Poisson Distribution and It's Applications

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Abstract

The three major discrete distributions are Binomial, Poisson and Hypergeometric distribution. In this study we discuss about Poisson distribution which was introduced by *Siméon Denis Poisson* in 1837. It has been since subject of numerous publications and practical applications. The Poisson distribution is used in science, real life and more specifically related to health care services. The purpose of this paper is to raise awareness of numerous application opportunities and to provide more complete case coverage of Poisson distribution. Formal definition, basic properties and applications of Poisson distribution are discussed in this paper.

KEYWORDS: Poisson distribution, Probability, Statistics, Applications.

INTRODUCTION:

In probability theory there are two types of distribution:- (**i**)Discrete probability distribution. (**ii**)Continuous probability distribution.

The Poisson distribution is one of the most important discrete probability distribution, which was introduced by *Siméon Denis Poisson* in 1837. Poisson distribution stands for the number of statistically independent events, occurring within a unit of space or time.

A random variable X is said to follow a Poisson distribution if it assumes only non-negative values and its probability mass function is given by:

$P(x; \lambda) = \{e^{-\lambda}\lambda^{x}/x!\}$ x=0, 1, 2………; $\lambda > 0$

Where,

- e (Euler's constant) =2.71828
- x is a non-negative integer which denotes the number of occurrence of an event-the probability of which is given by the function.
- x! is the factorial of x.
- λ is a positive real number, equal to the expected number of occurrences that occur during the given interval.

CONDITIONS UNDER WHICH POISSON DISTRIBUTION CAN BE USED:

- The random variable ‘X’ should be discrete.
- The occurrence of one event does not affect the probability for a second event to occur. That is, events occur independently.
- A dichotomy must exist, i.e., happening of the event must be of two alternatives as success and failure.
- Applicable are those cases where the number of trials ‘n’ is very large and the probability of success ‘p’ is very small but the mean ‘np’= λ is finite.

If these conditions are true than X is a Poisson random variable, and the distribution of X is a Poisson distribution

For example, if a particular event occurs on an average of 2 times per minute and we are interested in the number of events occurring in a 20 minute interval then we can use a Poisson distribution with $\lambda=20*2=40$. The given figure (*Figure-1*) shows the Poisson distribution for different value of λ. As a function of x, this is the probability mass function, which means it is a function that gives the probability that a discrete random variable is exactly equal to some value. The horizontal axis is the index x, i.e., the number of occurrences. λ is the expected number of occurrences. The vertical axis is the probability of x occurrences for a given value of λ. The function is defined only for integral values of x. The connecting lines are to be taken as a guide for the eye.

