Students' Errors in Mathematics

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Abstract

At the time of correction of different students' work like class work, achievement test, diagnostic test, home task in elementary school mathematics, it is found that students make different mistake or error due to different reasons. These errors or mistakes are classified in different ways by different educationists. Some of them classified the student's errors in mathematics in four types. These are factual error, procedural error, conceptual error and careless error. Others classified them under two types as slips and bugs. Factual error and careless error are also known as slips and procedural error & conceptual error are known as bugs. From different research studies, it is found that the reasons for students' errors may be students' lack of knowledge, misunderstanding, lack of adequate practice, lack of specific feedback from teacher regarding misunderstanding of specific concept, fatigue or tiredness, poor attention, carelessness etc. Theoretically it is found that students' errors or misconceptions arise due to gap between existing knowledge or schema and new knowledge or ideas or information. To resolve the student's errors, teacher may use error analysis method or RtI (Response to Intervention) framework and give error wise prevention.

KEYWORDS: Students' errors, Error analysis, Factual error, Procedural error, Conceptual error, Careless error, Slips, Bugs

Introduction:

Identification of students' errors in elementary mathematics is most important for the students with learning disabilities, and low performing students (Fuchs, Fuch & Hamlett, 1994; Salvia& Ysseldyke, 2004 cited on Lai, 2012) and the students who have difficulty in learning mathematics (Lai, 2012). Because, by pointing out the students' errors, teacher can provide appropriate/ proper instruction/ intervention/ remediation or take proper steps to improve student learning, and understanding of mathematics (Lai, 2012). To identify the students' errors, the teacher/ instructor should review each and every student's work through error analysis and look for the pattern of errors or type of errors. The errors occurring more than two times are considered as a pattern of error and it requires proper intervention to remediate these errors (Riccomini, 2016).

Students' errors and their types:

It is seen in students' works (class note, assignment, home task , achievement test, diagnostic test etc.) that some students have not followed the correct steps or procedures to solve a problems (Lai, 2012), some have not mastered in basic facts (Ginsburg, 1987 as cited in Lai, 2012) and some make mistake or errors due to misunderstanding of the basic concept, principle and ideas connected to the given mathematical problems (Brown, Skow & the IRIS Center, 2016) and some errors are made or committed by the students only for their poor attention, carelessness (Stein, Silbert & Carnine, 1997) and tiredness or fatigue (Fisher & Frey, 2012; Brown, Skow

& the IRIS Center, 2016). All the errors made by the students are classified in different ways-

Classification of errors -I

According to Lai, 2012; Fisher & Frey, 2012, Riccomini, 2014; Brown, Skow & the IRIS Center, 2016 et.al students' errors may be classified in four types

1) Factual error
2) Procedural error
3) Conceptual error
4) Careless error

Factual errors are made by the students when they cannot recall the required facts related to the given problems or they have not mastered in the basic facts related to the given problems (Ginsburg, 1987 as cited in Lai, 2012). It may happen due to lack of factual information like vocabulary, digit identification, place value identification etc. (Brown & Skow, 2016).

Procedural errors arise when students do not follow the correct steps or procedures to solve a given problem (Lai, 2012). i.e., it happens when a studentincorrectly applies a rule or an algorithm to solve a problem (Brown, Skow & the IRIS Center, 2016).

Conceptual error,the most important and serious error made by the student is **conceptual error**. This type of error occurs due to not understanding the specific math concept (Ginsburg, 1987 as cited in Lai, 2012) or it may happen due to misconception or a faulty understanding of the related or concerned principles and ideas connected to the problems(Brown, Skow & the IRIS Center, 2016). It may look like procedural error (Ginsburg, 1987 as cited in Lai, 2012). So, it is difficult to differentiate between them (Rittle-Johnson, Siegler & Alibali, 2001; Riccomini, 2014). Therefore it is important to know that conceptual knowledge is knowing the relationship between or among the related concepts (Wearne & Hiebert, 1988) and an understanding of why an algorithm (series of steps to solve a problem) or procedures work to solve a problem (Hiebert & Wearne, 1986 ; Bauer, 2016) on the other hand procedural knowledge is knowing a series of steps (algorithm) or techniques that a student follow to find the answer of a problem or completing a task not rich in relationship (Bauer, 2016 ; Hiebert, 1992).

