

Squaring and Cube of Numbers

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Abstract

In this paper result of cubes of two, three, four, five digit numbers are discussed. The result for cubes of (n+1) digit numbers is obtained.

Introduction: Squaring of two and three digit numbers is given by J. Trachtenbeg [1]. The Squaring of four and five digits is given by C.R. Bembelkar and D.B. Dhaigude [2]. The same logic can be applied for Cubes of two, three, four and five digit numbers. Attempt is made in this regard to explain cubes of two, three, four and five digit numbers. The result is also generated for squaring and cubes of (n+1) digit numbers.

(1) Result related to squaring of digit numbers:

Lemma 1.1:[1]

If a_0, a_1 are digits where $a_0 \neq 0$ then

$$\begin{aligned} (a_1 a_0)^2 &= (10a_1 + a_0)^2 \\ &= 10^2 a_1^2 + 10(2a_1 a_0) + a_0^2 \end{aligned}$$

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Example 1: $(65)^2 = (36)(60)(25)$

∩

$$= (36)(62)5$$

$$= 4225$$

6 2

$$\text{i.e. } (65)^2 = 4\ 2\ 2\ 5.$$

Remark : Result is obtained by collapsing the numbers.

Theorem 1.1: [1] If a_0, a_1, a_2 are digits where $a_2 \neq 0$ then

$$(a_2 a_1 a_0)^2 = 10^4 a_2^2 + 10^3 (2a_2 a_1) + 10^2 (a_1^2 + 2a_2 a_0) + 10(2a_1 a_0) + a_0^2$$

Proof : Let a_0, a_1, a_2 be digits where $a_2 \neq 0$ then

$$\begin{aligned} (a_2 a_1 a_0)^2 &= (10^2 a_2 + 10 a_1 + a_0)^2 \\ &= (10^2 a_2 + 10 a_1)^2 + 2(10^2 a_2 + 10 a_1)(a_0) + (a_0)^2 \\ &= 10^4 a_2^2 + 10^3(2a_2 a_1) + 10^2 a_1^2 + 10^2(2a_2 a_0) + 10(2a_1 a_0) + a_0^2 \\ &= 10^4 a_2^2 + 10^3(2a_2 a_1) + 10^2(a_1^2 + 2a_2 a_0) + 10(2a_1 a_0) + a_0^2 \end{aligned}$$

Example 2:

$$\begin{aligned} (345)^2 &= (9) (24) (46) (40) (25) \\ &= (9) (24) (46) (42)5 \\ &= (9) (24) (50)25 \\ &= (9) (29) 025 \\ &= (11)9025 \\ &= 119025 \\ &\quad \quad \quad 2 \ 5 \ 4 \ 2 \\ \text{i.e. } (345)^2 &= 1 \ 1 \ 9 \ 0 \ 2 \ 5 \end{aligned}$$

Remark : Result is obtained by collapsing the numbers.

Theorem 1.2: [2] If a_0, a_1, a_2, a_3 are digits where $a_3 \neq 0$ then

$$\begin{aligned} (a_3 a_2 a_1 a_0)^2 &= 10^6 a_3^2 + 10^5(2a_3 a_2) + 10^4(a_2^2 + 2a_3 a_1) + \\ &\quad 10^3(2a_3 a_0 + 2a_2 a_1) + 10^2(a_1^2 + 2a_2 a_0) + \\ &\quad 10(2a_1 a_0) + a_0^2 \end{aligned}$$

Proof: Let a_0, a_1, a_2, a_3 be digits where $a_3 \neq 0$ then

$$\begin{aligned} (a_3 a_2 a_1 a_0)^2 &= (10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0)^2 \\ &= [(10^3 a_3 + 10^2 a_2)^2] + 2[(10^3 a_3 + 10^2 a_2)(10 a_1 + a_0)] + [(10 a_1 + a_0)^2] \\ &= [10^6 a_3^2 + 2(10^5 a_3 a_2) + 10^4 a_2^2 + 2(10^4 a_3 a_1) + 2(10^3 a_3 a_0) + 2(10^3 a_2 a_1) + \\ &\quad 2(10^2 a_2 a_0) + 10^2 a_1^2 + 10(2a_1 a_0) + a_0^2] \\ &= 10^6 a_3^2 + 10^5(2a_3 a_2) + 10^4(a_2^2 + 2a_3 a_1) + 10^3(2a_3 a_0 + 2a_2 a_1) + \\ &\quad 10^2(a_1^2 + 2a_2 a_0) + 10(2a_1 a_0) + a_0^2 \end{aligned}$$

