Research in Mathematics Education

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Abstract

In this paper, we made an effort to find ways to motivate and to create interest on the use of technology for teaching and learning of mathematics among High School mathematics Teachers in Mizoram. A workshop with High School Teachers was conducted by Mizoram State Council of Educational Research & Training (SCERT) and we introduced an open source software, namely, GeoGebra in the workshop. We discussed some of the problems from their school text book. We analyzed the impact of the workshop through the pre and post questionnaires. The analysis showed that the training programme was successful in motivating and generating more interest among the teachers. We concluded that in classroom teaching, the teacher should put more emphasis in motivating the students through technology based teaching as far as possible.

Introduction

Today, in many locations around the world, there is a significant gap between the knowledge and skills students learned in school, and there is demand for knowledge and skilled workers in the workplaces and communities. Employers reported that they need students who are better prepared in skills such as professionalism and work ethic, oral and written communication, teamwork and collaboration, critical thinking and problem solving, application of information technology, and leadership.(Sources: The Partnership for 21st Century Skills, enGauge, and SCANS Report highlight different skills and call them “21stCentury Skills.”). So the emphasis in schools is increasingly on learning how to learn, rather than just acquiring specific technical skills that keep changing anyway. Mathematics, to most, is a complex and difficult subject. The tendency for most students is to consider the subject as one that is boring, thus, creating lack of interest in the topics being discussed. This poses a great challenge for teachers and educators, especially in the secondary and senior secondary levels, wherein a good study habit and a firm grasp of basic concepts should be developed.

Why the use of Technology

Technology can provide mechanisms to sustain assistance to mathematics teachers in their use of technology to implement mathematics education reforms in their classes. Technology enables mathematics education reform (Kaput, 1992). We must provide mathematics teachers extended opportunities to experience and do mathematics in an environment supported by diverse technologies (Dreyfus & Eisenberg, 1996). The heart of our approach is the development of mathematical power understanding, using, and appreciating mathematics. Our interest is in empowering teachers through the use of technology in mathematics exploration, open-ended problem solving, interpreting mathematics, developing understanding, and communicating about mathematics

(Bransford, et al, 1999; Schoenfeld, 1982, 1989, 1992; Silver, 1987). In most of the country especially in a developed country, now widely acknowledged that the earlier visions for how pupils' learning might be changed by the inclusion of technology have not translated into widespread changes in classroom practices. This is partly due to an underdeveloped knowledge of how teachers' practices are impacted by the use new of technologies, and subsequently how teachers drive in them within their professional lives, for the purpose of improving pupils' mathematical learning. More recent research has focused on the development of teachers' knowledge and practices within technology enhanced classroom environments. For example, the instrumental approach used in didactics of mathematics (Artigue 2002; Trouche 2005), initially used to analyze students' interactions with technology in mathematics learning, has been applied to the study of teachers' professional development through its central notion of "instrumental genesis", using the concept of orchestration and its extension (Drijvers et al. 2010; Trouche 2005). During PME37 the development of teacher's practices with technology has also been discussed extensively at a Research Forum on Meta-Didactical Transposition (MDT) (Aldon et al. 2013; Arzarello et al. 2014). Other ways to describe the use and knowledge of technologies by teachers is given by theories such as Pedagogic Technological Knowledge (PTK) (Hong and Thomas 2006; Thomas and Hong 2005), Technological, Pedagogical and Content Knowledge (TPACK) (Koehler and Mishra 2009; Mishra and Koehler 2006), and the Structuring Features of Classroom Practice framework (Ruthven 2009). A comprehensive discussion comparing TPACK, the Structuring Features of Classroom Practice Framework and the Instrumental Orchestration Approach can be found in (Ruthven 2014). Further to this, research on teacher identities has also contributed insights into how and why teachers develop their practice (or not) as users of technology based learning. From a socio-cultural perspective, teachers' learning is conceptualized as the evolution of their participation in practices that develop their pedagogical identities, which Wenger describes as "a way of talking about how learning changes who we are" (1998). As this Research Forum is focused on making visible the dynamic processes of teachers' development of their classroom practices with and through technology over time, the theoretical frameworks have been chosen as they enable this temporal element to be seen. However, our choices are not exhaustive!