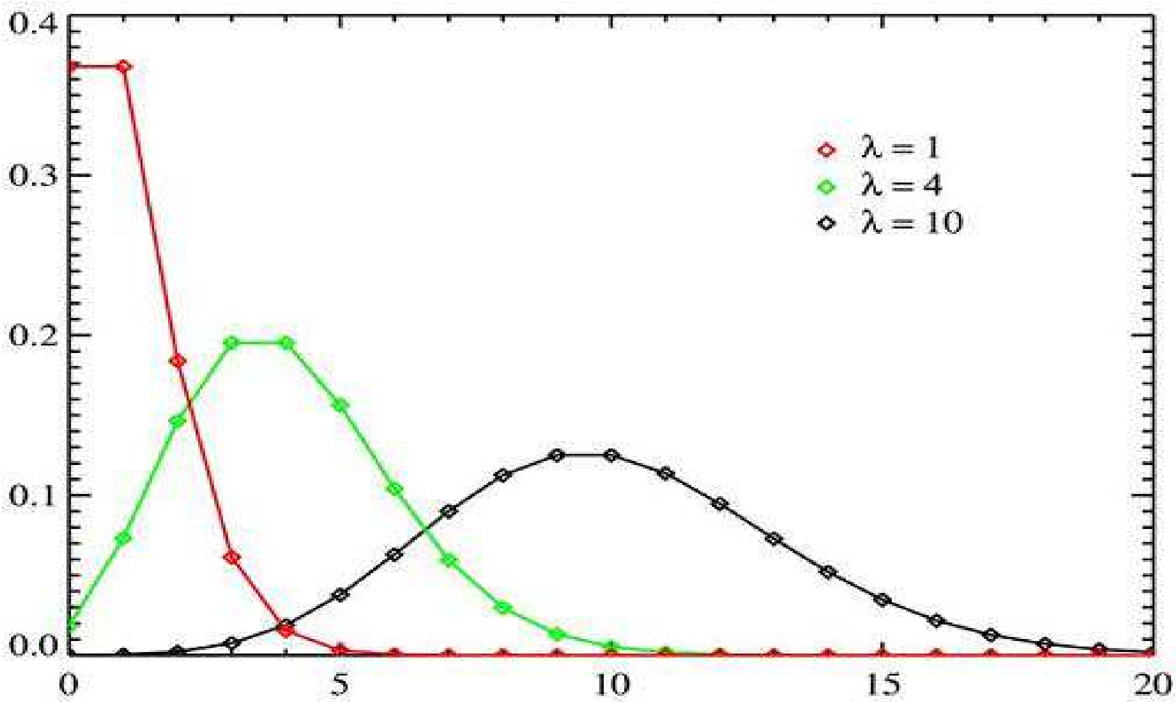

Figure-1

ANALYSIS:

Let X be a Poisson variate, then

$$\text{Mean}=\mu_1' = \sum_{X=0}^{\infty} x p(x, \lambda)$$

$$= \sum_{X=0}^{\infty} x\, [e^{-\lambda}\lambda^{x}/x!]$$

$$= \lambda\, e^{-\lambda}\, [\sum_{X=1}^{\infty} \lambda x^{-1}/(x-1)!]$$

$$= \lambda\, e^{-\lambda}\, [1+\lambda+\lambda^{2}/2! + \lambda^{3}/3! + ...]$$

$$= \lambda\, e^{-\lambda}.e^{\lambda}$$

$$= \lambda \text{--- (i)}$$

Hence the mean of the Poisson distribution is λ.

$$\mu_2' = E(X^2) = \sum_{x=0}^{\infty} x^{2} p(x, \lambda)$$

$$= \sum_{X=0}^{\infty} [x(x-1) + x]\, e^{-\lambda}\lambda^{x}/x!$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} x(x-1)\, \lambda^{x}/x! + \sum_{x=0}^{\infty} x.\, e^{-\lambda}\lambda^{x}/x!$$

$$= \lambda^{2}\, e^{-\lambda}\, [\sum_{x=2}^{\infty} .\lambda^{x-2}/(x-2)!] + \lambda$$

$$= \lambda^{2}\, e^{-\lambda}.e^{\lambda} + \lambda$$

$$= \lambda^{2} + \lambda \text{---------------------------- (ii)}$$

$$\mu_3' = E(X^3) = \sum_{X=0}^{\infty} x^{3} p(x, \lambda)$$

$$= \sum_{X=0}^{\infty} [x(x-1)(x-2)+3x(x-1)+x]\, e^{-\lambda}\lambda^{x}/x!$$

$$= \sum_{X=0}^{\infty} x(x-1)(x-2)\, e^{-\lambda}\lambda^{x}/x! + 3\sum_{X=0}^{\infty} x(x-1)\, e^{-\lambda}\lambda^{x}/x! + \sum_{X=0}^{\infty} x\, e^{-\lambda}\lambda^{x}/x!$$

$$= e^{-\lambda}\lambda^{3}\, [\sum^{\infty} \lambda^{x-3}/(x-3)!] + 3e^{-\lambda}\lambda^{2}\, [\sum^{\infty} \lambda^{x-2}/(x-2)!] + \lambda$$

$$= e^{-\lambda}\lambda^3 e^{\lambda} + 3e^{-\lambda}\lambda^2 e^{\lambda} + \lambda$$

$$= \lambda^3 + 3\lambda^2 + \lambda \text{ ------------------- (iii)}$$

$$\mu_4' = E(X^4) = \sum_{X=0}^{\infty} x^4 p(x, \lambda)$$

$$= \sum_{x=0}^{\infty} \{x(x-1)(x-2)(x-3) + 6x(x-1)(x-2) + 7x(x-1) + x\}\, e^{-\lambda}\lambda^x / x!$$

