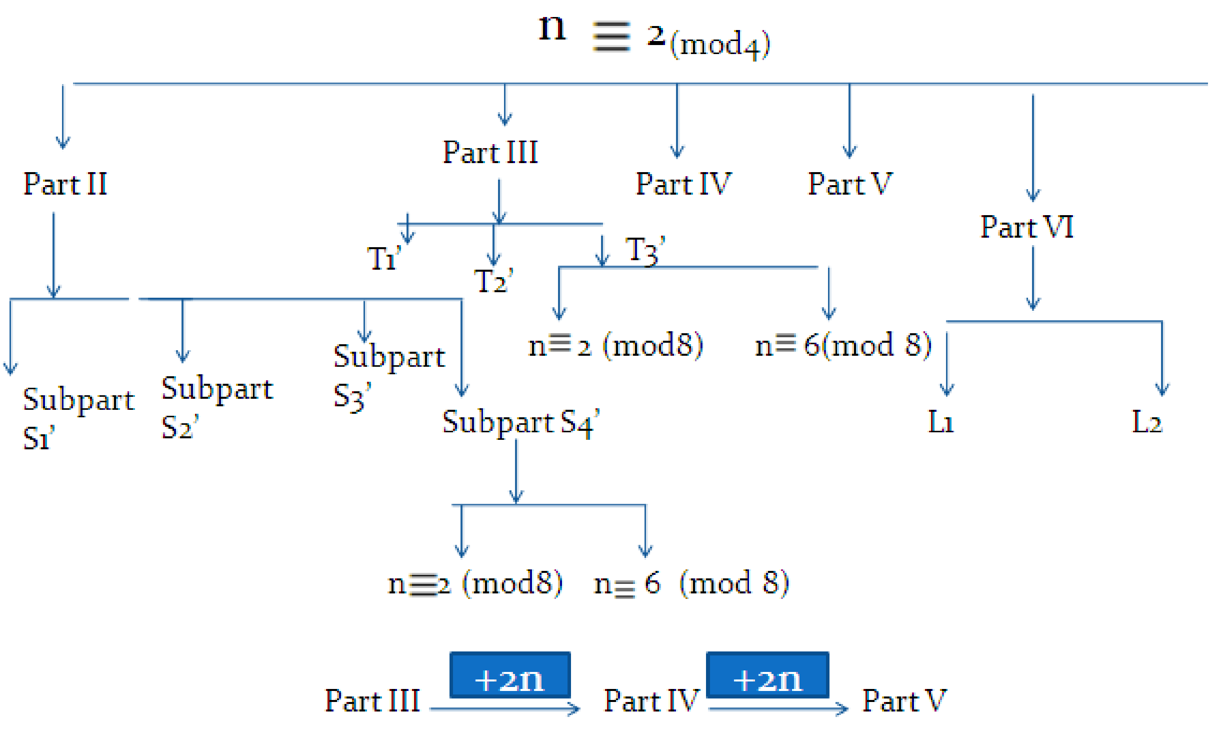

Figure 2

Part VI : The labeling function is defined by considering Two sub-parts L_1' and L_2'

Sub-part L_1': It contains 2 vertices $u_{\frac{5n}{2}+1}$ and $v_{\frac{5n}{2}+1}$. The labeling function is

f($u_{\frac{5n}{2}+1}$)= $\frac{17n}{2}+2$, f($v_{\frac{5n}{2}+1}$)= $\frac{19n}{2}+1$

