Kinematic and Dynamic Analysis of 2r Flexible Manipulator

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Abstract

Flexible robots are widely used in space applications for their quick response, lower energy consumption, lower overall mass and operation at high speeds. These robots are inherently flexible and kinematics of these cannot be solved using rigid body assumptions. The end point position of flexible robot will depend on amount of flexibility of the link under consideration. The flexibility in the link makes it a continuous dynamical system with infinite degrees of freedom. This paper presents a systematic and generalized approach for mathematical modelling of a planar manipulator with multi links. Flexible links are treated as Euler Bernoulli beams. The joints are assumed rigid throughout this work. The resulting generalized models are simulated in MSC ADAMS software, for a simplified case of two link flexible planar manipulator with end effectors motion in a horizontal straight line. To evaluate the effect of flexibility on the manipulators, first, both links are considered rigid, then first link is made flexible, finally both links are made flexible and analysis is performed for varying payload at the tip of the link. The variation of tip position for all these cases is simulated using MSC ADAMS, the results show flexibility significantly affects the system behaviour. The torque varies by more than 60% for a flexible link with payload when compared with the rigid link with payload. Therefore, it is imperative to take into account the effects of link flexibility while determining the required torque at the respective joints for manipulating the end effector.

KEYWORDS— Euler Bernoulli beams, link flexibility, Assumed Mode Method, kinematic and dynamic analysis

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I. Introduction

Robotic manipulators are widely used to help in dangerous, monotonous, and tedious jobs. Most of the existing robotic manipulators are designed and build in a manner to maximize stiffness in an attempt to minimize the vibration of the end-effector to achieve good position accuracy. This high stiffness is achieved by using heavy material and a bulky design. In order to improve industrial productivity, it is required to reduce the weight of the arms and/or to increase their speed of operation. For these purposes it is very desirable to build flexible robotic manipulators, but the greatest disadvantage of these manipulators is the vibration problem due to low stiffness. researchers worldwide are nowadays engaged in the investigation of dynamics and control of flexible manipulator.

II. Literature Review

When the rigid links are replaced by lightweight, they undergo some damped vibration before it comes to the steady state. if the flexible links are moved with low speed they behave as rigid joints.B. Subudhi and A.S. Morris [1] proposed dynamic modeling technique for a manipulator with multiple flexible links and flexible joints, based on a combined Euler–Lagrange formulation and assumed modes method.Zhang Ding-guo and Zhou Sheng-fen [2,4] developed the dynamic modeling and simulation of an N-flexible-link and N-flexible joint robot.Santosha Kumar Dwivedy and Peter Eberhard, [3] provided a survey of the literature related to dynamic analyses of flexible robots. Both link and joint flexibility are considered in this work and the results are applied to general class of problems. Khairudin et al [5] presented the dynamic modeling and characterization of a two-link flexible robot manipulator, incorporating structural damping, hub inertia and payload that moves in the horizontal plane. Gerasimos G. Rigatos [7] presented a comparative study on representative methods for model-based and model-free control of flexible-link robots.

III. Problem Statement

The objective of this paper is to obtain a mathematical model for n-link flexible manipulator. And there by evaluate the effect of flexibility on the kinematic and dynamic performance of the manipulators. As a case study, a 2 link manipulator is modelled using Msc Adams and it is made to move in a horizontal straight line, flexibility is first varied by varying the links(Rigid-Rigid/Flexible-Rigid/Flexible-Flexible) and second by varying the payloads ((M_L 10N and 50N) and its effect on the torque generated is found and tabulated. Link parameters assumed throughout the paper are tabulated in Table 1.

TABLE I. Link Parameters

Length of the link (L1)	300 mm
Length of the link (L2)	400 mm
Width of the link (w)	40 mm
Radius of holes (r)	10 mm
Depth of the cross section of link (d)	20 mm
Cross Section Area (A)	w x d mm^2
Moment of inertia (I)	$(w*d^3)/12$ mm^4
Young's Modulus of the material (E)	$2*10^5$ N/mm^2
Density (ρ)	$7.8*10^{-3}$ g/mm^3
Payload mass (M_p)	10N and 50N

IV. Research Methodology

IV.I Mathematical formulation for two-link manipulator:

Considering a single rigid link robotic arm, the tip position of the link can be easily expressed by equation 1.

$$(X_0, Y_0) = (L\cos(\theta t), L\sin(\theta t)) \quad (1)$$

Using Assumed Modes Method, the flexible link's tip positions were found. The Assumptions made for modelling a flexible link with Assumed mode method (AMM).

- The flexible link with uniform density and flexural rigidity is considered.
- Euler-Bernoulli beam is assumed i.e. vibration of beam considered only in transverse direction.
- The deflection of the flexible link is small compared to the length of the link..

When the link undergoes angular motion the tip of the link vibrates which dies down with time. Therefore, the position of any point and the tip of the link can be defined as

$$P_0(x, t) = A_R\, P(x, t) \qquad (2)$$

where $A_R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ the rotation matrix.

