# Types of Geometry and its Applications

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## Abstract

Geometry , an ancient Greek word ; Geo-" earth " , Metron-"measurement" is branch of mathematics concerned with questions of shapes , size , relative positions of figures , and properties of spaces . Euclid was a Greek mathematician , often referred to as the " **Founder of Geometry** " or the " **Father of Geometry** ".

A mathematician who works in the field of geometry is called a " geometer " .Geometry arose independently in a number of early cultures as a practical way for dealing with length , areas , volumes .Geometry began to see elements of formal mathematical science emerging in the west as early as the 6th century BC . By the 3rd century BC , geometry was put in to an axiomatic form by Euclid , whose treatment , Euclid's Elements ,set a standard for many centuries to follow .

Geometry arose independently in India , with texts providing rules for geometric constructions appearing as early as 3rd century BC . By the early 17th century , geometry had been put on a solid analytic footing by mathematicians such as Rene Descartes and Pierre de Fermat . Since then , and into modern times , geometry has expanded into non-Euclidean geometry and manifolds , describing spaces that lie beyond the normal range of human experience .While geometry has evolved significantly throughout the years , that are some general concepts that are more or less fundamental to geometry . These include the concepts of points , lines , planes , surface , angles and curves , as well as the more advanced notions of manifolds and topology .

Geometry has applications to many fields , including art , architecture , physics as well as to other branch of mathematics .

**KEYWORDS-** Euclidean geometry, Plane Geometry, non-Euclidean geometry, hyperbolic geometry, elliptic /spherical geometry

## Introduction

Geometry is a subject in mathematics , that focused on the study of shapes , size , relative configuration and spatial properties . It was first formally organized by the Greek mathematician Euclid . Geometry has been the subject of countless developments . As a result , many types of geometry exist , including Euclidean , non-Euclidean geometry . The discussion firstly focused on the properties of the lines , points , and angles . We will also place emphasis on geometric measurement including lengths , area and volumes of various shapes . By the end of this section it will not be hard to see that geometry is all around us .Through this project we decided to explain the different types of geometries especially spherical geometry , analytic geometry , hyperbolic geometry , absolute geometry , finite geometry , and 65properties and real life application of that geometry .

**TYPES OFGEOMETRY**

Since we are talking about geometry , we should first establish what we meant by " geometry " . In broad terms , geometry is the realm of math in which we talk about things like points , lines , angles , triangles , circles , squares , and other shapes as well as the properties of all these things .

**2.1 Euclidean geometry**

The types of geometry we typically learn in school – and the types of geometry we usually think of when we think of " geometry " –is known as **Euclidean geometry** . Euclidean geometry gets its name from the ancient Greek mathematician Euclid , who wrote a book called " **The Elements** " over 2000 years ago in which he outlined , derived , and summarized the geometric properties of objects that in flat two dimentional plane .This is why Euclidean geometry is also known as " **Plane Geometry**".

**2.2Non-Euclidean Geometry**

In mathematics , non-Euclidean geometry consist of two geometries based on axioms closely related to those specifying Euclidean geometry . As Euclidean geometry lies at the intersection of metric geometry and affine geometry , non-Euclidean geometry arises when either the metric geometry requirement is relaxed , or the parallel postulate is replaced with an alternative one . In the latter case one obtains **hyperbolic geometry** and **elliptic /spherical geometry** ,the traditional non-Euclidean geometries . When the metric requirement is relaxed , then there are affine planes associated with the planar algebras which give rise to kinematic geometries that have also been called non-Euclidean geometry .

**SOME EXOTIC GEOMETRIES**

**3.1SPHERICAL GEOMETRY**

Spherical geometry is defined as " the study of figures on the surface of a sphere " and is three dimensional , spherical analogue of Euclidean or planar geometry . It is an example of geometry that is not Euclidean . Two practical applications of the principles of spherical geometry are navigation and astronomy .

**3.1.1 Properties**

**3.1.1.1 The Basics Of Spherical Geometry**

A sphere is defined as a closed surface in 3D formed by a set of points an equal distance R from the centre of the sphere , O .The sphere's radius is the distance from the center of the sphere to the sphere's radius , so based on the definition given above, the radius of the sphere is R .