The workshop

We restate our perspective that 'professional development' encompasses a wide range of individual and collaborative activities across a broad range of structured and informal opportunities, which are constrained by country-specific and cultural boundaries and expectations. Central to all of these activities lies the development of a teacher's mathematical, pedagogical and technological knowledge and practice. Consequently, the notion of an explicit 'design' implies that there has been some fore-thought. While there have been some research studies that have sought to articulate the processes and outcomes of more informal professional development activities (Clark-Wilson et al. 2014), here we focus on professional development that has been constructed for the purpose of developing teacher's classroom uses of technology.

The workshop was conducted for Mathematics teachers from 20 selected High schools in Aizawl district, Mizoram. Before we start the workshop pre-questionnaires were filled up by teachers on the first day to investigate their beliefs about the use of technology based leaning, level of satisfaction about their jobs and about the current curricula and the method they are practicing on teaching and learning. We are happy that most of the participants in the workshop believe that technology based leaning may positively influence students attitude and beliefs towards learning mathematics. However, most of the participants never practice the use of technology for teaching and learning, this may be due to the fact that lack of knowledge on technology based learning; in fact all the participants did not attend such kind of workshop before. After having demonstration of the dynamics software Geogebra through power point presentation the following problem from their current textbook was explore in the workshop and followed with practical class.

Theorem of Thales: The angle in a semi circle is at right angle.

The classical Solution:

Given: AB is a diameter of a circle C (O, r) and $\angle ACB$ is an angle in a semicircle.

To Prove: $\angle ACB = 90^\circ$

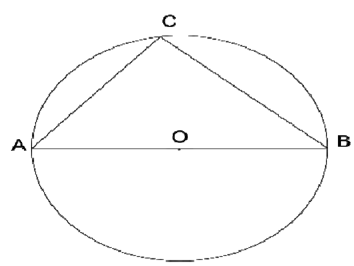


Figure 1

Proof: We know that the angle subtended by an arc at the centre of a circle is twice the angle form by it at any point on the remaining part of the circle.

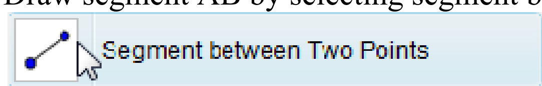
$$\therefore \angle AOB = 2\angle ACB$$

$$\Rightarrow 2\angle ACB = \angle AOB = 180^\circ [\because \angle AOB \textit{ is a straight angle}]$$

$$\Rightarrow \angle ACB = 90^\circ$$

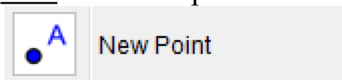
Solution (GeoGebra):

1. Draw segment AB by selecting segment between two point tool

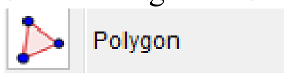


2. Construct a semicircle through points A and B
   <u>Hint</u>: The order of clicking points A and B determines the direction of the semicircle.

3. Create a new point C on the semicircle
   Hint: Check if point C really lies on the arc by dragging it with the mouse.