The position vector of any point on link with body attached frame of reference is given by

$$P(x, t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y(x, t) \end{bmatrix} \qquad (3)$$

The position vector along the length of the link depends on the lateral deformation ‘y’ of the link at that section at a given time. Value of ‘y’ can be found by Assumed Mode Method (AMM). The problem of flexible link can be solved assuming it as a Euler- Bernoulli’s cantilever beam with a payload ‘M_P’ at the tip of the beam undergoing free vibration, the governing equation to represent the vibration of link can be written as follows.

$$E\,I\,\frac{\partial^4 y(x,t)}{\partial x^4} + \rho\,A\,\frac{\partial^2 y(x,t)}{\partial t^2} = 0 \qquad (4)$$

Boundary Conditions*:* Since, the equation of motion Eq. 4 involves a second order derivative with time and a fourth order derivative with ‘x’, two initial conditions and four boundary conditions are needed for finding a unique solution for y(x, t) and they are given in following Equations 5 to 10

$$y(x, t = 0) = y_i \qquad (5)$$

$$\frac{\partial y(x,t=0)}{\partial t} = \left.\frac{\partial y(x,t)}{\partial t}\right|_{t=0} = y_i' \qquad (6)$$

$$y(x,t)|_{x=0} = 0; \qquad (7)$$

$$\left.\frac{\partial y(x,t)}{\partial x}\right|_{x=0} = 0; \qquad (8)$$

$$EI\left.\frac{\partial^2 y(x,t)}{\partial x^2}\right|_{x=L} = -J_L\frac{\partial^2}{\partial t^2}\left(\left.\frac{\partial y(x,t)}{\partial x}\right|_{x=L}\right) \qquad (9)$$

$$EI\left.\frac{\partial^3 y(x,t)}{\partial x^3}\right|_{x=L} = -M_L\frac{\partial^2}{\partial t^2}\left(y(x,t)|_{x=L}\right) \qquad (10)$$

The equation used for computing the tip position of a flexible two link manipulator for the given joint motions is given by

$$y(x,t) = \sum_{j=1}^{2} C_{1,j}\sin(\omega_j t)\left\{\left(\cos\left(\beta_j x\right) - \cosh\left(\beta_j x\right)\right) - \alpha\left(\sin\left(\beta_j x\right) - \sinh\left(\beta_j x\right)\right)\right\}$$
(11)

j = 1,2, and Where α is

$$\alpha = \frac{-\beta_j^{\,3}\cos(\beta_j L) - \beta_j^{\,3}\cosh(\beta_j L) + \frac{M_L}{\rho}\beta_j^{\,4}\sin(\beta_j L) - \frac{M_L}{\rho}\beta_j^{\,4}\sin h(\beta_j L)}{-\beta_j^{\,3}\sin(\beta_j L) - \beta_j^{\,3}\sinh(\beta_j L) - \frac{M_L}{\rho}\beta_j^{\,4}\cos(\beta_j L) + \frac{M_L}{\rho}\beta_j^{\,4}\cos h(\beta_j L)}$$

From these, the generated torque equations are

$$\tau_1 = [(\tfrac{1}{3}m_1 + m_2)\, L_1^{\,2} + \tfrac{1}{3}m_2\, L_2^{\,2} + m_2\, L_1\, L_2\, C_2]\, \ddot{\theta}_1 +$$

$$m_2\, [\tfrac{1}{3}\, L_2^{\,2} + \tfrac{1}{2}L_1\, L_2\, C_2]\, \ddot{\theta}_2 - m_2\, L_1\, L_2\, S_2\dot{\theta}_1\dot{\theta}_2 - \tfrac{1}{2}\, m_2\, L_1\, L_2\, S_2\dot{\theta}_2^{\,2} + (\tfrac{1}{2}m_1 + m_2)gL_1\, C_1 + (\tfrac{1}{2}m_2\, gL_2C_{12}) \qquad (12)$$

$$\tau_2 = m_2\, (\tfrac{1}{3}\, L_2^{\,2} + \tfrac{1}{2}L_1\, L_2C_2)\ddot{\theta}_1 + \tfrac{1}{3}\, m_2\, L_2^{\,2}\ddot{\theta}_2 + \tfrac{1}{2}\, m_2\, L_1\, L_2\, S_2\dot{\theta}_1^{\,2} + (\tfrac{1}{2}m_2\, gL_2C_{12}) \qquad (13)$$

A. Simulation

MSC ADAMS/View software is used to create static, kinematic and dynamic analysis of virtual prototypes by computer simulation. The link parameters assumed are tabulated in table 1. A manipulator with two links and an end effector is modelled, by making the end effector to move in a horizontal straight line as shown in fig 1.

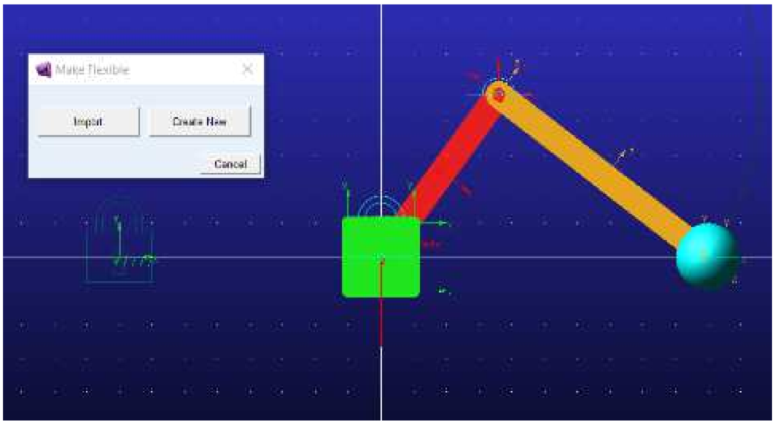

Fig.1. Horizontal Straight line motion as constraint

The links are made flexible by using five steps as shown in Fig. 2.