**3.1.1.2 Great Circle**

Great circles are defined as those circles of intersection which share the same radius R and the same centre O as the sphere it intersects . As their name implies, the great circles are the largest circles of intersection of one can obtain by passinga straight plane through a sphere . On the globe , a line or a meridian of longitude forms half of a great circle running from the pole to pole and with its centre at the centre of the earth . Another example of a great circle on the globe is the equator ,found at 0 degree latitude. See the illustration given below;

**3.1.1.3 Spherical Triangle**

When the arcs of three great circle intersect on the surface of a sphere , the lines enclose an area known as a **spherical triangle** . Angle between great circles are measured by calculating the angles between the planes on which the great circles themselves lie . The spherical angle formed by two intersecting arcs of great circle is equal to the angle between the tangent lines formed when the great circle planes touch the circle at their common point ( an antipode of the sphere since two great circles intersect each other in a line passing through the sphere's center) .

**3.1.1.4 Girard's Theorem**

The Girard's theorem is introduced by Albert Girard , it is used to find the area of a triangle with angles $\alpha$ , $\beta$ , $\gamma$ ; states that the area of a triangle with the three angles on a sphere of radius R can be find using the formula ;

$$\textbf{Area} = R^2 (\alpha + \beta + \gamma - \Pi)$$

**3.1.2 Latitude and Longitude Distance**

Latitude is the angle of a point above or below ( North and South ) the equator . See the illustration ;Small circles parallel to equator are called **parallel latitude .** And the angle between these small circles and the equator is called the latitude .

**3.1.2.1 Angular Distance**

The angular distance of latitude on opposite side of the equator can be found by adding angles and the angular distance of latitude on the same side of the equator can be found by subtracting the angles .

**3.1.2.2 Nautical Miles**

Note that **1° = 60 Nautical Mile ( M )**

Therefore ,

Nautical miles on opposite side , for example , if angular distance is 80°, then

$$80° = 80 \times 60 \text{ M} = 4800 \text{ M}$$

$$= 4800 \times 1.853 \text{ km} = 8894.4 \text{ km}$$

**3.1.3 Great Circle Distance**

We have

Earth Radius ≈ 6400 km ( 6371 km)

Therefore , the distance between two great circles is given by the formula ,

$$\mathbf{D = ( \theta \div 360) \times 2\pi R}$$

Where $\theta$ is the angular distance and R is the radius of earth .

**3.2 ANALYTIC GEOMETRY**

Analytical geometry is a branch of algebra that is used to model geometric object- points , ( straight ) lines , and circles being the most basis of these . Analytic geometry is a great invention of Descrates and Fermat . In classical mathematics , analytic geometry is also known as Coordinate geometry or Cartesian geometry , is the study of geometry using a coordinate system .

Analytic geometry is widely used in physics and engineering , and also in aviation , rocketry , space science , and spaceflight . It is the foundation of most modern fields of geometry , including algebraic , differential ,discrete and computational geometry .

**3.2.1 Coordinates**

In analytic geometry , the plane is given a coordinate system , by which every point has a pair of real number coordinates .similarly , Euclidean space is given coordinates where every point has three coordinates . There are variety of coordinate system used , but the most common are following :

**3.2.1.1 Cartesian coordinates**

The most common coordinates system to use is the Cartesian coordinates system , where each point has an x-coordinate representing its horizontal position , and a y-coordinate representing its vertical position . These are typically written as an ordered pair ( x , y ) . This system can also be used for three dimensional geometry , where every point in Euclidean space is represented by an ordered triple of coordinate ( x , y , z ) . See the illustration;

**3.2.1.2 Polar coordinates**

In polar coordinates , every point of the plane is represented by its distance r from the origin and its angle θ from the polar axis .See the illustration ;

**3.2.1.3 Cylindrical coordinates**

In cylindrical coordinates , every point of space is represented by its height z , its radius r from the z-axis and the angle θ its projection on the x-y plane makes with respect to the horizontal axis . See the illustration ;