Example 3: $(5634)^2 = (25) (60) (66) (76) (57) (24) (16)$

$$\begin{aligned} &= (25) (60) (66) (76) (57) (25)6 \\ &= (25) (60) (66) (76) (59)56 \\ &= (25) (60) (66) (81)956 \\ &= (25) (60) (74)1956 \end{aligned}$$

$$=(25) (67)41956$$

$$=(31)741956$$

$$6\ 7\ 8\ 5\ 2\ 1$$

$$\text{i.e. } (5634)^2=3\ 1\ 7\ 4\ 1\ 9\ 5\ 6$$

Remark : Result is obtained by collapsing the numbers.

Theorem 1.3: [2]If a_0, a_1, a_2, a_3, a_4 are digits where $a_4 \neq 0$ then

$$\begin{aligned} (a_4 a_3 a_2 a_1 a_0)^2 &= 10^8 a_4^2 + 10^7 (2a_4 a_3) + 10^6 (a_3^2 + 2 a_4 a_2) + \\ &10^5 (2a_4 a_1 + 2a_3 a_2) + 10^4 (a_2^2 + 2a_4 a_0 + 2a_3 a_1) + \\ &10^3 (2a_3 a_0 + 2a_2 a_1) + 10^2 (a_1^2 + 2a_2 a_0) + 10(2a_1 a_0) + a_0^2 \end{aligned}$$

Proof: Let a_0, a_1, a_2, a_3, a_4 be digits where $a_4 \neq 0$ then

$$\begin{aligned} (a_4 a_3 a_2 a_1 a_0)^2 &= (10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0)^2 \\ &= [(10^4 a_4 + 10^3 a_3)^2 + 2(10^4 a_4 + 10^3 a_3)(10^2 a_2 + 10 a_1 + a_0) + (10^2 a_2 + 10 a_1 + a_0)^2] \\ &= 10^8 a_4^2 + 2 \cdot 10^7 a_4 a_3 + 10^6 a_3^2 + 2(10^6 a_4 a_2 + 10^5 a_4 a_1 + 10^4 a_4 a_0 + 10^5 a_3 a_2 + \\ &10^4 a_3 a_1 + 10^3 a_3 a_0) + 10^4 a_2^2 + 2(10^3 a_2 a_1) + 10^2 a_1^2 + 2(10^2 a_2 a_0 + 10 a_1 a_0) + a_0^2 \\ &= 10^8 a_4^2 + 10^7 (2a_4 a_3) + 10^6 (a_3^2 + 2 a_4 a_2) + 10^5 (2a_4 a_1 + 2a_3 a_2) + \\ &10^4 (a_2^2 + 2a_4 a_0 + 2a_3 a_1) + 10^3 (2a_3 a_0 + 2 a_2 a_1) + 10^2 (a_1^2 + 2a_2 a_0) + 10(2a_1 a_0) + a_0^2 \end{aligned}$$

Example 4: $(25128)^2 = (04) (20) (29) (18) (53) (84) (20) (32) (64)$

$$= (04) (20) (29) (18) (53) (84) (20) (38)4$$

$$= (04) (20) (29) (18) (53) (84) (23)84$$

$$= (04) (20) (29) (18) (53) (86)384$$

$$= (04) (20) (29) (18) (61) 6384$$

$$= (04) (20) (29) (24)16384$$

$$= (04) (20) (31)416384$$

$$= (04) (23)1416384$$

$$= (06)31416384$$

$$2\ 3\ 2\ 6\ 8\ 2\ 3\ 6$$

$$\text{i.e. } (25128)^2 = 6\ 3\ 1\ 4\ 1\ 6\ 3\ 8\ 4$$

Remark : Result is obtained by collapsing the numbers.