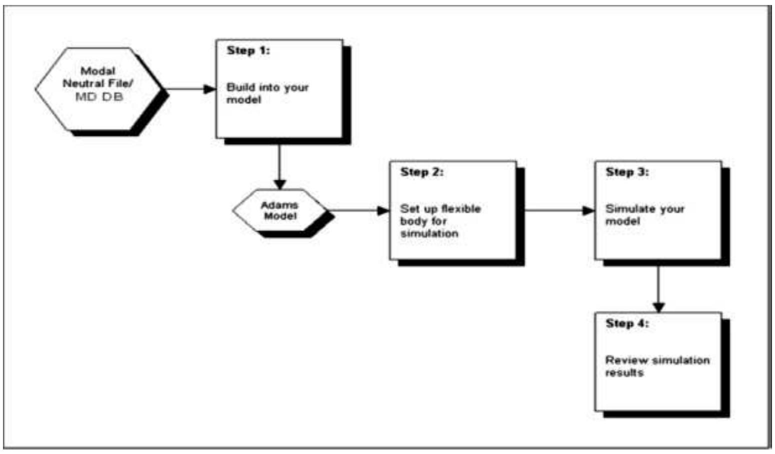

Fig. 2. Steps to convert rigid link to flexible links

V. Results and Discussion

- Horizontal Straight Line Motion as Constraint

A. The torque, force, position, velocity and acceleration differences observed for case 1(rigid-rigid), case 2(flex-rig) and case 3 (flex-flex) at joint 1, joint 2 and end effector are presented in Table 2. The time of simulation considered is 2sec. for 200 steps. To observe the effect of flexibility the simulations are performed with different load conditions (10Nand 50N payload mass).

Rigid-Rigid Case

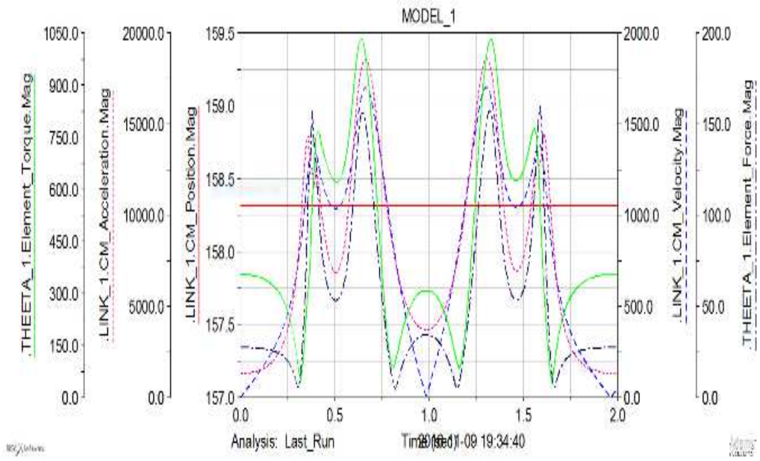

Fig.3. Position, Velocity, Acceleration, Force and Torque's generated at link 1

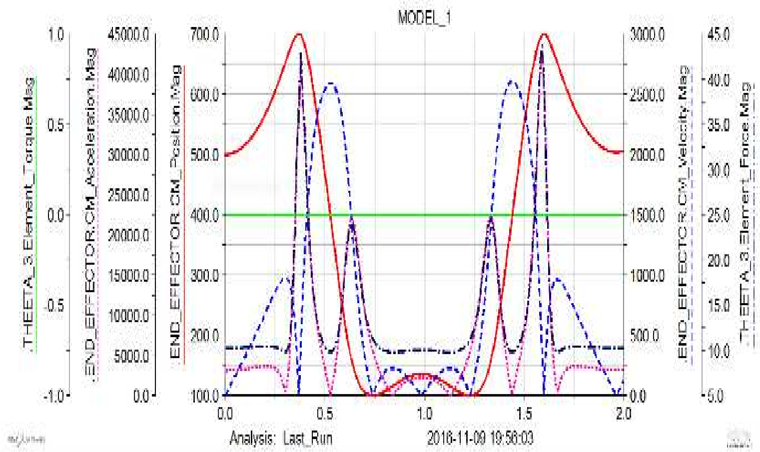

Fig.4. Position, Velocity, Acceleration, Force and Torque's generated at End Effector

The Torque, Force, Position, Velocity, Acceleration's generated at link 1 and end effector for (rigid-rigid) case with 10N payload are shown in the Figures 3 and 4. From the graphs it can be observed that magnitude of the position of centre of mass of link one is constant and there is a continuous increase and then decrease in the magnitude of the position of the Centre of mass of the end effector, as the end effector is made to move from (500,0) to (700,0) and then again back towards (0,0). And y coordinate is made constant. From the graph we can observe that initial and final velocities are maintained zero. The acceleration depends on the variation of velocity with time.

Flexible-Rigid Case

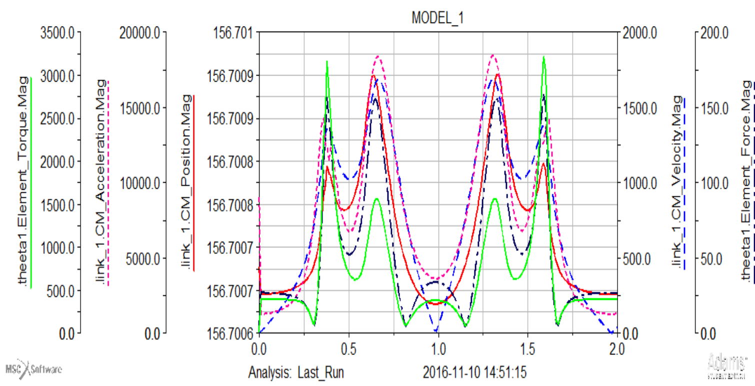

Fig.5. Position, Velocity, Acceleration, Force and Torque's generated at link 1

The Position, Velocity, Acceleration, Force and Torque's generated at link 1 with 10N payload for (flexible-rigid) case are shown in the Fig 5. The fluctuations of acceleration at the beginning from the graph fig5 for link1 is due to the vibrations of link due to flexibility. There is a slight fluctuation in position because of the Flexibility induced.