Careless erroris committed in spite of sufficient concept, skills and factual knowledge, by students due to poor attention and carelessness (Stein, Silbert & Carnine, 1997) or and tiredness or fatigue (Fisher & Frey, 2012; Brown, Skow & the IRIS Center, 2016). This type of error is called **careless error.**

Classification of errors -II

But Ketterlin-Geller & Yovanoff (2009) and Ginsburg (1987 as cited in McCall, 1999) classified the students' errors only in two types-

1. Slips
2. Bugs

Slips are random errors in students declarative or procedural knowledge that are not the result of inherent misunderstanding in the domain". (Ketterlin-Geller and

Yovanoff ,2009). Slips may arise due to memory deficits, impulsivity or visual-motor integration problem and also it is easier to identify (Lai, 2012) and it may be possible to fully recover or overcome from this kind of error through proper instruction or remediation provided by the subject teacher or instructor.

Bugsrepresent persistent misconceptions about domain specific knowledge or skills that consistently interfere with students' demonstration of their abilities (Ketterlin-Geller and Yovanoff ,2009). Bugs are more serious error than slips and it is more difficult to identify (Lai, 2012) and also to overcome. According to Ginsberg (1987; cited in Newton, 2015) students can self-correct the slips but students need help to correct the bugs. Since factual errorand careless error are not arising due to inherent misunderstanding and students can self-correct, so these errors are known as 'slips' (Ginsberg 1987; cited in Newton, 2015). Since procedural & conceptual knowledge are intertwined components such that these are complement to each other (Lai & Murray, 2012) and conceptual error and procedural error occur due to persistent misconceptions about the math concept and consistently interfere with students' demonstration of their abilities (Ketterlin-Geller and Yovanoff ,2009) and students need help to correct. So, this type of error comes under bugs. (Ginsberg 1987; cited in Newton, 2015).

Possible reasons of student errors:

From the several researches it is found that there are several reasons for which students make different mistakes or errors. Students' lack of knowledge or misunderstanding (Hudson & Miller, 2006; Fisher & Frey, 2012; Riccomini, 2014), misconceptions or a faulty understanding of the principles and ideas related to the mathematical problem (Brown & Skow, 2016), lack of adequate practice, lack of specific feedback from teachers regarding misunderstanding of specific concept of the students, mathematics anxiety of the students etc. (Lai, 2012) are the possible reasons. Not every error made by the student are happened only due to lack of knowledge or skills. Sometimes it is simply because of students are fatigued or distracted (Fisher & Frey, 2012) or because of poor attention and carelessness (Stein, Silbert & Carnine, 1997). It may happen because of poor vocabulary knowledge, limited reading skill, inability to identify relevant information, lack of prior knowledge. (Brown, Skow & the IRIS Center, 2016).

Theoretical background of student errors and misconceptions:

According to Piaget(as cited in Sarwadi & Shahrill, 2014)knowledge is not constructed only from experiences. It constructs through the combination of experience and present knowledge structure or existing schema of a child. Child's mental structure or schemata are constructed through two mental processes. These are assimilation and accommodation. Assimilation is a process by which new ideas or information or concepts are fitted directly with the existing schema (i.e., what a child already knows) without any change. On the other hand, when it is not possible to fit the new ideas or information directly with the existing schema of a child then accommodation process work. That is, by this process existing schema has to be restructured to fit the new ideas or information. So, child's present knowledge or existing schemata determine what will be learned by them from experiences.

Mathematics learning is cumulative in nature. i.e., there is a link between previous knowledge and newly gained knowledge. Therefore, if a child is unable to assimilate

as well as accommodate then there is a gap between previous knowledge and new knowledge. This gap lead to students' mathematical errors and misconceptions. But students' errors or misconceptions are not always bad. It is a part of knowledge construction and learning process. But for smooth academic progress it is important that it is dealt with diagnostically.

Identification of students' errors:

Teacher/educationist/researcher may comparethefollowingerror wise symptoms with the wrong response in students' work to determine different types of error made by them.

Careless error:

Since this kind of error arise due to working fast, poor attention, carelessness, (Stein, Silbert & Carnine, 1997; Math Greek Mama, 2016) tiredness, fatigue etc. (Fisher & Frey, 2012) So, following kind of mistake may happen in students' wrong responses.

i) Wrongly or incorrectly copying the numbers from the problems at the time of starting the solution of a problem.Not follow the directions or instructions of the given problems in the questions paper.(Math Greek Mama, 2016)
ii) Dropping the mathematical signs. (Math Greek Mama, 2016)
iii) Sometimes asking for one question but answer is given for another question or more questions.(Khan Academy, 2019)
iv) Use of improper unit. (The Asian Parent, n.d.)