4. Create triangle ABC



5. Create the interior angles of triangle ABC

6. Move the point C along the circumference and we get the GeoGebra figure below:

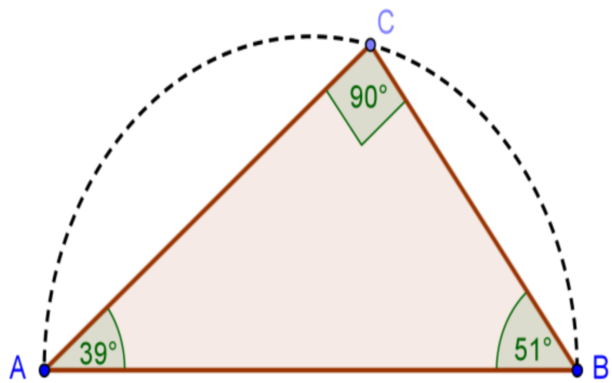
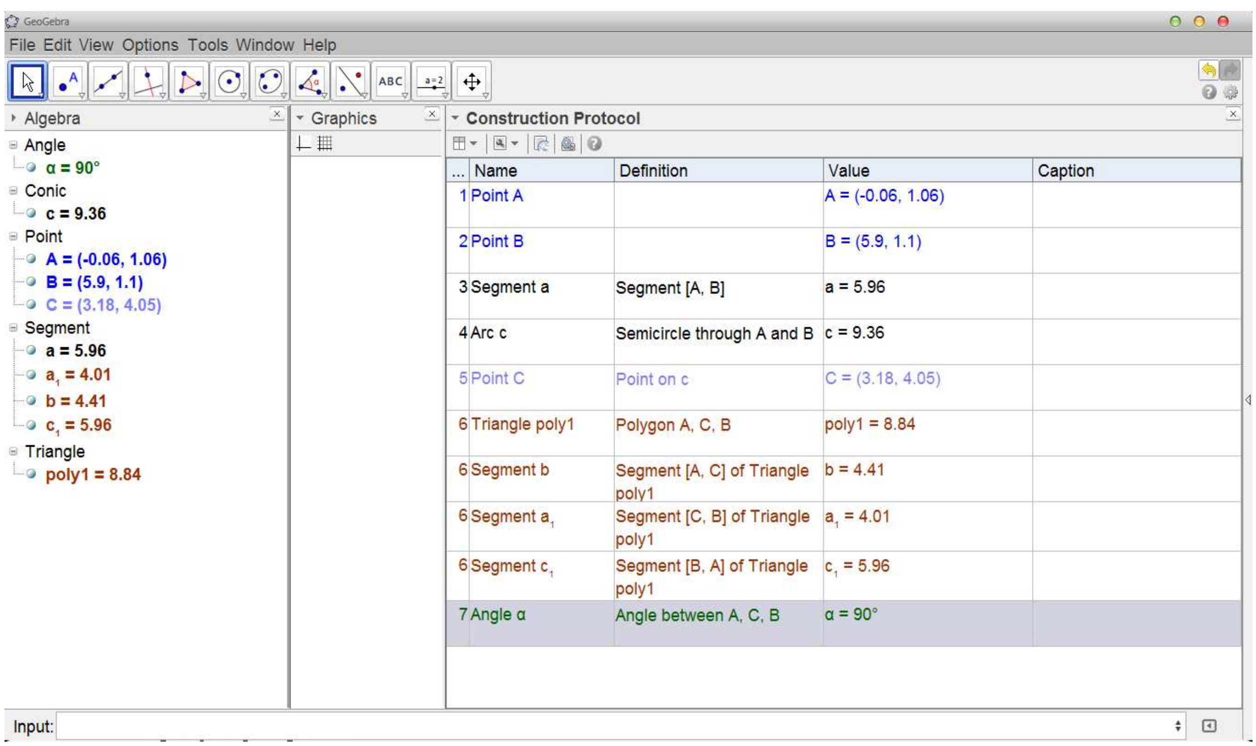


Figure 2

Table 1: Construction Protocol table



...	Name	Definition	Value	Caption
1	Point A		A = (-0.06, 1.06)	
2	Point B		B = (5.9, 1.1)	
3	Segment a	Segment [A, B]	a = 5.96	
4	Arc c	Semicircle through A and B	c = 9.36	
5	Point C	Point on c	C = (3.18, 4.05)	
6	Triangle poly1	Polygon A, C, B	poly1 = 8.84	
6	Segment b	Segment [A, C] of Triangle poly1	b = 4.41	
6	Segment a_1	Segment [C, B] of Triangle poly1	a_1 = 4.01	
6	Segment c_1	Segment [B, A] of Triangle poly1	c_1 = 5.96	
7	Angle α	Angle between A, C, B	α = 90°	

Discussion: Figure 2 shows a dynamic worksheet that allows students to explore the theorem of Thales. Vertex C of the triangle ABC lies on a semicircle over segment AB. Following the instructions above the dynamic figure, students are guided towards discovering the meaning of this theorem, stating that such a triangle is always a right triangle. After coming up with a conjecture, students can verify it for a number of different triangles, which can be created by moving points A and B with the mouse to another position. From the GeoGebra figure above one can easily identify the angle in a semi-circle is at right angle. If we move the point C by seclecting move or drag tools we can find there is no change in the magnitude of the angle C which is more attractive for student to agreed the theorem. The table above is the construction protocol and we can have a complete necessary informations.

Pythagorean Theorem: Let '*a*' and '*b*' be the lengths of sides of a right triangle, and let '*c*' be the length of the hypotenuse. Then the sum of the areas of the squares on '*a'* and *'b'* equal to the area of the sum of the square on '*c'*.

<u>Prove of Pythagorean Theorem</u>: To Prove

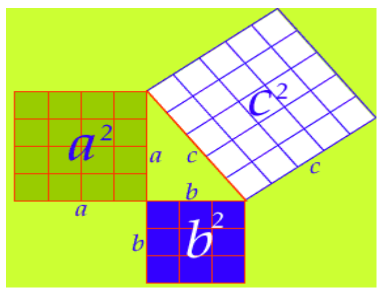


Figure 3

Algebraically we can write $a^2 + b^2 = c^2$.