$$= e^{-\lambda}\lambda^4 [\sum_{x=4}^{\infty} \lambda^{x-4}/(x-4)!] + 6e^{-\lambda}\lambda^3 [\sum_{x=3}^{\infty} \lambda^{x-3}/(x-3)!] + 7e^{-\lambda}\lambda^2 [\sum_{x=2}^{\infty} \lambda^{x-2}/(x-2)!] + \lambda$$

$$= \lambda^4 (e^{-\lambda} e^{\lambda}) + 6\lambda^3 (e^{-\lambda} e^{\lambda}) + 7\lambda^2 (e^{-\lambda} e^{\lambda}) + \lambda$$

$$= \lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda \text{ -------------- (IV)}$$

These are the four raw moments of Poisson distribution

The four central moments of the Poisson distribution are as follows:-

$\mu_1 = \mu_1'$

$\mu_2 = \mu_2' - \mu_1'^2$

$= (\lambda^2 + \lambda) - \lambda^2$ [from equation (i) and (ii)]

$= \lambda$

So the mean and variance of the Poisson distribution are each equal to λ.

$\mu 3 = \mu_3' - 3\mu_2' \mu_1' + 2\mu_1'^3$

$= (\lambda^3 + 3\lambda^2 + \lambda) - 3\lambda(\lambda^2 + \lambda) + 2\lambda^3$ [from (i), (ii), (iii)]

$= \lambda$

$\mu_4 = \mu_4' - 4\mu_3' \mu_1' + 6\mu_2' \mu_1'^2 - 3\mu_1'^4$

$= (\lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda) - 4\lambda(\lambda^3 + 3\lambda^2 + \lambda) + 6\lambda^2(\lambda^2 + \lambda) - 3\lambda^4$ [from (i), (ii), (iii), (iv)]

$= 3\lambda^2 + \lambda$

PROPERTIES OF POISSON DISTRIBUTION:

- The mean and variance of Poisson distribution are both equal to λ.
- The sum of independent Poisson variables is a further Poisson variable with mean equal to the sum of the individual means.
- The Poisson distribution provides an approximation for the Binomial distribution.

APPLICATIONS OF POISSON DISTRIBUTION:

A few applications of Poisson distribution are depicted below

- Telecommunication example: telephone calls arriving in a system.
- Astronomy example: photons arriving at a telescope.
- Chemistry example: the molar mass distribution of a living polymerization.
- Biology example: the number of mutations on a strand of DNA per unit length.
- Management example: customers arriving at a counter or call centre.
- Finance and insurance example: number of losses or claims occurring in a given period of time.
- Earthquake seismology example: an asymptotic Poisson model of seismic risk for large earthquakes.
- Radioactivity example: number of decays in a given time interval in a radioactive sample.
- Optics example: the number of photons emitted in a single laser pulse. This is a major vulnerability to most Quantum key distribution protocols known as Photon Number Splitting (PNS).
- Electronics: In a Poisson process, the number of observed occurrences fluctuates about its mean λ with a standard deviation $\sigma_{\hat{k}}=\sqrt{\lambda}$. These fluctuations are denoted as Poisson noise or (particularly in electronics) as shot noise. Shot noise or Poisson noise is a type of electronic noise which can be modeled by a Poisson process.
- Computer Science (Microsoft Excel): The POISSON.DIST function is categorized under Excel Statistical functions. It is helpful in calculating the Poisson probability mass function. As a financial analyst, POISSON.DIST is useful in forecasting revenue. Also, we can use it to predict the number of events occurring over a specific time, for example, the number of cars arriving at the mall parking per minute.

CONCLUSION:

We can say that Poisson distribution is useful in rare events where the probability of success (p) is very small and probability of failure (q) is very large and value of ‘n’ is very large. It gives the probability of a number of events in an interval generated by a Poisson process. The Poisson distribution is defined by the rate parameter, λ, which is the expected number of events in the interval. By giving the example on discussion on its applications it can be conclusively seen that the Poisson distribution has a strong theoretical background and very wide spectrum of practical applications. Bringing original and unusual cases, featuring Poisson processes and using softwares like 'Excel' to describe Poisson distribution may provide opportunities for increasing students' eagerness towards learning Probability and Statistics with Reliability, Queuing, and Computer science applications.

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