Mastery over Subject Matter and Pedagogical Knowledge of Teachers: Key Factors to Better Mathematics Learning

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Abstract

Mathematics has been known to be a most universal curriculum subject in the absence of which the basic education of an individual cannot be considered as complete. In the present paper an effort has been made to analyze various pedagogical issues related to the mathematics learning. From the very past, mathematics has been taken as a dull and dry subject and many students continually has encountered obstacles to deal with this subject. The knowledge of mathematics mainly influences the decision making of every individual in all areas of life so, it becomes imperative that how the teaching of mathematics can be made actually effective and what a teacher can do to break the past beliefs about mathematics. A number of experts of the field have highlighted the importance of preparing teachers to teach mathematics using appropriate technology. Teacher education materials should be developed using an approach that integrally develops teachers' understandings of content, technology, and pedagogy to prepare them to teach different topics using specific relevant technological tools. The important components of a mathematics teaching cycle include a teachers' knowledge of mathematics, mathematical activities and representations, students' learning of particular content and their hypotheses about students' current knowledge, and their personal theories about mathematics, learning, and teaching. Technology, pedagogy, and content knowledge collectively have been needed for teachers to understand how to use technology effectively to teach specific subject matter. With technology becoming a ubiquitous part of daily experiences, it is important for mathematics teachers, to know how to capitalize on the power of technology to create lessons that assist students in developing understandings of mathematics. Finally, by preparing the prospective teachers of mathematics in the remarkable manner in every respect, they will certainly be capable of dealing with each type of problems of every student and it will certainly break the past beliefs and prototypes prevalent about teaching and learning of the compulsory curriculum subject mathematics.

KEYWORDS: Mastery, Mathematics, Pedagogy, Curriculum, Technology, Content Knowledge, etc.

Introduction

Mathematics is a basic curriculum subject in the absence of which the basic education of an individual cannot be considered as complete. From the very past, it has been taken as a dull and dry subject and a number of students persistently feel encountered obstacles to deal with this subject. The knowledge of mathematics mainly influences the competence of every individual in all areas of life so; it becomes very

important to make mathematics education actually effective and to break the past discouraging beliefs about mathematics. Experts of the field highlight the importance of preparing teachers to teach mathematics using appropriate technology. Teacher education materials should be developed using an approach that integrally develops teachers' understandings of content, technology, and pedagogy to prepare them to teach mathematics using specific technological tools.

The researchers of the field may be sharing the common goal of improving mathematics education but have been differing in the methods employed to achieve it. The important role of content, pedagogy and technology has slowly moved to the forefront of mathematics education in recent years due to a confluence of many factors. Because mathematics is the underpinning of every aspect of mathematics education, including curriculum, pedagogy, and assessment. Although, in the early grades, the role of content knowledge tends to be not as dominant in affecting the quality of mathematics teaching even then the sound pedagogical decisions can only be based on sound content knowledge, so, its significance cannot be overlooked.

The National Council of Teachers of Mathematics (NCTM, 2000) has stated that "Technology is essential in teaching and learning mathematics; it influences what is taught and enhances students' learning". Whether technology will enhance or hinder students' learning depends on teachers' competence for using technological tools, which is based on the knowledge gained during a teacher preparation program. The Association of Mathematics Teacher Educators (AMTE, 2006) has recommended that teacher education programs should "provide opportunities for teachers to acquire the knowledge and experiences needed to incorporate technology in the context of teaching and learning mathematics". The types of knowledge and skills prospective teachers need to support students' learning needs through technology are further delineated in the International Society for Technology in Education as '*National Educational Technology Standards for Teachers'* originally released in 2000 and updated in 2008. There are strong recommendations for using technology to teach students who are learning mathematics and to prepare teachers for the same who will teach mathematics.

Mathematics education of any generation has been influenced by a larger network including the school, home, community, and wider education system. Teachers have to realize that some of the practices may be more applicable to the local circumstances of the stakeholders than others. The basic principles considered important for mathematics pedagogy have been such that it must: (i) be grounded in the general premise that all students have the right to access education and the specific premise that all have the right to access mathematical culture; (ii) acknowledge that all students, irrespective of age, can develop positive mathematical identities and become powerful mathematical learners; (iii) be based on interpersonal respect and sensitivity and be responsive to the multiplicity of cultural heritages, thinking processes, and realities typically found in the classrooms; (iv) be focused on optimizing a range of desirable academic outcomes that include conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning; (v) be committed to enhance a range of social outcomes within the mathematics classroom that will contribute to the holistic development of students for productive citizenship. Keeping in view the basic pedagogical principles of mathematics

education, the researcher in the present paper intended to analyze the various pedagogical issues in mathematics learning and following objectives have been focused:

A. To analyze various pedagogical issues in order to improve mathematics education.
B. To analyze the need of content and pedagogical knowledge for a teacher.