Theorem 1.4: [2]General Theorem for squaring of (n+1) digit numbers:

Statement: If $a_0, a_1, a_2, a_3, a_4, \dots, a_n$ are (n+1) digits where $a_n \neq 0$ then

$$(a_0 a_1 a_2 a_3 \dots a_n)^2 = 10^{2n} a_n^2 + 10^{2n-1} (2a_n a_{n-1}) + 10^{2n-2} (a_{2n-1} + 2a_n a_{n-2}) + \dots + 10^2 (a_1^2 + 2a_2 a_0) + 10(2a_1 a_0) + a_0^2$$

(2)Main Results: Now we state and prove a result related to cubes of two, three, four, five digit numbers:

Theorem 2.1: If a_0, a_1 are digits where $a_1 \neq 0$ then

$$(a_1 a_0)^3 = 10^3 a_1^3 + 10^2 (3a_1^2 a_0) + 10(3a_0^2 a_1) + a_0^3$$

Proof: Let a_0, a_1 be digits where $a_0 \neq 0$ then

$$\begin{aligned} (a_1 a_0)^3 &= (10a_1 + a_0)^3 = (10a_1 + a_0)^2 (10a_1 + a_0) \\ &= (10a_1)^3 + 3(10a_1)^2 (a_0) + 3(10a_1)(a_0)^2 + a_0^3 \\ &= 10^3 a_1^3 + 10^2 (3a_1^2 a_0) + 10(3a_0^2 a_1) + a_0^3 \end{aligned}$$

Example 5: $(65)^3 = (6^3) (3 \times 6^2 \times 5) (3 \times 5^2 \times 6) (5^3)$

$$\begin{aligned} &\cap \\ &= (216) (540) (450) (125) \\ &\cap \\ &= (216) (540) (462) 5 \\ &\cap \\ &= (216) (586) 25 \\ & \\ &= (274) 625 \\ &\quad 58 \quad 46 \quad 12 \\ \text{i.e. } (65)^3 &= 274 \quad 6 \quad 2 \quad 5 \end{aligned}$$

Remark : Result is obtained by collapsing the numbers.

Theorem 2.2: If a_0, a_1, a_2 are digits where $a_2 \neq 0$ then

$$(a_2 a_1 a_0)^3 = 10^6 a_2^3 + 10^5 (3a_2^2 a_1) + 10^4 (3a_1^2 a_2 + 3a_2^2 a_0) + 10^3 (a_1^3 + 6a_2 a_1 a_0) + 10^2 (3a_1^2 a_0 + 3a_0^2 a_2) + 10(3a_1 a_0^2) + a_0^3$$

Proof : Let a_0, a_1, a_2 be digits where $a_2 \neq 0$ then

$$\begin{aligned} (a_2 a_1 a_0)^3 &= (10^2 a_2 + 10a_1 + a_0)^3 = (10^2 a_2 + 10a_1 + a_0)^2 (10^2 a_2 + 10a_1 + a_0) \\ &= [10^4 a_2^2 + 10^3 (2a_2 a_1) + 10^2 (a_1^2 + 2a_2 a_0) + 10(2a_1 a_0) + a_0^2] (10^2 a_2 + 10a_1 + a_0) \text{---by th(1.1)} \end{aligned}$$

$$\begin{aligned}
 &= [10^6 a_2^3 + 10^5 (2a_2^2 a_1) + 10^4 (a_1^2 a_2 + 2a_2^2 a_0) + 10^3 (2 a_2 a_1 a_0) + 10^2 a_0^2 a_2] + \\
 & [10^5 a_2^2 a_1 + 10^4 (2a_2 a_1^2) + 10^3 (a_1^3 + 2a_2 a_1 a_0) + 10^2 (2a_1^2 a_0) + 10 a_0^2 a_1] + \\
 & [10^4 a_2^2 a_0 + 10^3 (2a_2 a_1 a_0) + 10^2 (a_1^2 a_0 + 2a_2 a_0^2) + 10 (2a_1 a_0^2) + a_0^3] \\
 &= 10^6 a_2^3 + 10^5 (3a_2^2 a_1) + 10^4 (3a_1^2 a_2 + 3a_2^2 a_0) + 10^3 (a_1^3 + 6 a_2 a_1 a_0) + 10^2 (3a_1^2 a_0 + 3a_2 a_0^2) \\
 &+ 10 (3a_1 a_0^2) + a_0^3
 \end{aligned}$$