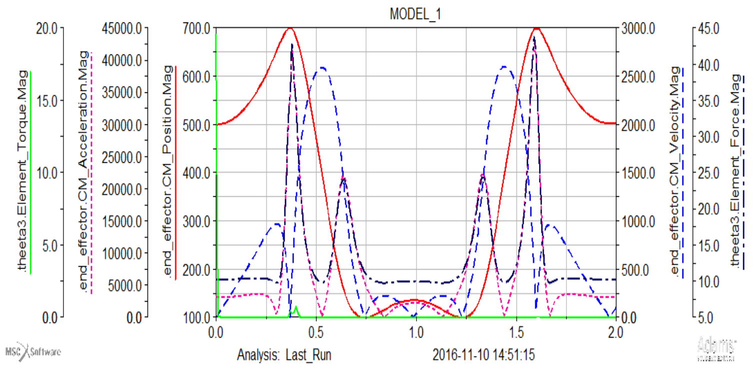

Fig.6. Position, Velocity, Acceleration, Force and Torque's generated at End Effector

The Position, Velocity, Acceleration, Force and Torque's generated at end effector with 10N payload for (flexible-rigid) case are shown in the Fig 6. There is a continuous increase and then decrease in the magnitude of the position of the Centre of mass of the end effector, as the end effector is made to move from (500,0) to (700,0) and then again back towards (0,0). And y coordinate is made constant. From the graph we can observe that initial and final velocities are maintained zero. The acceleration depends on the variation of velocity with time.

Flexible-Flexible case

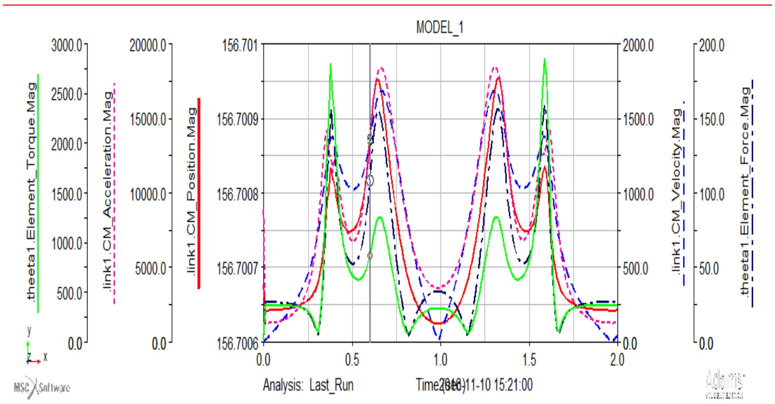

Fig.7. Position, Velocity, Acceleration, Force and Torque's generated at Link 1

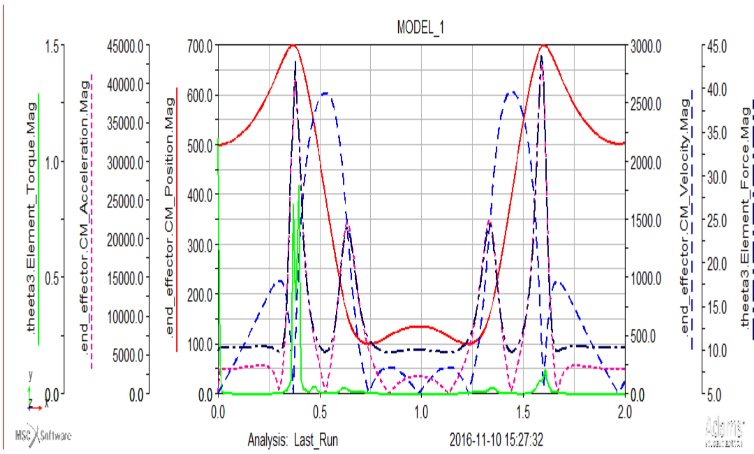

Fig.8. Position, Velocity, Acceleration, Force and Torque's generated at End Effector

The Position, Velocity, Acceleration, Force and torque's generated at link 1 and End effector for (flexible-flexible) case are shown in the Fig 7 and 8. The fluctuations of acceleration at the beginning from the graphs Fig 7 and Fig 8 for link1 and end effector are due to the vibrations of link due to flexibility.

TABLE 2: Torque, Force, Position, Velocity, Acceleration generated at Link1, Link 2 for R-R/F-R/F-F cases

Rigid-Rigid Case

MASS OF END EFFECTOR = 1000 g

PARAMETERS	LINK 1			LINK 2			END EFFECTOR		
	MIN	MAX	AVG	MIN	MAX	AVG	MIN	MAX	AVG
TORQUES (Nmm)	46	1034	441	20	2633	659	2*10^-15	2*10^-15	2*10^-15
FORCES (N)	4.6	160	54	1	131	32	9.8	43	13
POSITION (mm)	158.312	158.312	158.312	96.2958	503.7085	285.0529	100.0081	699.9999	383.0754
VELOCITIES(mm/sec)	0	1704	774	0	2377	1074	0	2605	831
ACCELERATIONS(mm/s2)	1330	18591	7707	2719	33695	9797	15	42620	7582

MASS OF END EFFECTOR = 5000 g

PARAMETERS	LINK 1			LINK 2			END EFFECTOR		
	MIN	MAX	AVG	MIN	MAX	AVG	MIN	MAX	AVG
TORQUES (Nmm)	38	957	443	17	5462	922	6*10^-16	1.6*10-15	1*10-15
FORCES (N)	4.6	298	65	0.8	273	46	49	198	60
POSITION (mm)	158.3182	158.3182	158.3182	96.2926	503.7067	276.7982	100.0021	699.9974	371.0341
VELOCITIES (mm/s)	0	1615	726	0	1873	994	0	1787	747
ACCELERATIONS (mm/s2)	623	17273	6836	1274	30843	7936	63	38523	5052