The important components of a mathematics teaching cycle include a teachers' knowledge of mathematics, mathematical activities and representations, students' learning of particular content and their hypotheses about students' current knowledge, and their personal theories about mathematics, learning, and teaching. Technology, pedagogy, and content knowledge collectively have been needed for teachers to understand how to use technology effectively to teach specific subject matter. It is important for mathematics teachers, to know how to capitalize on the power of technology to create lessons that assist students in developing understandings of mathematics.

A. PEDAGOGICAL ISSUES ELEVATING MATHEMATICS EDUCATION

(1) DEVELOPING MATHEMATICAL PROFICIENCY

Teachers, who truly care about their students, work hard at developing trusting classroom communities. Also, they ensure that their classrooms have a strong mathematical focus and they have high as well as realistic expectations about what their students can achieve. In such a climate, students find themselves able to think, reason, communicate, reflect upon, and critique the mathematics they encounter; their classroom relationships become a resource for developing their mathematical competencies and identities.

Students want to learn in a harmonious environment. Teachers can help create such an environment by respecting and valuing the mathematics and the cultures that students expect in the classroom. Teachers ensure safety and make it easier for all their students to get involved. It is important, however, that they avoid the kind of caring relationships that encourage dependency among students. Rather, they need to promote classroom relationships that allow students to think on their own, ask questions, and take intellectual risks. Teachers are the single most important resource for developing students' mathematical identities. Teachers can help students develop a positive attitude towards mathematics by attending to the differing needs that originate from home environments, languages, capabilities, and perspectives. A positive attitude raises comfort levels and gives students greater confidence in their capacity to learn and to understand mathematics.

(2) INDEPENDENCE AND COLLABORATION

It can be difficult to grasp a new concept or solve a problem when distracted by the views of others. For this reason, teachers should ensure that all students are given opportunities to think and work quietly on their own, where they are not required to process the varied and sometimes conflicting perspectives of others. Teachers are the primary resource for nurturing patterns of mathematical reasoning in the minds of the

students. They manage, facilitate, and monitor student participation and record their solutions, emphasizing efficient ways. Teachers should invite students to explain their solutions to others and also encourage them to listen to and respect one another, accept and evaluate different viewpoints, and engage in an exchange of thinking and perspectives.

Working collaboratively with partners and in small groups can help students to understand mathematics better. Such arrangements can often provide the emotional and practical support that students need to simplify the nature of a task and identify possible ways forward to accomplish it. Pairs and small groups are not only useful for enhancing engagement; they also facilitate the exchange and testing of ideas and encourage higher level thinking. In small, supportive groups, students learn to make guesses and engage in mathematical argumentation and validation. For maximum effectiveness groups should have only four or five members. Students of varying mathematical achievements, being in a group, integrate the insight of different levels and enhance overall understanding of the whole group.

(3) THINKING-LEARNING CONNECTION

Students' competencies should be used by the teacher as starting points for their planning and decision making. Existing competencies of the students like language, reading and listening skills, ability to cope with complexity, and mathematical reasoning, become resources for the better learning of the subject matter. The personal experiences of the students provide them a way to think on the right direction and hence advancing the understanding and learning of mathematics. When students can envisage the situations or events in which a problem is embedded, they can use their own experiences and knowledge as a basis for developing context-related strategies that they can later refine into generalized strategies. When students are engaged in tasks, they focus on the thinking and their teachers get a chance to pose new questions or design new tasks that helps challenge and extend thinking.

(4) USING MISCONCEPTIONS AND ERRORS

Mistakes can be made by learners due to the insufficient time or care. But errors may also arise from consistent, alternative interpretations of mathematical ideas that represent the learner's attempts to create meaning. Therefore, rather than dismissing such ideas as "errors" or "wrong thinking", teachers have to view them as a natural and often necessary stage in a learner's conceptual development. Teachers must analyze such misconceptions and use them as building blocks for developing deeper understandings of the subject.

There are many ways in which teachers can provide opportunities for students to learn from their errors. Firstly, by organizing discussion that focuses student attention on difficulties that have surfaced. Secondly, asking students to share their interpretations or solution strategies so that they can compare and re-evaluate their thinking. And thirdly, posing questions that create pressure, which needs to be resolved.