Example 6: $(345)^3 = (27) (108) (279) (424) (465) (300) (125)$

$$= (27) (108) (279) (424) (465) (312) 5$$

$$= (27) (108) (279) (424) (496) 25$$

$$= (27) (108) (279) (473) 625$$

$$= (27) (108) (326) 3625$$

$$= (27) (140) 63625$$

$$= 41063625$$

$$14 \quad 32 \quad 47 \quad 49 \quad 31 \quad 12$$

$$\text{i.e. } (345)^3 = 41 \quad 0 \quad 6 \quad 3 \quad 6 \quad 2 \quad 5$$

Remark : Result is obtained by collapsing the numbers.

Theorem 2.3: If a_0, a_1, a_2, a_3 are digits where $a_3 \neq 0$ then

$$\begin{aligned}
 (a_3 a_2 a_1 a_0)^3 &= 10^9 a_3^3 + 10^8 (3a_3^2 a_2) + 10^7 (3a_3 a_2^2 + 3 a_3^2 a_1) + \\
 & 10^6 (a_2^3 + 3 a_3^2 a_0 + 6a_3 a_2 a_1) + 10^5 (3a_2^2 a_1 + 3a_1^2 a_3 + 6a_3 a_2 a_0) + \\
 & 10^4 (3a_1^2 a_2 + 3a_2^2 a_0 + 6a_3 a_1 a_0) + 10^3 (a_1^3 + 3 a_0^2 a_3 + 6a_2 a_1 a_0) + \\
 & 10^2 (3a_1^2 a_0 + 3 a_0^2 a_2) + 10 (3a_0^2 a_1) + a_0^3
 \end{aligned}$$

Proof : Let a_0, a_1, a_2, a_3 be digits where $a_3 \neq 0$ then

$$\begin{aligned}
 (a_3 a_2 a_1 a_0)^3 &= (10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0)^3 \\
 &= (10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0)^2 (10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0) \\
 &= [10^6 a_3^2 + 10^5 (2a_3 a_2) + 10^4 (a_2^2 + 2 a_3 a_1) + 10^3 (2a_3 a_0 + 2a_2 a_1) + \\
 & 10^2 (a_1^2 + 2a_2 a_0) + 10 (2a_1 a_0) + a_0^2] (10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0) \quad \text{----by th(1.2)} \\
 &= [10^9 a_3^3 + 10^8 (2a_3^2 a_2) + 10^7 (a_2^2 a_3 + 2 a_3^2 a_1) + 10^6 (2a_3^2 a_0 + 2 a_3 a_2 a_1) + \\
 & 10^5 (a_1^2 a_3 + 2 a_3 a_2 a_0) + 10^4 (2 a_3 a_1 a_0) + 10^3 a_0^2 a_3] + [10^8 a_3^2 a_2 + \\
 & 10^7 (2a_3 a_2^2) + 10^6 (a_2^3 + 2 a_3 a_2 a_1) + 10^5 (2a_3 a_2 a_0 + 2a_2^2 a_1) + \\
 & 10^4 (a_1^2 a_2 + 2a_2^2 a_0) + 10^3 (2 a_2 a_1 a_0) + 10^2 a_0^2 a_2] + \\
 & [10^6 a_3^2 a_0 + 10^5 (2a_3 a_2 a_0) + [10^7 a_3^2 a_1 + 10^6 (2a_3 a_2 a_1) + \\
 & 10^5 (a_2^2 a_1 + 2 a_3 a_1^2) + 10^4 (2a_3 a_1 a_0 + 2a_2 a_1^2) + 10^3 (a_1^3 + 2a_2 a_1 a_0) + \\
 & 10 (2a_1^2 a_0) + a_0^2 a_1] + [10^6 a_3^2 a_0 + 10^5 (2a_3 a_2 a_0) + 10^4 (a_2^2 a_0 + 2 a_3 a_1 a_0) + \\
 & 10^3 (2a_3 a_0^2 + 2a_2 a_1 a_0) + 10^2 (a_1^2 a_0 + 2a_2 a_0^2) + 10 (2a_1 a_0^2) + a_0^3]
 \end{aligned}$$