Factual error:

The incorrect responses come under factual error if the following errors are found in students' work sheet that students

i) Do not know or recall the meaning of mathematical vocabulary and terminology.
ii) Do not identify digits and sign.
iii) Making counting error.
iv) Use incorrect formula.
v) Do not identify place value.
vi) Making mistake or error in computation.
vii) Do not know or have not mastered in basic number facts with relation to addition, subtraction, multiplication and division like 4+2=5, 4×2=5, 5-4=3, 8÷4=3 etc.
(Riccomini, 2016; Brown, Skow & the IRIS Center, 2016)

Procedural error:

The mistake or error made by the students comes under procedural error if the following types of errors are found in students' work

i) Unable to use or follow the correct steps or procedure to solve a problem. (Lai, 2012).
ii) Unable to use a formula or step by step procedure for solving a problem (Riccomini, 2016).

iii) Incomplete calculation. (Muthukrishnan, Kee & Sidhu, 2019).
iv) Forgetting to regroup (carry) when adding, subtracting or multiplying (Brown, Skow & the IRIS Center, 2016).
v) Unable to regrouping with 0 (zero)

For example,

```
2 0 4
   -4 1
 ---------
   2 4 3
```

Here subtracts the 0 (zero) from 4 instead of borrowing.

vi) Unable to correct alignment. (Riccomini, 2016)

Conceptual error:

Wrong responses or wrong answer given by the students are come under conceptual error when student make the following type of mistake in their work sheet.

i) They use incorrect logic to solve any problem (Math Greek Mama, 2016)
ii) It may be possible that all computations are correct in each step but still get the wrong answer. (Math Greek Mama, 2016)
iii) Misunderstanding of place value. i.e., students are unable to put the digit in appropriate place value position when they solve this kind of problems(Brown, Skow & the IRIS Center, 2016)
iv) Students always subtract the lesser number from greater number in every situation where it is not required. (Brown, Skow & the IRIS Center, 2016)
For example,

```
 2 5 3
 -  1 7 5
 ---------
     1 2 2
```

Here subtracts the 3 from 5 and 5 from 7 instead of borrowing.

vii) Skip regrouping (carry) when needed (Muthukrishnan, Kee & Sidhu, 2019).
For example,

```
 7 6
    +  5
 ---------
     7 1
```

Here add6 with 5 but do not carry 1 to next step.

viii) Inappropriate generalization (Irwin &Britt, 2004 as cited in Lai & Murray, 2014).
For example, students have misconception that always less number is subtracted from greater number
For example,

```
 2 5 3
 -  1 7 5
 ---------
     1 2 2
```

Here subtracts the 3 from 5 and 5 from 7 instead of borrowing.

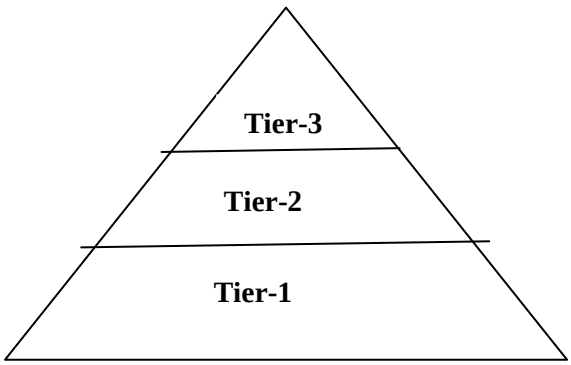
ix) Overgeneralization of the principle or rule that "Division always make smaller", "the dividend must be larger than the divisor" (Bell, Fischbein & Greer,1984; Okazaki & Koyama, 2005).

x) Unable to choose a correct operation for a word problem. (Tirosh & Glover, 1989; Graeber & Tirosh, 1990).

Prevention of studenterrors:

There are different ways found in different research studies to resolve the students' error in elementary mathematics. Some of the ways are discussed here-

1. **Through error analysis:**Every student's workisreviewed, identified and looked for error patterns or types. Then analyse theprobable causes of the errors. After this teacher should provide proper remediation to overcome the errors made by the students (Lai, 2012). Riccomini (2016) echoessimilar things. That is at first find out or identify the errors through error analysis of the student's independent work. Then categorize the student's errors into factual error, procedural error, conceptual error and careless error or slips and bugs and then look for error patterns that is errors occurring more than two times then provide appropriate intervention. Brown, Skow & the IRIS Center (2016) stated four steps to conduct an error analysis. These four steps are step-1-collect data, step-2-identify error patterns, step-3- determine reasons for errors and step-4 is, use the data to address error patterns. In this last step teacher should decide the proper instructional strategy to reduce causes of their errors or deficits or misunderstanding.
2. **Through RtI(Response to Intervention) frame work** given by Gersten, Beckmann, Clarke, Foegen, Marsh, Star & Witzel(2009), three levels of interventions are given to the students who are struggling with mathematics on the basis of evidence-based recommendation. These three levels are Tier-1, Tier-2 and Tier-3. Tier-1 is the mathematics instruction that all students in a classroom receive.In tier-2 interventions, schools provide additional assistance for those who do not progress adequately in tier-1. Tier-3 interventions are provided to the students who are not adequately progressing or benefiting from tier-2 intervention. The RtI frame work or three tiers of Response to Intervention is like a pyramid where tier-1 is in the bottom and tier-3 is in the top of the pyramid as given below diagrammatically.