**3.2.1.4 Spherical coordinates**

In spherical coordinates , every point in space is represented by its **distance** ρ from the origin , the angle θ its projection on the x-y plane makes with respect to the horizontal axis , and the angle φ that it makes with respect to the z-axis . The names of the angles are often reversed in physics . See the illustration ;

**3.2.2 Equations and Curve**

In analytic geometry , any equation involving the coordinates specifies a subset of plane , namely the solution set for the equation , or locus . For example , the equation y=x corresponds to the set of all the points on the plane whose x-coordinate and y-coordinate are equal . These points from a line , and y=x is said to be the equation for this line .In general , linear equation involving x and y specify lines , quadratic equations specify conic sections , and more complicated equations describe more complicated figures.

**3.2.3 Lines and Planes**

Lines in a cartesian plane or , more generally , in affine coordinates , can be described algebraically by linear equations . In two dimensions , the equation for non-vertical lines is often given in the **slope-intercept form** : y=mx+b

where : m is the slope or gradient of the line .

bis the y-intercept of the line .

x is the independent variable of the function y=f(x) .

**3.2.4 Conic Section**

In Cartesian coordinate system , the graph of the quadratic equation in two variables is always a conic section –though it may degenerate , and all conic section arise in this way . The equation will be of the form ;$\mathbf{Ax^2+Bxy+Cy^2+Dx+Ey+F=0}$ ; where A , B , C not all zero.

**3.2.4.1 Ellipse**

An ellipse obtained as the intersection of a cone with an inclined plane . In mathematics , an ellipse is a curve in a plane surrounding two focal point such that the sum of the distances of the two focal point is constant for every point on the curve . As such , it is a generalization of a circle , which is a special type of an ellipse having both focal point on the same location . The elongation of an ellipse is represented by its eccentricity , which for an ellipse can be any number from 0 to arbitrary close to but less than 1 .

The equation of an ellipse is given by ;

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$;where a and b are the semi-major and semi-minor axis .

**3.2.4.2 Circle**

Circle is a special kind of ellipse with both focal point at the same location . As a conic section , the circle is the intersection of a plane perpendicular to the cone's axis . The standard equation for a circle with center ( h , k ) and radius r is $(x-h)^2+(y-k)^2=r^2$

**3.2.4.3 Parabola**

A parabola is the curve formed by the intersection of a right circular conical surface and a plane which is parallel to another plane that is tangential to the conical surface , when the plane is at the same slant as the side of cone . A parabola can also be defined as the set of all point in a plane which are an equal distance away from a given point ( called the focus of the parabola ) and a given line ( called directrix of the parabola ) . The focus doesn't lie on the directrix . The standard equation of parabola is given by ,

$$y^2=4ax$$

**3.2.4.4 Hyperbola**

In mathematics , a hyperbola is a type of smooth curve lying in a plane , defined by its geometric properties or by equations for which it is the solution set . A hyperbola has two pieces , called connected components or branches , that are mirror image of each other and resemble two infinite bows . The hyperbola is one of the three kind of conic section , formed by the intersection of a plane and a double cone . If the plane intersect both halves of the double cone but doesn't passes through the apex of the cone , then the conic is hyperbola .The standard equation of hyperbola is given by ,

**3.3 HYPERBOLIC GEOMETRY**

In mathematics ,**hyperbolic geometry** ( also called Bolyai-Lobachevskian geometry )is a non-Euclidean geometry . In the 18$^{th}$ century , Johann Heinrich Lambert introduced what are today called hyperbolic functions and computed the area of hyperbolic triangle .

In the 19th century , hyperbolic geometry was extensively explored by the Hungarian mathematician Janos Bolyai and the Russian mathematician Nikolai Ivanovich Lobachevsky , after whom it is sometimes named . Lobachevsky published a paper entitled " On the principles of geometry " in 1829-30 , while Bolyai discovered hyperbolic geometry ad published his independent account of non-Euclidean geometry in the paper " The absolute science of space " in 1832 . The term " **hyperbolic geometry** " was introduced by Felix Klein in 1871 .