Consider the given figure below, we take the square with side of length $a+b$ and dissect it into two ways. First into two squares of areas a^2 and b^2, and four triangles of area $\frac{1}{2}ab$. Then into a square of area c^2 and four triangles of area $\frac{1}{2}ab$.

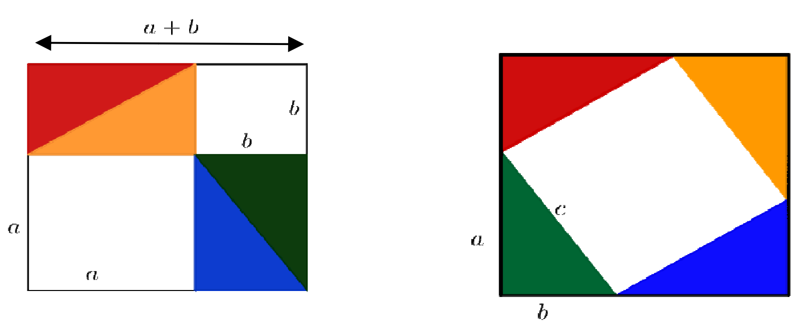


Figure 4

Now, since the two squares are the same, they must have equal area. Thus

$$a^2 + b^2 + 4.\frac{1}{2}ab = c^2 + 4.\frac{1}{2}ab$$

By cancelling $4.\frac{1}{2}ab$ on both sides, we get

$$a^2 + b^2 = c^2$$

Which proves the theorem.

Prove of Pythagorean Theorem (GeoGebra)

To Prove: The sum of the areas of the squares on two sides equal to the area of the sum of the square on the hypotenuse.

1) Start the new sketch and hide the axis.
2) Select segment between two point tool and draw a line segment AB.
3) Select semi-circle through two point tool and draw a semicircle through the line segment.
4) Choose any point on the circle called as C.
5) Select segment between two point tool, joint AC and BC.
6) Select the angle tool find angle C.
7) Hide the semi-circle
8) Select the distance tool and click all the line.
9) Select regular polygon tool and draw a square from all the sides.
10) Select area tool and find the area of a squares.