Flexible-Rigid Case

MASS OF END EFFECTOR = 1000 g

PARAMETERS	LINK 1			LINK 2			END EFFECTOR		
	MIN	MAX	AVG	MIN	MAX	AVG	MIN	MAX	AVG
TORQUES (Nmm)	24	3218	725	5	1085	324	9*10^-5	19.5	0.1
FORCES (N)	2.2	159	53	0.3	132	32	9.8	44	13.6
POSITION (mm)	156.7006	156.7009	156.707	96.7405	503.2567	284.816	99.999	699.9956	382.7906
VELOCITIES (mm/s)	0	1690	767	0	2373	1075	0	2598	831
ACCELERATIONS(mm/s2)	1310	18466	7675	2706	33745	9808	14	42699	7564

MASS OF END EFFECTOR = 5000 g

PARAMETERS	LINK 1			LINK 2			END EFFECTOR		
	MIN	MAX	AVG	MIN	MAX	AVG	MIN	MAX	AVG
TORQUES(Nmm)	23.6	6393	879	4	2265	411	2*10^-5	17.8	0.1
FORCES(N)	2	310	65	0.6	285	46	49	207	60
POSITION(mm)	156.7006	156.701	156.707	96.7463	503.089	277.1377	100.0106	699.7591	371.555
VELOCITIES(mm/s)	0	1597	718	0	1868	994	0	1779	747
ACCELERATIONS(mm/s2)	612	17377	6828	1265	31948	7986	13	40319	5066

Flexible-Flexible Case

MASS OF END EFFECTOR: 1000 g

PARAMETERS	LINK 1			LINK 2			END EFFECTOR		
	MIN	MAX	AVG	MIN	MAX	AVG	MIN	MAX	AVG
TORQUES(Nmm)	19	2853	643	4	1283	343	1*10^-4	1	0. 026
FORCES (N)	1.9	159	53	0.3	131	32	9.8	43	13
POSITION (mm)	156.7006	156.701	156.7007	96.7406	503.2568	284.8172	99.997	699.9959	382.7914
VELOCITIES(mm/s)	0	1689	766	0	2375	1075	0	2601	831
ACCELERATIONS(mm/s2)	1312	18457	7674	2709	33742	9840	23	42724	7555

MASS OF END EFFECTOR = 5000 g

PARAMETERS	LINK 1			LINK 2			END EFFECTOR		
	MIN	MAX	AVG	MIN	MAX	AVG	MIN	MAX	AVG
TORQUES(Nmm)	19	5602	757	4	2727	438	3*10^-5	3	0.07
FORCES (N)	2	311	65	0.3	286	46	49	207	60
POSITION(mm)	156.7006	156.7011	156.7007	96.7465	503.0896	277.1387	100.0106	699.7591	371.555
VELOCITIES (mm/s)	0	1597	718	0	1870	995	0	1779	747
ACCELERATIONS(mm/s2)	613	17303	6824	1265	31912	8016	5.8	40395	5063

B. Comparision of Position of Link 1 and End Effector for Rigid-Rigid, Flexible-Rigid and Flexible-Flexible Case:

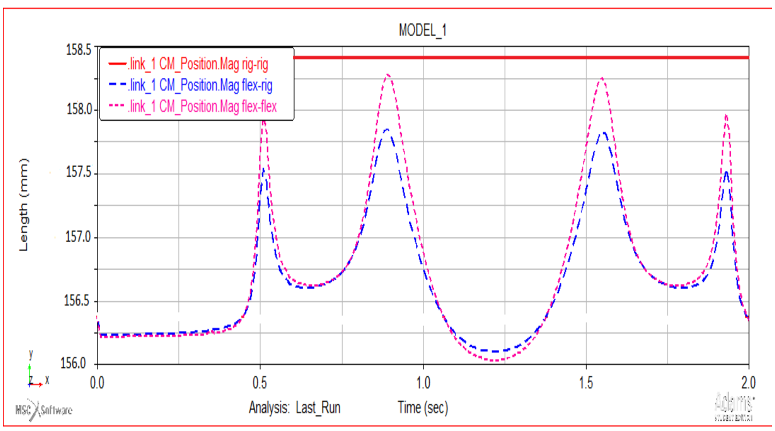

Fig.9. Comparision of position between rig-rig, flex-rig and Flex-Flex case for Link 1

The deviation of link 1 for flex-flex case is compared with Flex-rig and rig-rig case for horizontal motion. Center of mass of link 1 for rig-rig case is moving at 158.312 mm, for flex-rig case, it is varying form 156.7005 mm to 156.704 mm whereas for flex-flex case, it is varying from 156.7006 mm to 156.705 mm. when the link is made flexible, there is a 1.5mm deviation when compared to rig-rig case.

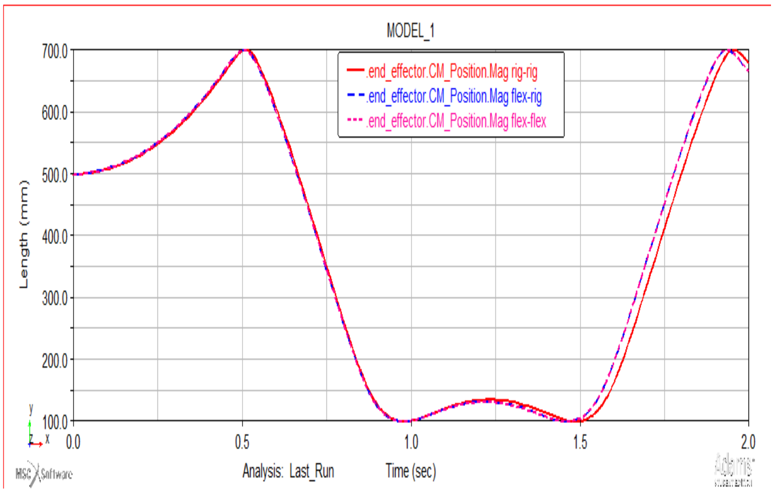

Fig.10. Comparision of position between rig-rig, flex-rig and Flex-Flex case for End Effector

There is a small deviation for flex-flex body when compared to rig-rig body in between 1.5 to 2 sec.