3. **Error wise prevention** may be given. That is, at first teacher should find out what type of error (i.e., careless error, factual error, procedural error and conceptual error) are made by the students. Then teacher should give the

remediation according to their error. Error wise probable prevention are as follows

i) For careless error:

- **a)** Before moving to the solution double read or carefully read the question and under line what will be found out.
- **b)** Circle the key words or information in the problems on the questions paper. (Math Greek Mama, 2016).
- **c)** Since careless error may arise due to working fast, student may slow down the speed of their work and do carefully (Math Greek Mama, 2016) or use time management procedure (The Asianparent, n.d)the total working time or examination or test time are distributed as how much time spent for reading the questions, how much time is taken to give the answer to each question, how much time is taken to finish up the all questions and how much time is taken to revise the answers of all questions.
- **d)** Do not do many steps in the mind or head (Ian, 2002as cited in Peterson, 2018). Or do not skip many steps at once (Khan Academy, 2019).
- **e)** Always check the answer (Ian, 2002as cited in Peterson, 2018)
- **f)** Students should keep the record of their mistakes (Ian, 2002as cited in Peterson, 2018).
- **g)** Carefully use the proper unit in proper place. (The Asianparent, n.d)
- **h)** Learn from previous mistakes. (Rick, n.d. as cited in Peterson, 2018)
- **i)** Make a habit to determine what is given and what are asked to find in the questions. (Rick, n.d. as cited in Peterson, 2018)

ii) For factual error:

- **a)** Slow down the working speed and working more carefully. (Math Greek Mama, 2016)
- **b)** Check the answer after solving each problem if the final answer is wrong. i.e., students should go through the solution again to check the computational error. (Math Greek Mama, 2016)

iii) Procedural error:

- **a)** After determining the procedural error teacher should teach explicitly the steps to complete the math problems and then increase the speed with which the problems are complied. (Burns & Klingbeil, 2010as cited in Burns, 2011)
- **b)** Repeated practices.
- **c)** Problem-solving approaches followed by appropriate learning methods. (Nuraini, Cholifah & Laksono, 2018)
- **d)** Matching math intervention should be given and avoid mismatched intervention (i.e., conceptual intervention for a student who need a procedural intervention). Burns, 2011)

iv) Conceptual error:

- **a)** After determining the conceptual error or understanding of a particular math concept or skills of the students, teacher should teach this math skill with manipulative items and then student

should be allowed to independently practice this skill. (Burns & Klingbeil, 2010 as cited in Burns, 2011)

b) Teaching the concept in deferent ways. (Math Greek Mama, 2016)

c) By offering a suitable learning model like problem-solving models, the conceptual errors that often occur in students' mathematical work sheet will be minimised (Nuraini, Cholifah & Laksono, 2018)

d) To develop good concepts, good examples are required for corresponding concept. (Skemp, 1976 as cited in Sarwadi & Shahrill, 2014)

e) Matching math intervention should be given and mismatched intervention should be avoided (i.e., procedural intervention for a student who need a conceptual intervention). (Burns, 2011)

Conclusion:

Errors made by the students do not always show the weakness of the students rather it is an opportunity to a student to get a deeper knowledge on this concept.Errors made by students are a part of a learning process if it is examined diagnostically. On the other hand, identification and analysis of students' error are the powerful mechanism for understanding students' mathematical thinking (An & Wu, 2012 cited in Lai & Murray, 2014)So, as a teacher or researcher our primary aim should be to examine each and every students' written work diagnostically and look for pattern of their errors. Such that we can find out the probable cause of errors and misconceptions and develop proper strategies to prevent students' error and misconception through the connection or making a bridge between existing knowledge and new ideas. For these reasons severalresearchers and educationist (Riccomini, 2005; Sherman, Richardson & Yard, 2005; Yong et al, 2011) concluded that error analysis is an important skill to a teacher for teaching mathematics. But unfortunately, it is shown in different research studies that teachers are not equipped to design and implement proper educational or teaching intervention based on the students' errors (Riccomini, 2005) and teacher feel difficulty to distinguish betweenprocedural error and conceptual error(Rittle-Johnson, Siegler & Alibali, 2001; Riccomini, 2014) or 'slips' and 'bugs' (Ketterlin-Geller and Yovanoff ,2009). So, curriculum maker, educationist, teacher educators should consider the diagnostic value of students' errors and error analysis and addressed it within the syllabus of teacher education programmes (Herholdt & Sapire, 2014).

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