**3.3.1 Models of Hyperbolic Plane**

There are three models commonly used for hyperbolic geometry ; the Poincare disk model , the Klein model , the Lorentz or hyperboloid model . These models define a hyperbolic plane which satisfies the axioms of hyperbolic geometry . All these models are extendable to more dimensions .

**3.3.1.1 Poincare Disk Model**

Although Lobachevsky and Bolyai discovered the geometry , they can't come up with a genuine model to prove its truth . One of the main model of hyperbolic geometry is called the **Poincare Disk Model** , which is named after the mathematician Henri Poincare .

**3.3.3 Properties**

**3.3.3.1 Lines**

The definition of a line using Poincare model is an arc of a circle that is orthogonal to the circumference of the disk . Basically it means that each line on the Poincare model forms a right angle at the circumference .

**3.3.3.2 Hyperbolic Triangles**

Hyperbolic triangle can be created from the intersection of three orthogonal lines in hyperbolic geometry , just a triangle can be made by lines in Euclidean geometry . Unlike Euclidean triangles , where the angles always add up to 180° , in hyperbolic geometry sum of the angles of hyperbolic triangle is always less than 180° .

**3.3.3.3 Ideal Triangles**

Ideal triangles are special kind of hyperbolic triangles , whose vertices lie on the boundary of Poincare disk . Since these vertices are lie in the boundaries of disk , the lines get infinitely close together making each angle 0° , that means the area of triangle is Π unit and the perimeter is infinity .

**3.4 ABSOLUTE GEOMETRY**

Absolute geometry is a geometry based on an axiom system for Euclidean geometry without the parallel postulate or its alternatives. Traditionally, this has meant using only the first four of Euclid's postulates, but since these are not sufficient as a basis of Euclidean geometry, other systems, such as Hilbert's axioms without the parallel axiom, are used .The term was introduced by Janos Bolyai in 1832.It is sometimes referred to as **neutral geometry,** as it is neutral with respect to the parallel postulate .

**3.4.2 Theorems in absolute geometry**

Exterior angle theorem , Saccheri – Legendre theorem , Alternate Interior Angle Theorem

### 3.5 FINITE GEOMETRY

A **finite geometry** is any geometric system that has only a finite number of points. The familiar Euclidean geometry is not finite, because a Euclidean line contains infinitely many points. A geometry based on the graphics displayed on a computer screen, where the pixels are consideredto be the points would be a finite geometry. While there are many systems that could be called finite geometries, attention is mostly paid to the finite projective and affine spaces because of their regularity and simplicity. Finite geometries may be constructed via linear algebra, starting from vector spaces over a finite field; the affine and projective planes so constructed are called Galois geometries. Finite geometries can also be defined purely axiomatically. Most common finite geometries are Galois geometries, since any finite projective space of dimension three or greater is isomorphic to a projective space over a finite field. However, dimension two has affine and projective planes that are isomorphic to Galois geometry .Similar results holds for other kinds of finite geometries.

## APPLICATIONS

### 4.1 Application of Spherical Geometry

### 5.1.1 Derivation of Cosine Formula

$$\boldsymbol{Cos}\ (\boldsymbol{a}) = \boldsymbol{Cos}\ (\boldsymbol{a})\ \boldsymbol{Cos}\ (\boldsymbol{c}) + \boldsymbol{Sin}\ (\boldsymbol{b})\ \boldsymbol{Sin}\ (\boldsymbol{c})\ \boldsymbol{Cos}\ (\boldsymbol{A})$$

$$\boldsymbol{Cos}\ (\boldsymbol{b}) = \boldsymbol{Cos}\ (\boldsymbol{c})\ \boldsymbol{Cos}\ (\boldsymbol{a}) + \boldsymbol{Sin}\ (\boldsymbol{c})\ \boldsymbol{Sin}\ (\boldsymbol{a})\ \boldsymbol{Cos}\ (\boldsymbol{B})$$

$\boldsymbol{Cos}\ (\boldsymbol{c}) = \boldsymbol{Cos}\ (\boldsymbol{a})\ \boldsymbol{Cos}\ (\boldsymbol{b}) + \boldsymbol{Sin}\ (\boldsymbol{a})\ \boldsymbol{Sin}\ (\boldsymbol{b})\ \boldsymbol{Cos}\ (\boldsymbol{C})$ **.** This formula is called Cosine Formula .