C. Comparision of Velocities of Link 1 and End Effector for Rig-Rig, Flex-Rig and Flex-Flex Case:

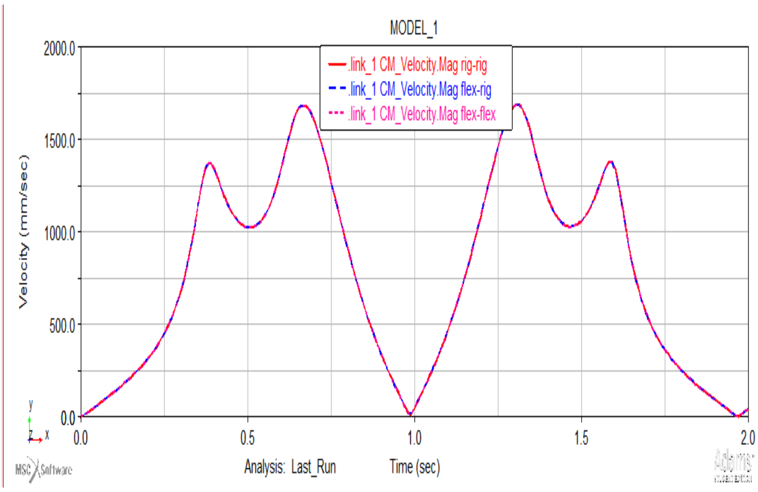

Fig.11. Comparision of velocities between rig-rig, flex-rig and Flex-Flex case for Link 1

The velocity deviation of link 1 for flex-flex case is compared with Flex-rig and rig-rig case for horizontal motion. The velocity of link 1 for rig-rig case is varying from 0 to 1704 mm/s, for flex-rig case, it is varying from 0 to 1690 mm/s, whereas for flex-flex case, it is varying from 0 to 1689 mm/s. When the body is made flexible, velocity is decreased by 15mm/s when compared to rig-rig case.

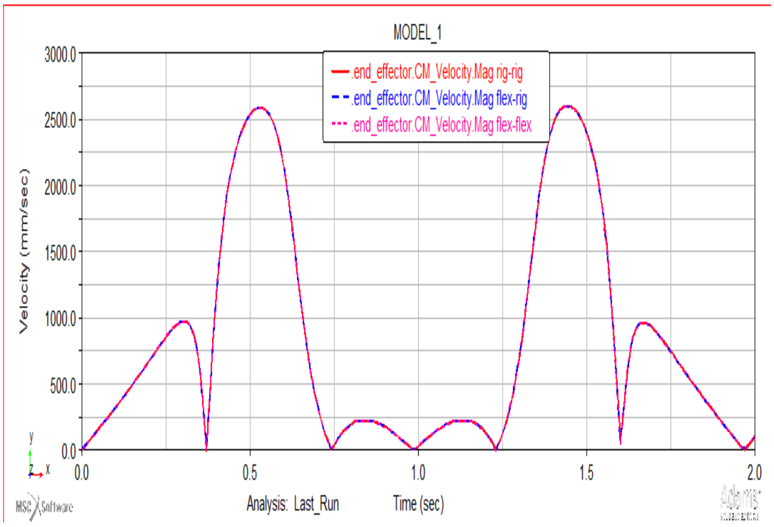

Fig.12. Comparision of velocities between rig-rig, flex-rig and Flex-Flex case for End Effector

When the link is made flexible, velocity is decreased by 4mm/s when compared to rig-rig case.

D. Comparision of Accelerations of Link 1 and End Effector for Rig-Rig, Flex-Rig and Flex-Flex Case:

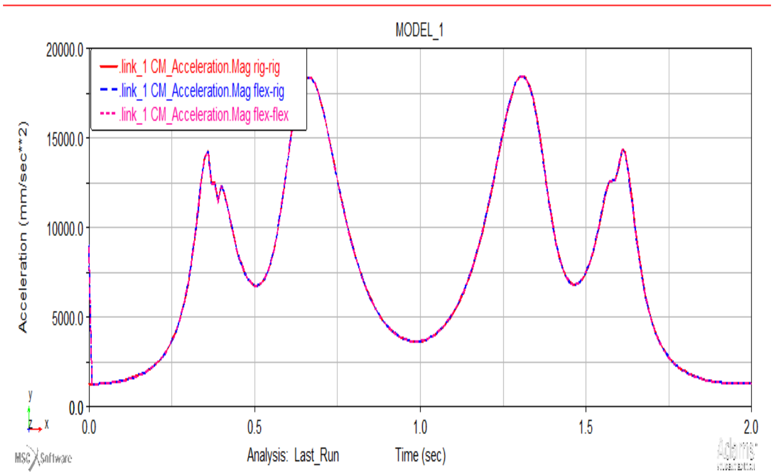

Fig.13. Comparision of acceleration between rig-rig, flex-rig and Flex-Flex case for Link 1