Sub-part L_2' :

f($u_{\frac{5n}{2}+2i}$)= $\frac{19n}{2}+2i$ and f($v_{\frac{5n}{2}+2i}$)= $\frac{19n}{2}-2(i-1)$;

f($u_{\frac{5n}{2}+2i+1}$)= $\frac{19n}{2}-(2i-1)$ and f($v_{\frac{5n}{2}+2i+1}$)= $\frac{19n}{2}+2i+1$;

for $1 \leq i \leq [\frac{n}{4}]$

Part VII : This part contains vertices u_i, v_i, $3n + 1 \leq i \leq \frac{7n}{2}$. Here consider four cases corresponding to n≡ 0,2,4,6(mod 8),n≠14,18,20,22.For n=14,18,20,22 labeling function is slightly different. The case n=14 is mentioned separately at the end and for n=18,20,22 can be obtained in similar way

<u>Case : n≡ 0 (mod 8)</u> : In this case Part VII is divided into four Sub-parts A1,A2,A3 and A4

Sub-part A1: It contains six vertices u_i, v_i, $3n + 1 \leq i \leq 3n+3$. The labeling function is :

$f(u_{3n+1}) = 9n + 1$, $f(v_{3n+1}) = \frac{35n}{4}$; $f(u_{3n+2}) = \frac{17n}{2} + 2$, $f(v_{3n+2}) = \frac{17n}{2} - 1$

$f(u_{3n+3}) = \frac{17n}{2} - 2$ and $f(v_{3n+3}) = \frac{17n}{2}$

Sub-part A2: The labeling function is defined as follows:

$f(u_{3n+2i+2}) = \frac{17n}{2} + 2i + 1$, $1 \leq i \leq \frac{n}{16} - 1$, when n≡ 0 (mod 16)

and $1 \leq i \leq [\frac{n}{16}]$, when n≡ 8 (mod 16),

$f(v_{3n+2i+2}) = \frac{17n}{2} - (2i + 1)$, $1 \leq i \leq \frac{n}{16} - 1$, when n≡ 0 (mod 16)

and $1 \leq i \leq [\frac{n}{16}]$, when n≡ 8 (mod 16),

$f(u_{3n+2i+3}) = \frac{17n}{2} - 2(i + 1)$, $1 \leq i \leq [\frac{n}{16}] - 1$,

$f(v_{3n+2i+3}) = \frac{17n}{2} + 2i + 2$, $1 \leq i \leq [\frac{n}{16}] - 1$

Sub-part A3: In this sub-part the labeling function is different for n≡ 0 (mod 16) and n≡ 8 (mod 16).

For n≡ 0 (mod 16) , the labeling function is

$f(u_{\frac{25n}{8}+2i}) = \frac{67n}{8} - (2i - 1)$, $f(v_{\frac{25n}{8}+2i}) = \frac{69n}{8} + 2i - 1$ for $1 \leq i \leq \frac{n}{16}$, and

$f(u_{\frac{25n}{8}+2i+1}) = \frac{69n}{8} + 2i$, $f(v_{\frac{25n}{8}+2i+1}) = \frac{67n}{8} - 2i$, $1 \leq i \leq \frac{n}{16} - 1$.

For n≡ 8 (mod 16) , the labeling function is

$f(u_{\frac{25n}{8}+2i}) = \frac{69n}{8} + 2i - 1$, $f(v_{\frac{25n}{8}+2i}) = \frac{67n}{8} - (2i - 1)$ and f($u_{\frac{25n}{8}+2i+1}$)= $\frac{67n}{8} - 2i$, f($v_{\frac{25n}{8}+2i+1}$)= $\frac{69n}{8} + 2i$,

for 1≤ i ≤ [$\frac{n}{16}$].

Sub-part A4: In this sub-part the labeling function is same for n ≡ 0 (mod 16) and n≡ 8 (mod 16).

f($u_{\frac{26n}{8}+2i-1}$)= $\frac{35n}{4} + 2i - 1$, f($v_{\frac{26n}{8}+2i-1}$)= $\frac{33n}{4} - 2(i-1)$, and

f($u_{\frac{26n}{8}+2i}$)= $\frac{33n}{4} - (2i - 1)$, f($v_{\frac{26n}{8}+2i}$)= $\frac{35n}{4} + 2i$, for 1≤ i ≤ $\frac{n}{8}$.