### 4.2 Application of Analytical Geometry

Mathematical application of analytical geometry lie mainly in relating algebra and geometry , but the concept are also useful for solving problems such as calculus on the Cartesian plane . Analytical geometry can also be used in practical world . One very important example is **cartography** , it can be used in the location of places and points on a topographical map . The concept of longitude and latitude is also based on analytical geometry ; which is also used in navigation . Archeologists use Cartesian coordinates system to identify the location of artifacts .

### 4.2.1 Conic Sections

#### 4.2.1.1 Ellipse

The ellipse without doubt one of the most popular curves to be seen .Even if you look around your room in at the moment you will see numerous examples of an ellipse . For example , every time you look at a circular figure at an oblique angle ( one that is not right angle ), the curve you see is an ellipse .

Whispering Galleries

Elliptical Billiard Tables

Satellites

Logos

#### 4.2.1.2 Parabola

Like the ellipse the parabola and its application can be seen extensively in the world around us . The shapes of the car headlight , mirror in reflecting telescope and radio antennae are examples of applications of parabola .

Car Headlight
Television and Radio Antennae
Parabolic Skis
Natural Occurring Phenomena

- A jet of water , like that formed by a fountain , forms the shapes of parabola .
- When a ball is struck in the air , it travels along a path in the shape of parabola .

**4.2.1.3 Hyperbola**

Hyperbolas are widely used in real life . They are used in combat in " sound ranging " to locate the position of enemy guns by detecting the sound of gunfire . If a quantity varies inversely as another quantity , such as pressure and volume of Boyle's Law for a perfect gas at a constant temperature , the graph is a hyperbola. Some astronomical bodies revolve around the sun in a hyperbolic path . One interesting application is if a torch light is placed against the wall its shadow on the wall forms a hyperbolic curve .

**4.3 Application of Hyperbolic Geometry**

**4.3.1 Theory of Special Relativity**

The special theory of relativity , which received its initial formulation by Poincare and Einstein in 1905 , gained general acceptance in 1908 about the same time as Minkowski's interpretation in terms of the 4 dimensional world . Soon after , in the year 1910-14 , Yugoslav mathematician Vladimir Varicakshowed that this theory finds a natural interpretation in hyperbolic geometry , an idea also put forward in less detail by a few other writers about the same time , notably Robb and Borel . Despite its apparently fundamental nature , this hyperbolic interpretation remains little known and has not yet found its way into standard texts on relativity theory , even after nearly a century .

**4.3.2 Hyperbolic Art**

The Dutch artist M.C.Escher was known for this geometric art and for repeating pattern in particular . Escher created a few design that could be interpreted as pattern in hyperbolic geometry . Figure given below is rendition of Escher's best known hyperbolic pattern , *Circle Limit* □ . Escher created his hyperbolic pattern by hand , which was very tedious and time consuming process , since the motifs were of different sizes and slightly different shapes.

**4.3.3 Hyperbolic Tessellation**

A tessellation refers to a uniform tiling of a plane with polygons , such that an equal number of identical polygons meet at each vertex . For example , the tiles in bathroom , the squares of linoleum on an office floor , or the honeycomb pattern in bees' nest are all tessellation of the Euclidean pane .

**Conclusion**

Geometry is a wide and interesting topic in mathematics and we can find even a single application of geometries among the whole things around us. Geometry is involved even in the structure of earth.

In mathematics , there are different type of geometries , relating to different types of objects . For example , earth is almost equated as sphere , and we explain spherical geometry on earth's sphere . Geometry has applications to many fields , including art , architecture , physics , as well as to other branches of mathematics .

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