$$\begin{aligned}
 &= 10^9 a_3^3 + 10^8 (3a_3^2 a_2) + 10^7 (3a_2^2 a_3 + 3a_3^2 a_1) + 10^6 (a_2^3 + 3a_3^2 a_0 + 6a_3 a_2 a_1) + \\
 &10^5 (3a_2^2 a_1 + 3a_1^2 a_3 + 6a_3 a_2 a_0) + 10^4 (3a_1^2 a_2 + 3a_2^2 a_0 + 6a_3 a_1 a_0) + \\
 &10^3 (a_1^3 + 3a_3 a_0^2 + 6a_2 a_1 a_0) + 10^2 (3a_1^2 a_0 + 3a_2 a_0^2) + 10(3a_1 a_0^2) + a_0^3
 \end{aligned}$$

Example 6:

$$\begin{aligned}
 (5247)^3 &= (125) (150) (360) (773) (708) (1020) (1135) (630) (588) (343) \\
 &= (125) (150) (360) (773) (708) (1020) (1135) (630) (622) 3 \\
 &= (125) (150) (360) (773) (708) (1020) (1135) (692) 23 \\
 &= (125) (150) (360) (773) (708) (1020) (1204) 223 \\
 &= (125) (150) (360) (773) (708) (1140) 4223 \\
 &= (125) (150) (360) (773) (822) 04223 \\
 &= (125) (150) (360) (855) 204223 \\
 &= (125) (150) (445) 5204223 \\
 &= (125) (194) 55204223 \\
 &= 144455204223
 \end{aligned}$$

$$19 \quad 44 \quad 85 \quad 82 \quad 114 \quad 120 \quad 69 \quad 62 \quad 34$$

$$\text{i.e. } (5247)^3 = 144 \quad 4 \quad 5 \quad 5 \quad 2 \quad 0 \quad 4 \quad 2 \quad 2 \quad 3$$

Remark : Result is obtained by collapsing the numbers.

Theorem 2.4: If a_0, a_1, a_2, a_3, a_4 are digits where $a_4 \neq 0$ then

$$\begin{aligned}
 (a_4 a_3 a_2 a_1 a_0)^3 &= 10^{12} a_4^3 + 10^{11} (3a_4^2 a_3) + 10^{10} (3a_3^2 a_4 + 3a_4^2 a_2) + 10^9 (a_3^3 + 6a_4 a_3 a_2 + 3a_4^2 a_1) + \\
 &10^8 (3a_2^2 a_4 + 3a_3^2 a_2 + 3a_4^2 a_0 + 6a_4 a_3 a_1) + 10^7 (3a_2^2 a_3 + 3a_3^2 a_1 + 6a_4 a_3 a_0 + \\
 &6a_4 a_2 a_1) + 10^7 (3a_2^2 a_3 + 3a_3^2 a_1 + 6a_4 a_3 a_0 + 6a_4 a_2 a_1) + 10^6 (a_2^3 + 3a_1^2 a_4 + \\
 &3a_3^2 a_0 + 6a_4 a_2 a_0 + 6a_3 a_2 a_1) + 10^5 (3a_2^2 a_1 + 3a_1^2 a_3 + 6a_4 a_1 a_0 + 6a_3 a_2 a_0) + \\
 &10^4 (3a_1^2 a_2 + 3a_2^2 a_0 + 3a_0^2 a_4 + 6a_3 a_1 a_0) + 10^3 (a_1^3 + 3a_3 a_0^2 + 6a_2 a_1 a_0) + \\
 &10^2 (3a_0^2 a_2 + 3a_1^2 a_0) + 10(3a_1 a_0^2) + a_0^3
 \end{aligned}$$