<u>**Case : n≡ 2 (mod 8)**</u> **:** In this case Part VII is divided into five Sub-parts,namely A1’,A2’,A3’. A4’ and A5’

Sub-part A1’: It contains eight vertices u_i,v_i, $3n + 1$≤ i ≤ 3n+4. The labeling function is :

f(u_{3n+1})= $9n + 1$, f(v_{3n+1})= $\frac{35n+2}{4}$; f(u_{3n+2})= $\frac{17n}{2} + 1$, f(v_{3n+2})= $\frac{17n}{2} - 1$

f(u_{3n+3})= $\frac{17n}{2} - 2$, f(v_{3n+3})= $\frac{17n}{2} + 3$, f(u_{3n+4})= $\frac{17n}{2} + 4$, f(v_{3n+4})= $\frac{17n}{2}$

Sub-part A2’: Labeling function is defined as

f($u_{3n+3+2i}$)= $\frac{17n}{2} - (2i + 1)$, f($v_{3n+3+2i}$)= $\frac{17n}{2} + 3 + 2i$,

f($u_{3n+4+2i}$)= $\frac{17n}{2} + 4 + 2i$, f($v_{3n+4+2i}$)= $\frac{17n}{2} - 2(i + 1)$

for 1≤ i ≤ [$\frac{n}{16}$] − 2 when n≡ 2 (mod 16) and 1≤ i ≤ [$\frac{n}{16}$] − 1 when n≡ 10 (mod 16)

Sub-part A3’: This sub-part contains two vertices. But these two vertices and the labeling function for these vertices are different for n≡ 2 (mod 16) and n≡ 10 (mod 16).

f($u_{\frac{25n+6}{8}}$)= $\frac{67n+10}{8}$, f($v_{\frac{25n+6}{8}}$)= $\frac{67n+2}{8}$ when n≡ 2 (mod 16) and

f($u_{\frac{25n+14}{8}}$)= $\frac{67n-6}{8}$, f($v_{\frac{25n+14}{8}}$)= $\frac{67n+2}{8}$ when n≡ 10 (mod 16)

Sub-part A4’: In this sub-part also vertices and the labeling function are different for n≡ 2 (mod 16) and n≡ 10 (mod 16).

For n≡ 2 (mod 16)

f($u_{\frac{25n-2}{8}+2i}$)= $\frac{69n-10}{8} + 2i$, f($v_{\frac{25n-2}{8}+2i}$)= $\frac{67n+10}{8} - 2i$

f($u_{\frac{25n+6}{8}+2i}$)= $\frac{67n+2}{8} - 2i$, f($v_{\frac{25n+6}{8}+2i}$)= $\frac{69n-2}{8} + 2i$; for 1≤ i ≤ [$\frac{n}{16}$]

For n≡ 10 (mod 16)

$f(u_{\frac{25n+6}{8}+2i}) = \frac{69n-2}{8} + 2i$, $f(v_{\frac{25n+6}{8}+2i}) = \frac{67n+2}{8} - 2i$

$f(u_{\frac{25n+14}{8}+2i}) = \frac{67n-6}{8} - 2i$, $f(v_{\frac{25n+14}{8}+2i}) = \frac{69n+6}{8} + 2i$; for $1 \le i \le [\frac{n}{16}]$

Sub-part A5': In this sub-part the labeling function is same for both n≡ 2 (mod 16) and n≡ 10 (mod 16).It is as follows:

$f(u_{\frac{13n-2}{4}+2i}) = \frac{35n-2}{4} + 2i$, $f(v_{\frac{13n-2}{4}+2i}) = \frac{33n+6}{4} - 2i$

$f(u_{\frac{13n+2}{4}+2i}) = \frac{33n+2}{4} - 2i$, $f(v_{\frac{13n+2}{4}+2i}) = \frac{35n+2}{4} + 2i$; for $1 \le i \le [\frac{n}{8}]$

<u>Case : n≡ 4 (mod 8)</u> : In this case Part VII is divided into five Sub-parts,namely A1",A2",A3", A4" and A5"

Sub-part A1": It contains two vertices u_{3n+1}, v_{3n+1}. The labeling function is :

$f(u_{3n+1}) = 9n + 1$, $f(v_{3n+1}) = \frac{35n}{4} + 1$.