(5) IMPROVING FOCUSSED DOMAIN

Teachers are required to design learning experiences and tasks that are based on sound principles and approaches of mathematics learning. Students must be given tasks to help them improve their understanding in the domain that is currently the focus. Students should not expect that tasks will always involve practicing the sums they have just been taught; rather, they should be encouraged take up the tasks sincerely and think productively with and about important mathematical ideas. Mathematical thinking involves the use of formulas, algorithms, and procedures to help understand the meaning and concepts of the subject. Experiences or activities that make students to think deeply about the subject and encourage them to think on their own instead of always relying on the teacher to accomplish the task. Such accomplishments may help students feel mathematics as a relevant, interesting and enjoyable curriculum subject.

Students must be provided with opportunities to practice whatsoever they are learning, it will certainly improve their computational fluency, problem solving skills as well as conceptual understanding. Skill development can often be incorporated into "doing" mathematics i.e. learning by doing; for example, learning about perimeter and area offers opportunities for students to practice multiplication and fractions. Games can also be a means of developing fluency and automaticity. Instead of using games as time-pass, teachers must choose and use these for meeting specific mathematical purposes and providing appropriate feedback and challenge for all participants.

(6) ASSESSMENT AND LEARNING

Teachers are required to use a wide range of formal and informal assessment techniques to monitor learning progress, diagnose learning issues, and determine the further learning needs of the students. In the course of regular classroom activity, they collect information about how students learn, what they seem to know and be able to do, and what interests them. In this way, they know what is working and what is not, and are able to make informed teaching and learning decisions.

During every lesson, teachers make a number of instructional decisions. Every moment assessment of student progress helps them decide what questions to ask, when to intervene, and how to respond to questions. They can gain a lot from observing students as they work and by talking with them: they can gauge students' understanding, see what strategies they prefer, and listen to the language they use. Teachers must use this information as a basis for deciding what examples and explanations they will focus on in classroom discussion. One-on-one interviews can also provide important insights: a thinking-aloud problem-solving interview will often reveal more about what is going on in a student's mind than a written test. With the help of interviews teachers can find surprising results about what students know and do not know. Interviews can make teachers more responsive to their students' diverse learning needs, as they take their expectations and assumptions as challenge.

(7) QUESTIONING AND FEEDBACK

Teachers, in order to have remarkable outcomes ensure complete participation of students in the process of mathematical thinking and problem solving. By allowing sufficient time for students to explore responses and by pressing for explanation and

understanding, teachers can ensure that students are productively engaged. Questioning is a powerful means of assessing students' knowledge and exploring their thinking. A key indicator of good questioning is how teachers listen to student responses. Teachers should not only pay attention to whether an answer is correct or not, but also to the student's mathematical thinking. They must take care of the fact that a wrong answer might indicate unexpected thinking rather than lack of understanding; equally, a correct answer may be arrived at via faulty thinking. To explore students' thinking and encourage them to engage at a higher level, teachers can use questions that start at the solution; for example, *If the area of a rectangle is 24 cm2 and the perimeter is 22 cm, what are its dimensions?* Questions that have a variety of solutions or can be solved in more than one way have the potential to provide valuable insight into student thinking and reasoning (Anthony and Walshaw, 2010).

It is a well known fact that feedback focuses on the task and not on marks or grades. It explains systematically that why something is right or wrong and also describes that what to do next or it may suggest some strategies for improvement. Teachers must support students, not by giving full solutions, but by prompting them to search for more information, try another method, or discuss the problem with classmates. In this manner the teacher can encourage the student to do further thinking or accomplish task before getting the progress report checked by the teacher.

(8) SELF AND PEER ASSESSMENT

Teachers should provide opportunities for students to evaluate their own work. This may include having students design their own test questions, share success criteria, write mathematical journals, or present portfolio evidence of growing understanding. When feedback is used to encourage continued student–student and student–teacher dialogue, self-evaluation becomes a regular part of the learning process and students develop greater self-awareness.

(9) TOOLS AND NEW TECHNOLOGIES

Teachers must utilize various types of tools and technologies to support their students' mathematical development. These include the number system itself, algebraic symbolism, graphs, diagrams, models, equations, notations, images, analogies, metaphors, stories, textbooks, and technology. Such tools provide vehicles for representation, communication, reflection, and argumentation. They are most effective when they cease to be external aids, instead becoming integral parts of students' mathematical reasoning. As tools become increasingly invested with meaning, they become increasingly useful for furthering learning.

An increasing array of technological tools is available for use in mathematics classrooms. These include calculator and computer applications, presentation technologies such as the interactive whiteboard, mobile technologies such as clickers and data loggers, and the Internet. These dynamic graphical, numerical, and visual applications provide new opportunities for teachers and students to explore and represent mathematical concepts. With guidance from teachers, technology can support independent inquiry and shared knowledge building. When used for mathematical

investigations and modelling activities, technological tools can link the student with the real world, making mathematics more accessible and relevant.