Proof: Let a_0, a_1, a_2, a_3, a_4 be digits where $a_4 \neq 0$ then

$$\begin{aligned}
 (a_4 a_3 a_2 a_1 a_0)^3 &= (10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0)^3 \\
 &= (10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0)^2 (10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0) \\
 &= [10^8 a_4^2 + 10^7 (2a_4 a_3) + 10^6 (a_3^2 + 2a_4 a_2) + 10^5 (2a_4 a_1 + 2a_3 a_2) + \\
 &10^4 (a_2^2 + 2a_4 a_0 + 2a_3 a_1) + 10^3 (2a_3 a_0 + 2a_2 a_1) + 10^2 (a_1^2 + 2a_2 a_0) + 10(2a_1 a_0) \\
 &+ a_0^2] (10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10 a_1 + a_0) \quad \text{----- by th(1.4)} \\
 &= [10^{12} a_4^3 + 10^{11} (2a_4^2 a_3) + 10^{10} (a_3^2 a_4 + 2a_4^2 a_2) + 10^9 (2a_4^2 a_1 + 2a_3 a_4 a_2) + \\
 &10^8 (a_2^2 a_4 + 2a_4^2 a_0 + 2a_4 a_3 a_1) + 10^7 (2a_3 a_4 a_0 + 2a_2 a_4 a_1) + 10^6 (a_1^2 a_4 + \\
 &2a_4 a_2 a_0) + 10^5 (2a_4 a_1 a_0) + 10^4 a_0^2 a_4] + [10^{11} a_4^2 a_3 + 10^{10} (2a_4 a_3^2)
 \end{aligned}$$

$$\begin{aligned}
 &+10^9(a_3^3+2 a_4a_3a_2)+10^8(2a_4a_3a_1+2a_3^2a_2)+ 10^7(a_2^2a_3+2a_4a_3a_0+2a_3^2a_1)+ \\
 &10^6(2a_3^2a_0+2 a_2a_3a_1) +10^5(a_1^2a_3+2a_3a_2 a_0)+10^4(2a_3a_1 a_0)+ 10^3 a_0^2a_3] \\
 &+ [10^{10}a_4^2a_2+10^9(2a_4a_3a_2)+10^8(a_3^2a_2+2 a_4a_2^2)+ 10^7(2a_4a_2a_1+2a_3a_2^2)+ \\
 &10^6(a_2^3+2a_4a_2a_0+2a_3a_2a_1)+10^5(2a_3a_2a_0+2 a_2^2a_1) +10^4(a_1^2a_2+2a_2^2a_0)+ \\
 &10^3(2a_2a_1 a_0)+ 10^2a_0^2a_2]+ [10^9a_4^2a_1+10^8(2a_4a_3a_1)+10^7(a_3^2a_1+2 a_4a_2a_1) \\
 &+10^9a_4^2a_1+10^8(2a_4a_3a_1)+10^7(a_3^2a_1+2 a_4a_2a_1)+ [10^9a_4^2a_1+10^8(2a_4a_3a_1) \\
 &+10^7(a_3^2a_1+2 a_4a_2a_1)+ 10^6(2a_4a_1^2+2a_3a_2a_1)+ 10^5(a_2^2a_1+2a_4a_1a_0+ \\
 &2a_3a_1^2)+10^4(2a_3a_1a_0+2 a_2a_1^2) +10^3(a_1^3+2a_2a_1 a_0) + 10^2(2a_1^2a_0)+ \\
 &10 a_0^2a_1] + [10^8a_4^2a_0+10^7(2a_4a_3a_0) +10^6(a_3^2a_0+2 a_4a_2a_0) + \\
 &10^5(2a_4a_1a_0+2a_3a_2a_0)+ 10^4(a_2^2a_0+2a_4a_0^2+2a_3a_1a_0)+ \\
 &10^3(2a_3a_0^2+2 a_2a_1a_0) +10^2(a_1^2a_0+2a_2 a_0^2)+10(2a_1 a_0^2)+ a_0^3] \\
 = &10^{12}a_4^3+10^{11}(3a_4^2a_3)+10^{10}(3a_3^2a_4+3 a_4^2a_2)+ 10^9(a_3^3+6a_4a_3a_2+ 3a_4^2a_1)+ \\
 &10^8(3a_2^2a_4+3a_3^2a_2+3a_4^2a_0+6a_4a_3 a_1)+ 10^7(3a_2^2a_3+3a_3^2a_1+6a_4a_3 a_0+ \\
 &6a_4a_2 a_1)+ 10^6(a_2^3+3a_1^2a_4+3a_3^2a_0+6a_4a_2 a_0+6a_3a_2 a_1)+10^5(3a_2^2a_1+ \\
 &3a_1^2a_3+6a_4a_1 a_0+ 6a_3a_2 a_0)+10^4(3a_1^2a_2+3a_2^2a_0+3a_0^2a_4+6a_3a_1 a_0)+ \\
 &10^3(a_1^3+3a_3a_0^2+ 6a_2a_1a_0)+ 10^2(3a_0^2a_2+3 a_1^2a_0)+ 10(3a_1a_0^2)+a_0^3
 \end{aligned}$$