Sub-part A2": In this sub-part there are two sub-cases corresponding to n≡ 4,12 (mod 16) . Here not only the labeling functions but also the number of vertices are different for n≡ 4 (mod 16) and n≡ 12 (mod 16).

Sub-case : n≡ 4 (mod 16) : In this sub-case A2" contains only six vertices u_i, v_i, $3n + 2 \le i \le 3n+4$. The labeling function is :

$f(u_{3n+2}) = \frac{17n}{2}$, $f(v_{3n+2}) = \frac{17n}{2} - 2$, $f(u_{3n+3}) = \frac{17n}{2} - 3$, $f(v_{3n+3}) = \frac{17n}{2} + 2$, $f(u_{3n+4}) = \frac{17n}{2} + 3$,

and $f(v_{3n+4}) = \frac{17n}{2} - 1$

Sub-case : n≡ 12 (mod 16) : In this sub-case the labeling function is :

$f(u_{3n+2i}) = \frac{17n}{2} - 2(i-1)$, $f(v_{3n+2i}) = \frac{17n}{2} + 2i$, $f(u_{3n+2i+1}) = \frac{17n}{2} + 2i + 1$,

$f(v_{3n+2i+1}) = \frac{17n}{2} - (2i-1)$; for $1 \le i \le [\frac{n}{16}]$

Sub-part A3": In this sub-part also there are two sub-cases corresponding to n≡ 4,12 (mod 16) .

Sub-case : n≡ 4 (mod 16) : In this sub-case the labeling function is :

$f(u_{3n+3+2i}) = \frac{17n}{2} - 2(i+1)$, $f(v_{3n+3+2i}) = \frac{17n}{2} + 2(i+1)$, for $1 \le i \le [\frac{n}{16}] - 1$

f($u_{3n+4+2i}$)= $\frac{17n}{2} + 3 + 2i$, f($v_{3n+4+2i}$)= $\frac{17n}{2} - 3 - 2i$; for 1≤ i ≤ [$\frac{n}{16}$] − 2

Sub-case : n≡ 12 (mod 16) : In this sub-case A3" contains only two vertices $u_{\frac{25n+4}{8}}$ and $v_{\frac{25n+4}{8}}$

f($u_{\frac{25n+4}{8}}$)= $\frac{67n+12}{8}$, f($v_{\frac{25n+4}{8}}$)= $\frac{67n+4}{8}$

Sub-part A4": In this sub-part again there are two sub-cases corresponding to n≡ 4,12 (mod 16)

Sub-case : n≡ 4 (mod 16) : In this sub-case the labeling function is :

f($u_{\frac{25n-4}{8}+2i}$)= $\frac{67n+12}{8} - 2i$, f($v_{\frac{25n-4}{8}+2i}$)= $\frac{69n-12}{8} + 2i$; for 1≤ i ≤ [$\frac{n}{16}$] + 1,

f($u_{\frac{25n+4}{8}+2i}$)= $\frac{69n-4}{8} + 2i$, f($v_{\frac{25n+4}{8}+2i}$)= $\frac{67n+4}{8} - 2i$; for 1≤ i ≤ [$\frac{n}{16}$]