Teachers need to make right and appropriate decisions about when and how they use technology to support learning. Successful teachers take time to share with their students the reasoning behind these decisions. They also monitor their own use of technology i.e. including overuse or underuse. Teachers need to be exposed to certain ongoing professional training programmes so that they may become capable of using new technologies in an appropriate manner to advance the mathematical thinking and learning of their students.

B. IMPORTANT REQUIREMENTS FOR A MATHEMATICS TEACHER

(1) CONTENT AND PEDAGOGICAL KNOWLEDGE

There can never be any other opinion regarding the issue that teachers must have a sound grasp of relevant content as well as the sound knowledge of the pedagogy needed to teach the same. They must know the huge philosophy, technology and psychology of the learners and the skills needed to teach effectively the subject in hand. They can think of, model, and use examples and metaphors to advance student thinking power. They can critically evaluate students' processes, solutions, and understanding and give appropriate and helpful feedback.

Pedagogical content knowledge is crucial at all levels of mathematics and with all groups of students. Teachers with in-depth knowledge have clear ideas about how to build procedural proficiency and how to extend and challenge student ideas. They use their knowledge to make the multiple decisions about tasks, classroom resources, talk, and actions that feed into or arise out of the learning process. Teachers with limited knowledge tend to structure teaching and learning around discrete concepts instead of creating wider connections between facts, concepts, structures, and practices.

To teach mathematical content effectively, teachers need a grounded understanding of students as learners. With such understanding, they may be aware of likely conceptions and misconceptions. They may use this type of awareness to make instructional decisions that strengthen conceptual understanding.

(2) UPDATING TEACHER COMPETENCY

There is no doubt in the fact that, teachers need to be exposed to certain ongoing professional training programmes so that they may become capable of using new technologies in an appropriate manner to advance the mathematical thinking and learning of their students.

Sound knowledge enables the teachers to listen and ask question more perceptively and effectively preparing them for on-the-spot classroom decision making. The development of teacher knowledge is greatly enhanced by efforts within the wider educational community. Teachers need the support of others—particularly material, systems, and human and emotional support. While teachers can learn a great deal by

working together with a group of supportive mathematics colleagues, professional development initiatives are often a necessary catalyst for major change.

CONCLUSION

Current research findings show that the nature of mathematics teaching significantly affects the nature and outcomes of student learning. This highlights the huge responsibility that teachers have for their students' mathematical well-being. In this paper, different pedagogical issues have been discussed as a starting point for discussing and dealing with changes, innovations, and reforms in mathematics education. These issues and pedagogical principles should be viewed and dealt with as a whole and not in isolation. Teaching is a complex process and many interrelated factors have an impact on student learning. The present paper offers the ways to address that complexity, and to make mathematics teaching more effective. Major innovations and genuine reforms require the efforts of all the teachers, principals, teacher educators, researchers, parents, specialist support services, school boards, policy makers, and the students themselves. Changes need to be negotiated and carried through classrooms, teams, departments, and faculties, and in teacher education programmes. Innovation and reform must be provided with adequate resources. Schools, communities, and nations need to ensure that their teachers have the knowledge, skills, resources, and incentives to provide students with the very best of learning opportunities. In this way, all the students will develop their mathematical proficiency and will have the opportunity to view themselves as powerful learners of mathematics. Finally, by preparing the prospective teachers of mathematics in the remarkable manner in every respect, they will certainly be capable of dealing with each type of problems of every student and it will certainly break the past beliefs and prototypes prevalent about teaching and learning of the compulsory curriculum subject mathematics.

REFERENCES

Anthony, G.; Walshaw, M. 2007. *Effective pedagogy in mathematics/p,ngarau: Best evidence synthesis iteration [BES]*. Wellington: Ministry of Education.

Anthony G. and Walshaw M. 2010. Effective Pedagogy in Mathematics. *International Bureau of Education*. Michigan State University. USA.

Association of Mathematics Teacher Educators. (2006). *Preparing teachers to use technology to enhance the learning of mathematics: A position of the Association of Mathematics Teacher Educators*. Retrieved from http://www.amte.net/Approved%20AMTE%20Technology%20Position%20Paper.pdf

Hill, H.; Rowan, B.; Ball, D. 2005. Effects of teachers' mathematical knowledge for teaching on student achievement. *American Education Research Journal*, no. 42, pp. 371–406.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Watson, A.; Mason, J. 2006. Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning*, no. 8, pp. 91–111.

Zevenbergen, R.; Lerman, S. 2008. Learning environments using interactive whiteboards: New learning spaces or reproduction of old technologies. *Mathematics Education Research Journal*, vol. 20, no. 1, pp. 107–125.