Example 6:

$$\begin{aligned}
 (25128)^3 &= (8)(60) (162) (209)(297) (669) (781) (498) (900) (1064) (288) (384) (512) \\
 &= (8)(60) (162) (209)(297) (669) (781) (498) (900) (1064) (288) (435)2 \\
 &= (8)(60) (162) (209)(297) (669) (781) (498) (900) (1064) (331)52 \\
 &= (8)(60) (162) (209)(297) (669) (781) (498) (900) (1097)152 \\
 &= (8)(60) (162) (209)(297) (669) (781) (498) (1009)7152 \\
 &= (8)(60) (162) (209)(297) (669) (781) (598)97152 \\
 &= (8)(60) (162) (209)(297) (669) (840)897152 \\
 &= (8)(60) (162) (209)(297) (753)0897152 \\
 &= (8)(60) (162) (209)(372) 30897152 \\
 &= (8)(60) (162) (246)2 30897152 \\
 &= (8)(60) (186)6230897152 \\
 &= (8)(78)66230897152 \\
 &= 15866230897152
 \end{aligned}$$

7 18 24 37 75 84 59 100 109 33 43 51

i.e. (25128)³=15 8 6 6 2 3 0 8 9 7 1 5 2

Remark : Result is obtained by collapsing the numbers.

Lastly we generalized the result for cubes of (n+1) digit number.

Theorem 2.5: General Theorem for Cube of (n+1) digit numbers:

Statement: If $a_0, a_1, a_2, a_3, a_4, \dots, a_n$ are (n+1) digits where $a_n \neq 0$ then

$$\begin{aligned} (a_0 a_1 a_2 a_3 a_4 \dots a_n)^3 = & 10^{3n} a_n^3 + 10^{3n-1} (3a_n^2 a_{n-1}) + 10^{3n-1} (3a_{n-1}^2 a_n + 3a_n^2 a_{n-2}) + \\ & 10^{3n-3} (a_{n-1}^3 + 6a_n a_{n-1} a_{n-2} + 3a_n^2 a_{n-3}) + \dots + \\ & 10^3 (a_1^3 + 3a_3 a_0^2 + 6a_2 a_1 a_0) + 10^2 (3a_0^2 a_2 + 3a_1^2 a_0) + 10(3a_1 a_0^2) + a_0^3 \end{aligned}$$

This generalized result is obtained using principle of mathematical induction.

Discussion and conclusion:

From the discussion of the Squaring of two and three digit numbers is discussed in [1] and squaring of four and five digits is discussed in [2], it is clear that, these results are more useful as far as the time required for calculation. The student taking competitive examinations where the use of calculators is strictly prohibited, will definitely welcome the method discussed in this paper. In present article the same logic can be applied for Cubes of two, three, four and five digit numbers. Attempt is made in this regard to explain cubes of two, three, four and five digit numbers. The result is also generated for cubes of (n+1) digit numbers.

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