Sub-case : n≡ 12 (mod 16) : In this sub-case the labeling function is :

f($u_{\frac{25n-4}{8}+2i}$)= $\frac{69n-12}{8} + 2i$, f($v_{\frac{25n-4}{8}+2i}$)= $\frac{67n+12}{8} - 2i$;

f($u_{\frac{25n+4}{8}+2i}$)= $\frac{67n+4}{8} - 2i$, f($v_{\frac{25n+4}{8}+2i}$)= $\frac{69n-4}{8} + 2i$;

for 1≤ i ≤ [$\frac{n}{16}$] + 1

Sub-part A5": Here the function is same for both n≡ 4 (mod 16) and n≡ 12 (mod 16)

f($u_{\frac{13n}{4}+2i}$)= $\frac{35n}{4} + 2i$, f($v_{\frac{13n}{4}+2i}$)= $\frac{33n}{4} - (2i - 1)$

f($u_{\frac{13n}{4}+2i+1}$)= $\frac{33n}{4} - 2i$, f($v_{\frac{13n}{4}+2i+1}$)= $\frac{35n}{4} + 2i + 1$;

for 1≤ i ≤ [$\frac{n}{8}$]

Case : n≡ 6 (mod 8) : In this case Part VII is divided into four Sub-parts, namely A1''',A2''',A3'''and A4'''

Sub-part A1''': It contains eight vertices u_i,v_i, $3n + 1$≤ i ≤ 3n+4. The labeling function is :

f(u_{3n+1})= $9n + 1$, f(v_{3n+1})= $\frac{35n+6}{4}$; f(u_{3n+2})= $\frac{17n}{2} - 1$, f(v_{3n+2})= $\frac{17n}{2} + 1$

f(u_{3n+3})= $\frac{17n}{2} + 3$, f(v_{3n+3})= $\frac{17n}{2} - 2$, f(u_{3n+4})= $\frac{17n}{2} - 3$, f(v_{3n+4})= $\frac{17n}{2}$

Sub-part A2''': The labeling function is defined as follows:

f($u_{3n+3+2i}$)= $\frac{17n}{2} + 2(i+1)$, 1≤ i ≤ $[\frac{n}{16}] - 1$, f($v_{3n+3+2i}$)= $\frac{17n}{2} - 2(i+1)$, 1≤ i ≤ $[\frac{n}{16}] - 1$,

f($u_{3n+4+2i}$)= $\frac{17n}{2} - 3 - 2i$, 1≤ i ≤ [$\frac{n}{16}$] − 2, when n≡ 6 (mod 16)

and 1≤ i ≤ [$\frac{n}{16}$] − 1, when n≡ 14 (mod 16),

f($v_{3n+4+2i}$)= $\frac{17n}{2} + 3 + 2i$, 1≤ i ≤ $[\frac{n}{16}]$ − 2, when n≡ 6 (mod 16)

and 1≤ i ≤ [$\frac{n}{16}$] − 1, when n≡ 14 (mod 16),

Sub-part A3’’’: In this sub-part the labeling function is different for n≡ 6 (mod 16) and n≡ 14 (mod 16).

For n≡ 6 (mod 16) , the labeling function is

f($u_{\frac{25n-6}{8}+2i}$)= $\frac{69n-14}{8} + 2i$, f($v_{\frac{25n-6}{8}+2i}$)= $\frac{67n+14}{8} - 2i$ and

f($u_{\frac{25n+2}{8}+2i}$)= $\frac{67n+6}{8} - 2i$, f($v_{\frac{25n+2}{8}+2i}$)= $\frac{69n-6}{8} + 2i$,

for 1≤ i ≤ [$\frac{n}{16}$] + 1.

For n≡ 14 (mod 16) , the labeling function is

f($u_{\frac{25n-6}{8}+2i}$)= $\frac{67n+14}{8} - 2i$, f($v_{\frac{25n-6}{8}+2i}$)= $\frac{69n-14}{8} + 2i$ for 1≤ i ≤ [$\frac{n}{16}$] + 2, and

f($u_{\frac{25n+2}{8}+2i}$)= $\frac{69n-6}{8} + 2i$, f($v_{\frac{25n+2}{8}+2i}$)= $\frac{67n+6}{8} - 2i$, 1≤ i ≤ $[\frac{n}{16}]$ + 1.