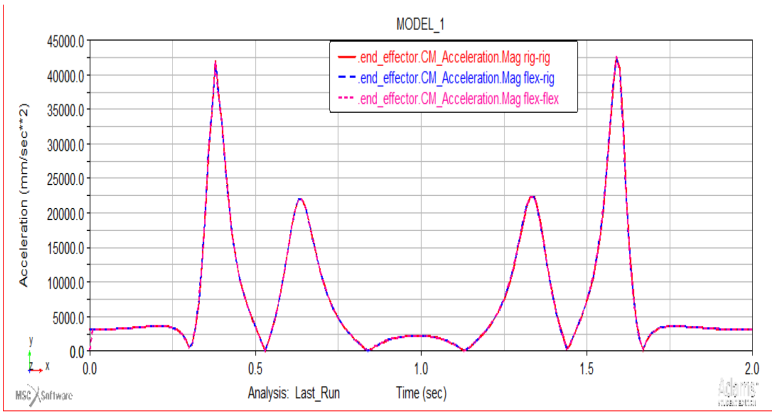

Fig.14. Comparision of acceleration between rig-rig, flex-rig and Flex-Flex case for End Effector

When flexibility is induced, acceleration is decreased by $134 mm/s^2$ for link 1 and increased by $104 mm/s^2$ for end effector. The fluctuations of acceleration at the beginning in the Fig.13 for link1 is due to the vibrations of link due to flexibility.

E. Comparision of Forces at theta 1 and theta 2 for Rig-Rig, Flex-Rig and Flex-Flex Case:

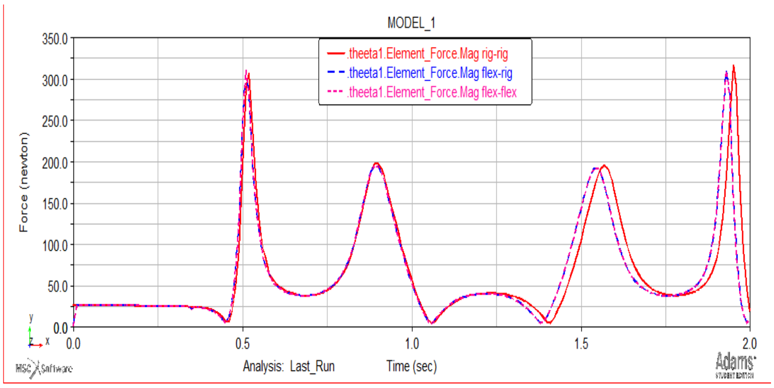

Fig.15.Comparision of Forces at theta1 between rig-rig, Flex-Rig and Flex-Flex case

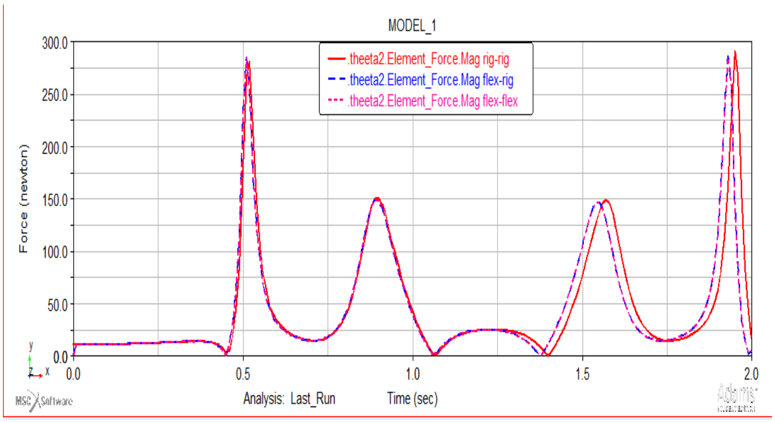

Fig.16.Comparision of Forces at theta2 between rig-rig, Flex-Rig and Flex-Flex case

There is deviation of 13 Newton for flex-flex body when compared to rig-rig body from 1.5s to 2s due to the flexibility induced.

F. Comparision of Torques generated at theta 1 and theta 2 for Rigid-Rigid, Flex-Rig and Flexible-Flexible Case:

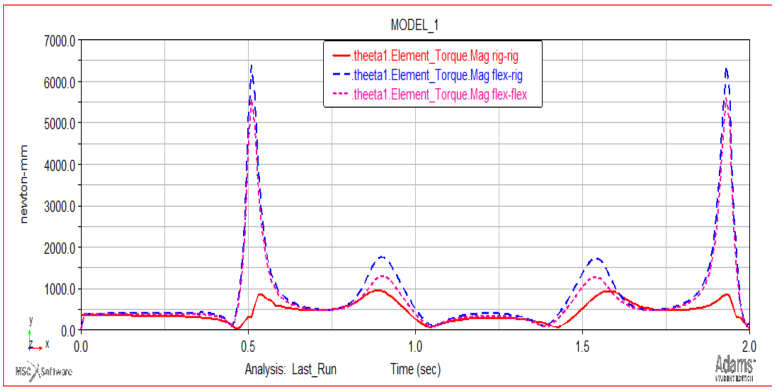

Fig.17.Comparision of torques generated at theta 1 between rig-rig, flex-rig and Flex-Flex case

When the links are made flexible, there is a sudden increase in torque values at theta 1, this is because of the impact produced while the link is changing direction.

