Existence of $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-type and $[(t_1+t_2)/z\sqrt{2}]$-type plane waves in V_5 for bimetric relativity

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Abstract

The bimetric relativity admits $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-type and $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational waves in five dimensional space-times V_5 having two time axes where in the later case the space-times can be reduced to conformal one.

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§1.Introduction

Reformulating Karade's (1994) definition of plane wave, we have obtained the plane wave solutions of the field equations of the field equations $N_i^j = 0$ in bimetric theory of gravitation proposed by Rosen(1973,74) where at each point of the space-times there are two line elements

$$ds^2 = g_{ij}dx^i dx^j \qquad \text{and} \qquad d\sigma^2 = f_{ij}dx^i dx^j \tag{1.1}$$

and established the existence of $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-type and $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational waves in four dimensional space-time V_4 having two time axes in the paper refer it to [1] and [2] respectively. This work has been extended to higher five dimensional space-time in the paper refer it to [3] where we have obtained the plane wave solutions of the field equation $N_i^j = 0$ in V_5 having two time axes are given by g_{ij} which satisfied

$$Q\rho_i^j + P\sigma_i^j = 0 \tag{1.2}$$

which further breaks in

$$\bar{\bar{w}}_3\rho_i^j + \bar{w}_3\sigma_i^j = 0 = \bar{\bar{\phi}}_3\rho_i^j + \bar{\phi}_3\sigma_i^j, \quad \bar{\bar{w}}_4\rho_i^j + \bar{w}_4\sigma_i^j = 0 = \bar{\bar{\phi}}_4\rho_i^j + \bar{\phi}_4\sigma_i^j, \tag{1.3}$$

where $w_3 = t_2 + \phi_3 z$, $\quad w_4 = t_2 + \phi_4 t_1$, $\quad \phi_3 = \frac{Z_{,3}}{Z_{,5}}$, $\quad \phi_4 = \frac{Z_{,4}}{Z_{,5}}$,

$$M_3 = \bar{w}_3 - \bar{\phi}_3 z, \qquad M_4 = \bar{w}_4 - \bar{\phi}_4 t_1,$$

$$N_3 = \bar{\bar{w}}_3 - \bar{\bar{\phi}}_3 z, \qquad N_4 = \bar{\bar{w}}_4 - \bar{\bar{\phi}}_4 t_1,$$

$$\rho_i^j = [(\phi_3^2 - \phi_4^2) - 1] g^{hj} \bar{g}_{hi} \quad \text{and} \quad \sigma_i^j = \frac{d}{dZ}\{[1 - (\phi_3^2 - \phi_4^2)] g^{hj} \bar{g}_{hi}\}.$$

In the present paper, we have studied these solutions (1.2) in detail for $[z - \frac{1}{\sqrt{2}}(t_1 + t_2)]$-type and $[(t_1 + t_2)/z\sqrt{2}]$-type plane waves in higher five dimensional space-times V_5 having two time axes for biometric relativity.

§ 2. $[z - \frac{1}{\sqrt{2}}(t_1 + t_2)]$-type plane wave in V_5

Let $Z = [z - \frac{1}{\sqrt{2}}(t_1 + t_2)]$ $\quad \Rightarrow Z_{,3} = 1$, $\quad Z_{,4} = -\frac{1}{\sqrt{2}}$, $\quad Z_{,5} = -\frac{1}{\sqrt{2}}$.

Then $\phi_3 = \frac{Z_{,3}}{Z_{,5}} = -\sqrt{2}$, $\quad \phi_4 = \frac{Z_{,4}}{Z_{,5}} = 1$.

Also $w_3 = t_2 + \phi_3 z = -Z\sqrt{2} - t_1$, $\quad w_4 = t_2 + \phi_4 t_1 = -Z\sqrt{2} + z\sqrt{2}$

$$\Rightarrow \bar{w}_3 = -\sqrt{2}, \qquad \bar{w}_4 = -\sqrt{2}.$$

Hence $M_3 = \bar{w}_3 - \bar{\phi}_3 z = -\sqrt{2}$, $\quad M_4 = \bar{w}_4 - \bar{\phi}_4 t_1 = -\sqrt{2}$

and $N_3 = \bar{\bar{w}}_3 - \bar{\bar{\phi}}_3 z = 0$, $\quad N_4 = \bar{\bar{w}}_4 - \bar{\bar{\phi}}_4 t_1 = 0$

$$\Rightarrow Q = 0 \qquad \because N_2 = N_3 = Q$$

$$\Rightarrow \sigma_i^j = 0. \tag{2.1}$$

With the above values, the L.H.S. of the field equations (1.2) become zero and hence the equation is identically satisfied. Therefore, it implies that $[z - \frac{1}{\sqrt{2}}(t_1 + t_2)]$-type plane gravitational wave in five dimensional space-time V_5 having two time axes exists in biometric relativity.

§ 3. $[(t_1+t_2)/z\sqrt{2}]$-type plane wave in V_5

Let $Z=[(t_1+t_2)/z\sqrt{2}]$

$$\Rightarrow Z_{,3}=-(t_1+t_2)/z^2\sqrt{2}, \qquad Z_{,4}=\frac{1}{z\sqrt{2}}, \qquad Z_{,5}=\frac{1}{z\sqrt{2}}$$

Then $\phi_3=\frac{Z_{,3}}{Z_{,5}}=-Z\sqrt{2}, \qquad \phi_4=\frac{Z_{,4}}{Z_{,5}}=1.$

Also $w_3=t_2+\phi_3 z=-t_1, \qquad w_4=t_3+\phi_4 t_1=zZ\sqrt{2},$

$$\Rightarrow \overline{w}_3=0, \qquad \overline{w}_4=z\sqrt{2}.$$

Hence $M_3=\overline{w}_3-\overline{\phi}_3 z=z\sqrt{2}, \qquad M_4=\overline{w}_4-\overline{\phi}_4 t_1=z\sqrt{2}$

$$\Rightarrow P=z\sqrt{2}, \qquad \because M_3=M_4=P$$

and $N_3=\overline{\overline{w}}_3-\overline{\overline{\phi}}_3 z=0, \qquad N_4=\overline{\overline{w}}_4-\overline{\overline{\phi}}_4 t_1=0$

$$\Rightarrow Q=0 \qquad \because N_3=N_4=Q$$

$$\Rightarrow \sigma_i^j=0. \tag{3.1}$$

and the field equation (1.2) reduces to

$$\{[1-(\phi_3^2-\phi_4^2)]g^{hj}\overline{g}_{hi}=c_i^j \qquad \text{i.e.,} \qquad 2[1-Z^2]g^{hj}\overline{g}_{hi}=c_i^j$$

where c_i^j are constants.

If we choose δ_i^j in particular, we get

$$[1-Z^2]g^{hj}\overline{g}_{hi}=\delta_i^j \qquad \text{i.e., } [1-Z^2]\overline{g}_{ki}=g_{ki}$$

and then $g_{ki} = D_{ki}[\frac{1+Z}{1-Z}]^{1/2}$ where D_{ki} are constants.

Noting (1.1), the space-times V_5 admitting $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational waves becomes

$$ds^2 = \left[\sqrt{\frac{z\sqrt{2}+(t_1+t_2)}{z\sqrt{2}-(t_1+t_2)}}\right] D_{ij}dx^i dx^j$$

which is reducible to a conformal space-times.

Conclusion : *The biometric relativity admits* $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$*-type and* $[(t_1+t_2)/z\sqrt{2}]$*-type plane gravitational waves in five dimensional space-times* V_5 *having two time axes where in the later case the space-times can be reduced to conformal one.*

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