Sub-part A4’’’: Here the function is same for both n≡ 6 (mod 16) and n≡ 14 (mod 16)

f($u_{\frac{13n+2}{4}+2i}$)= $\frac{35n+2}{4} + 2i$, f($v_{\frac{13n+2}{4}+2i}$)= $\frac{33n+2}{4} - 2i$

f($u_{\frac{13n+6}{4}+2i}$)= $\frac{33n-2}{4} - 2i$, f($v_{\frac{13n+6}{4}+2i}$)= $\frac{35n+6}{4} + 2i$;

for 1≤ i ≤ [$\frac{n}{8}$]

Labeling functions for Part VIII,Part IX and Part X are obtained by subtracting 2n,4n and 6n respectively from that of respective labels of Part VII, that is,

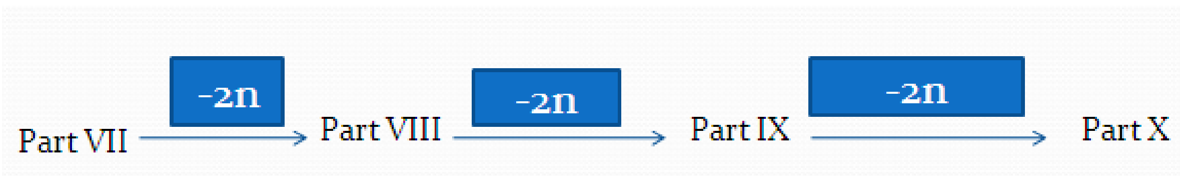

Figure 3

Illustration:1) As mentioned earlier a suitable labeling for Parts VII to X when n=14,that is, for $C_{70}*K_1$ is mentioned in Figure 4.