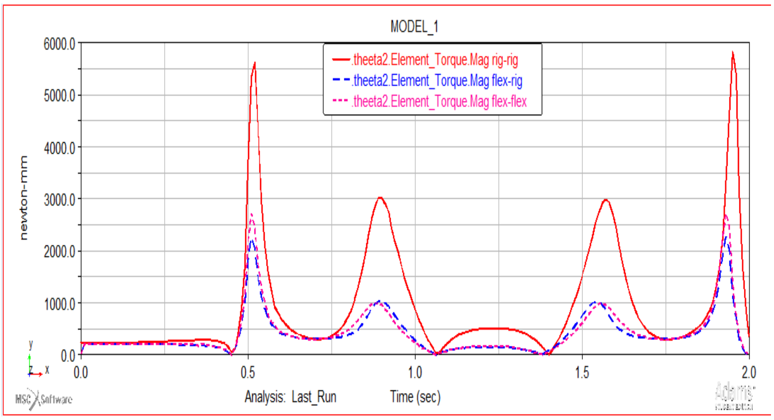

Fig.18.Comparision of torques at theeta 2 between rig-rig, flex-rig and Flex-Flex case

G. Torques at Joints for Straight Line Motion as Constraint with varying payloads

Rigid-Rigid Case

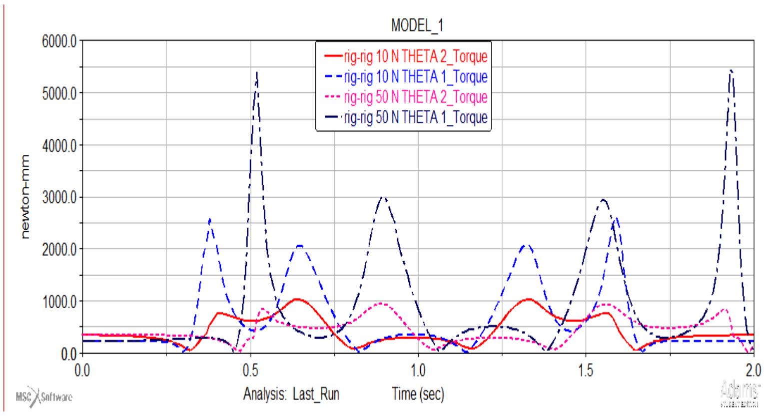

Fig.19. Torques at joint 1 and 2 for varying payloads at End Effector for Rigid-Rigid Case

Flexible- Rigid Case

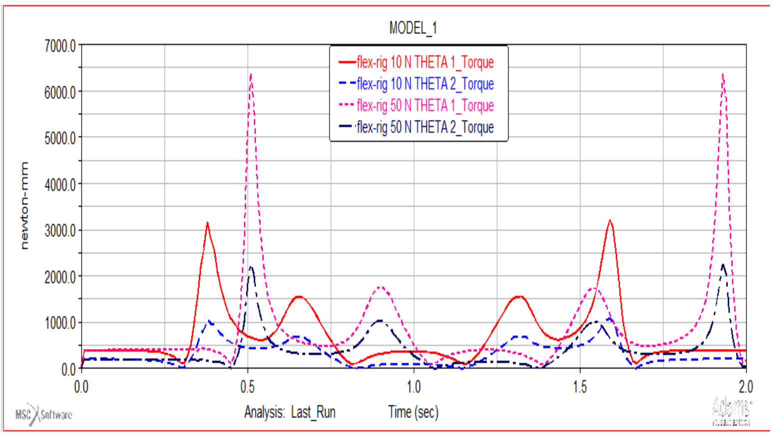

Fig.20. Torques at joint 1 and 2 for varying payloads at End Effector for Flexible-Rigid Case

Flexible-Flexible case

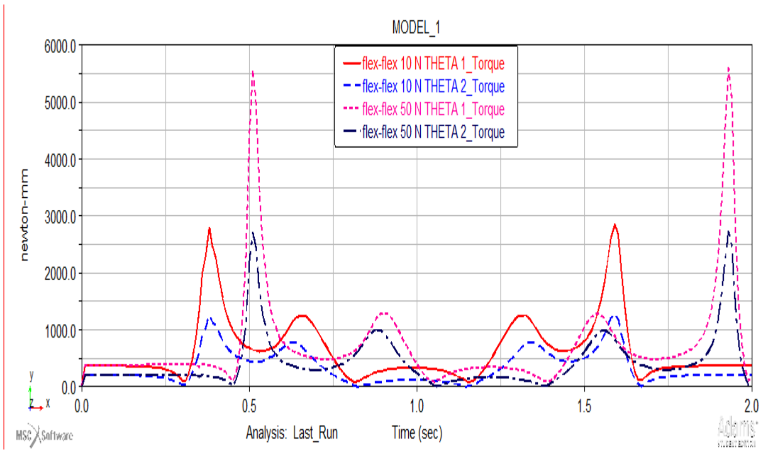

Fig.21. Torques at joint 1 and 2 for varying payloads at End Effector for Flexible-Flexible case

As the payload mass at End Effector is increased, torque values are increased. The torque at joint 1 is found to be more compared to that at joint 2, as the torque acting at joint 1 depends on both the links, whereas at joint 2 the torque acting is only due to the effect second link only.

VI. Conclusion

The mathematical models for rigid and flexible-link manipulators are presented. flexibility deteriorates the accuracy, stability and repeatability, mathematical models of flexibility are to be provided to design the effective controller for the flexible robots. The accuracy of the model will have the significant influence on the controller performance. It is found that the flexibility of link significantly affects the system behaviour.

The conclusions inferred are

1. There is a 1.01% decrease in position of link 1 and 0.03% decrease in position of end effector when both the links are made flexible.
2. There is a 0.88% decrease in velocity of link 1 and 0.15% decrease in velocity of end effector when both the links are made flexible.
3. There is a 0.72% decrease in acceleration of link 1 and 0.24% increase in acceleration of end effector when both the links are made flexible.
4. There is a 0.625% decrease in force at theta 1 and no change in force at theta 2 when both the links are made flexible.
5. There is 63% increase in torque at theta 1 and 51.2% decrease in torque at theta 2 when both the links are made flexible.
6. As the payload mass in increased from 10N to 50N, there was an increase of 96.3% at theta 1 and 112.5% increase at theta 2 for flexible-flexible case.

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