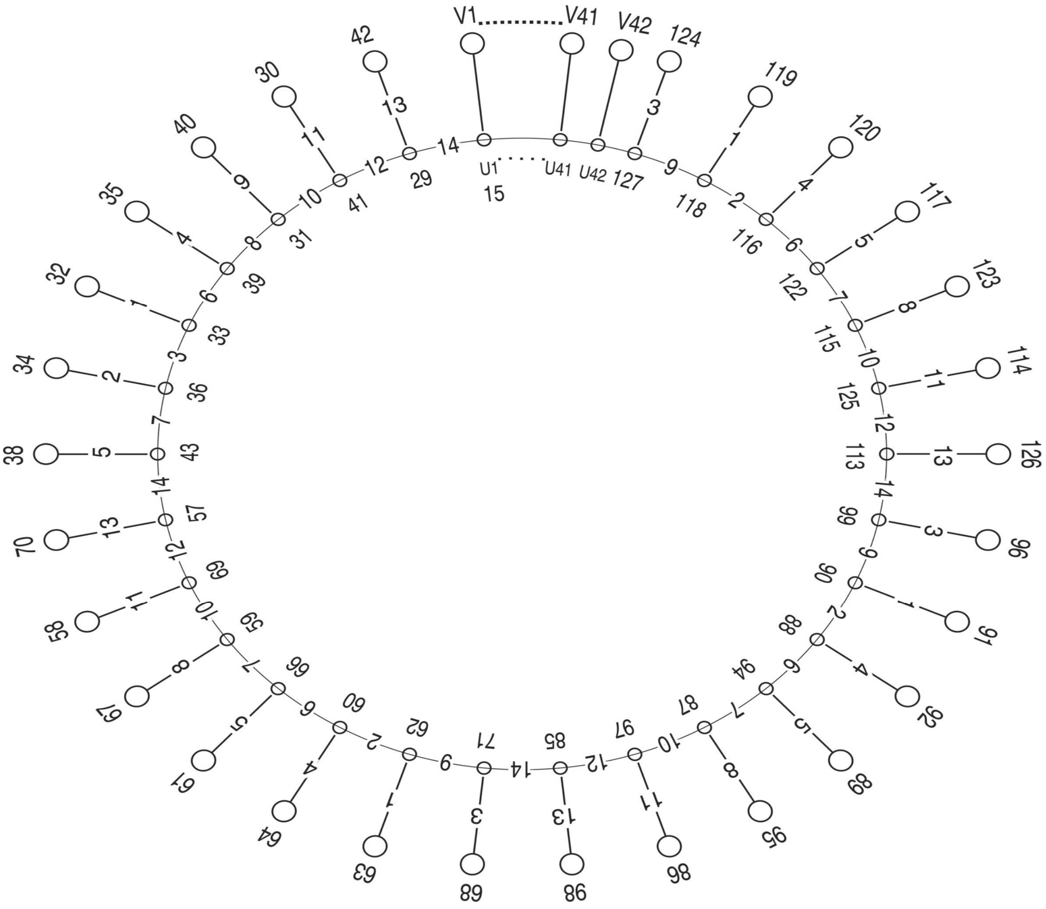

Figure 4

2) A suitable labeling when n=4,that is, for $C_{20}*K_1$ is mentioned in Figure 5.

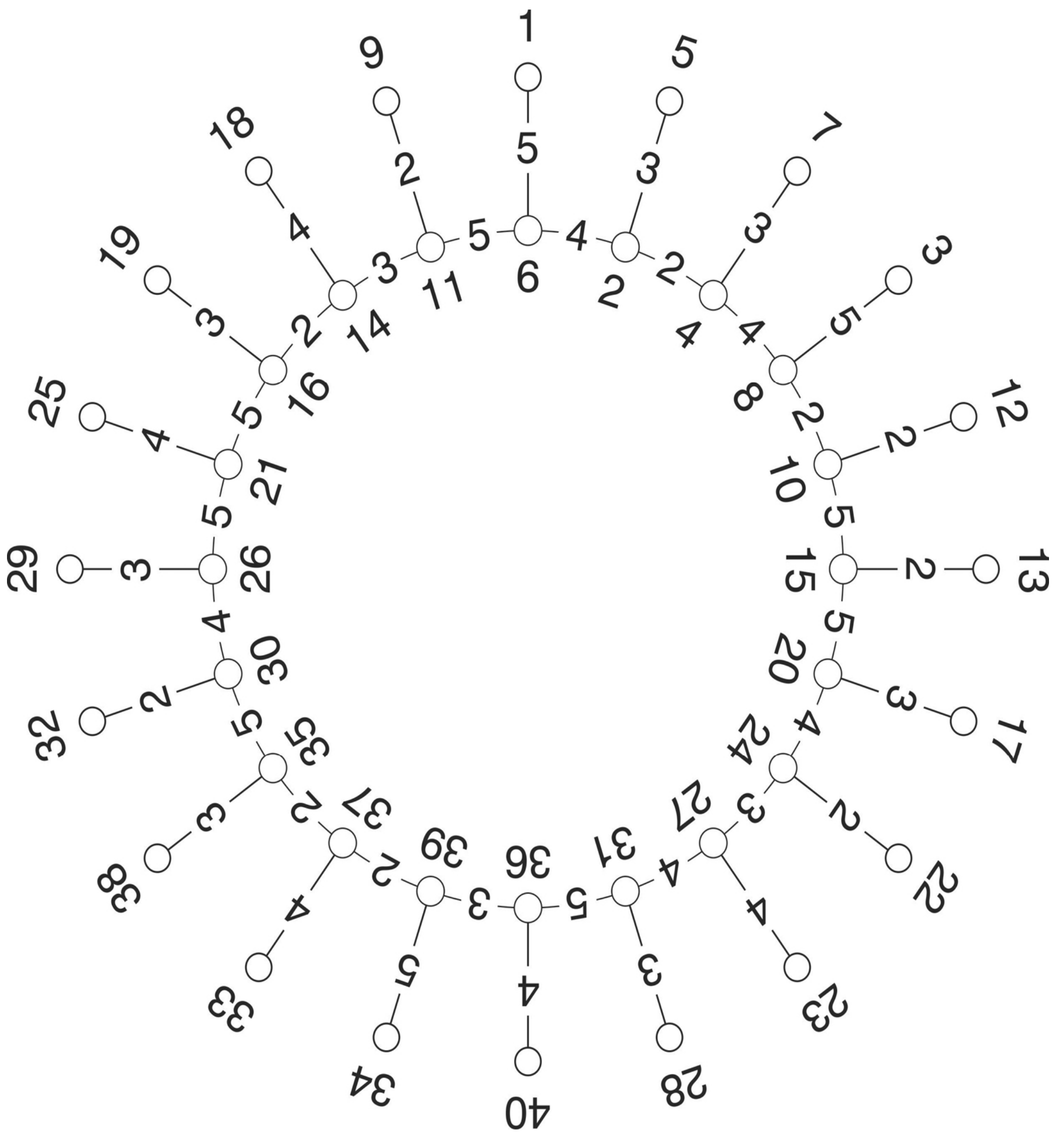

Figure 5

3) For remaining values of n, that is, for n = 2,6,8 and10 figures are not drawn but labels of u_i and v_i **are mentioned below. Where the labels in the first row are those of** v_i **and labels in the second row are those of** u_i **.The labels of** u_i ,v_i **are given one below other.**

Minimal 10-Equitable labeling of C_{10}*K_1**:**

1	**3**	**7**	**11**	**20**	**13**	**19**	**15**	**9**	**2**
5	**4**	**8**	**12**	**16**	**17**	**18**	**14**	**10**	**6**

Minimal 10-Equitable labeling of C_{30}*K_1**:**

1 6 3 9 8 11 17 22 28 31 35 41 42 49 48 57 54 59 58 52 47 46 37

7 2 5 4 10 14 20 24 29 33 32 38 43 44 50 53 56 60 55 51 45 40 36

39 25 26 23 18 15 12

34 30 27 21 16 19 13

Minimal 10-Equitable labeling of $C_{40}*K_1$**:**

1 8 3 6 13 14 11 10 29 30 27 26 45 46 43 42 61 62 59 58 77 78 75

9 2 7 4 5 12 15 16 21 28 31 32 37 44 47 48 53 60 63 64 69 76 79

80 67 71 70 66 51 55 54 50 35 39 38 34 19 23 22 18

74 73 68 72 65 57 52 56 49 41 36 40 33 25 20 24 17

Minimal 10-Equitable labeling of $C_{50}*K_1$**:**

1 10 3 8 5 15 18 17 14 12 36 35 38 33 32 56 55 58 53 52 76 75 78

11 2 9 4 7 6 16 13 19 20 27 37 34 39 40 47 57 54 59 60 67 77 74

73 72 96 95 98 93 100 84 82 85 88 90 64 62 65 68 70 44 42 45 48 50

79 80 87 97 94 99 92 91 83 89 86 81 71 63 69 66 61 51 43 49 46 41

24 22 25 28 30

31 23 29 26 21

Conclusion : It can be verified that all labels from 1 to 10n are used only once and induced edge-weights from 1 to n are such that every edge-weight is repeated exactly 10-times. Hence Corona $C_{5n}*K_1$ is minimally 10-equitable for all $n \geq 6$, when n is even.

Whereas for n=2 and n=4, it can be observed that C_{10}*K_1 ; C_{20}*K_1 are minimally 10-equitable with edge-weights {1,4} and {2,3,4,5} respectively.

Hence C_{5n}*K_1 is minimally 10-equitable for all n, when n is even.

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