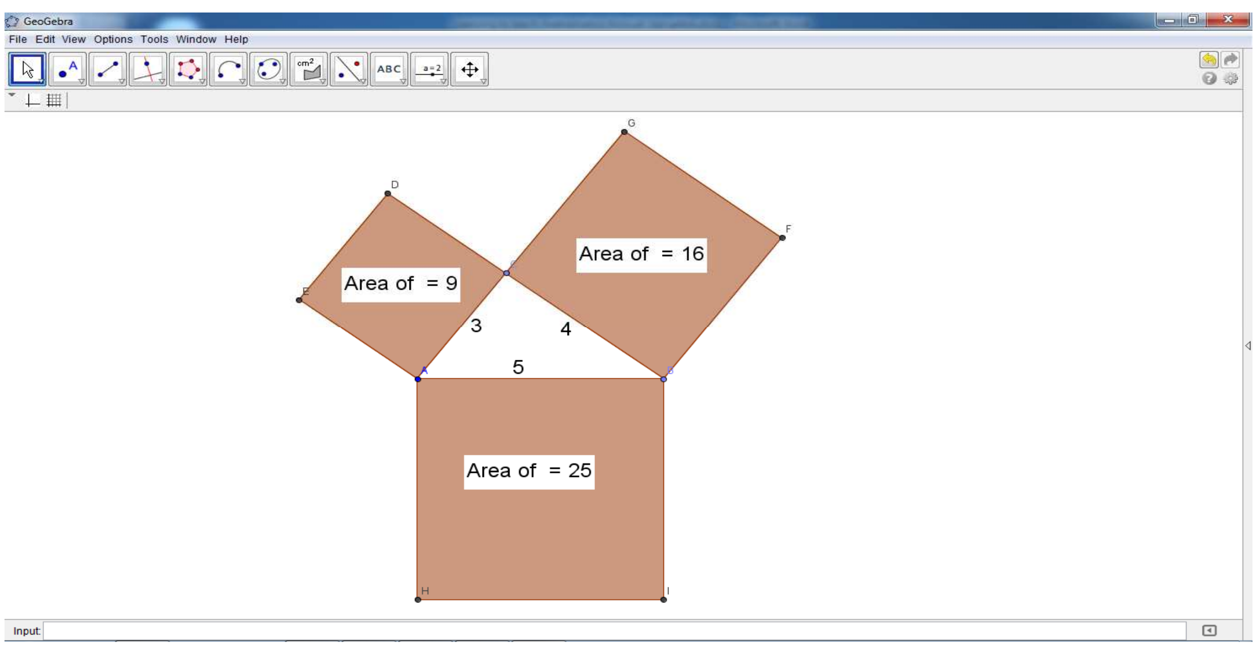


Figure 5

Discussion: There are over 10 proofs of Pythagoras's theorem with GeoGebra inter activities. The goal of each inter activity is to show the different types of thinking and reasoning in a colorful dynamic way. Pythagoras lived in the 500's BC, and was one of the first Greek mathematical thinkers. Pythagoreans were interested in Philosophy, especially in Music and Mathematics? The statement of the Theorem was discovered on a Babylonian tablet circa 1900− 1600 B.C. Professor R. Smullyan in his book 5000 B.C. and Other Philosophical Fantasies tells of an experiment he ran in one of his geometry classes. He drew a right triangle on the board with squares on the hypotenuse and legs and observed the fact the square on the hypotenuse had a larger area than either of the other two squares. Then he asked,"Suppose these three squares were made of beaten gold, and you were offered either the one large square or the two small squares. Which would you choose?" Interestingly enough, about half the class opted for the one large square and half for the two small squares. Both groups were equally amazed when told that it would make no difference. Like this example we can consider some real life applications to Pythagorean Theorem: The Pythagorean Theorem is a starting place for trigonometry, which leads to methods, for example, for calculating length of a lake, Height of a Building and length of a bridge etc.

The Sum of angles in a Triangle is 180 degree

Geometry is called flat if it is assumed that the three interior angles of any triangle will sum to 180 degrees. Geometry at the high-school level is assumed to be flat, and in our geometry course, it is our first fundamental postulate.

Euclid's Proof that the sum of angles in a triangle is 180°:

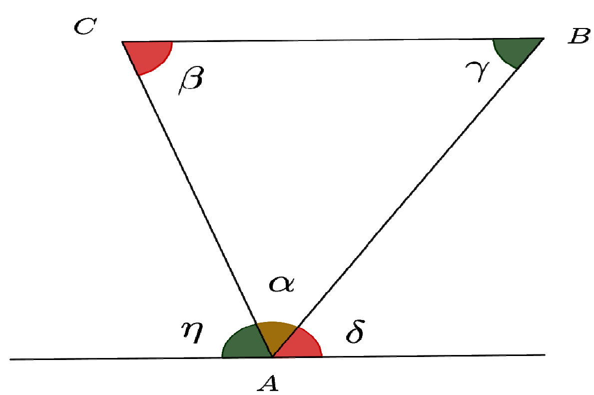


Figure 6

To prove : $\alpha+\beta+\gamma=180^\circ$

Given: A triangle ABC .

Construction : Draw a line parallel to BC through A

Proof : $\eta=\beta$ (Alternate interior angles)

$\delta=\gamma$ (Alternate interior angles)

$\because \alpha+\beta+\gamma=\alpha+\eta+\delta$

$\Rightarrow \alpha+\beta+\gamma=\alpha+\eta+\delta=180^\circ$ (Angles in a linear pair)

Hence, proved.

GeoGebra view

1. Open a new GeoGebra file.
2. Hide the Algebra View and coordinate axes (*View* menu).
3. Show the Input Bar (*View* menu).
4. Set the number of decimal places to 0 (menu *Options – Decimal places*) and use the following steps.

a) Create a triangle ABC with counter clockwise direction.
b) Create the angles α, β and γ of triangle ABC.
c) a = 2 Create slider for angle δ with interval 0˚ to 180˚ and increment 10˚.
d) a = 2 Create slider for angle ε with interval 0˚ to 180˚ and increment 10˚.
e) Create mid-point D of AC and E of AB.
f) Rotate the triangle around point D by angles δ (setting *clockwise*).
g) Rotate the triangle around point E by angles ε (setting *counter clockwise*).
h) Draw a line parallel to BC through A
i) Join BC' and CE' by selecting line through two pint tools cutting the line parallel to BC at F and C'.
j) Hide D, E and E' by selecting show object and also hide lines CF, BC' and AF.
k) Join CF, AF, AC' and BC' by selecting segment between two point tools.
l) ABC Create dynamic text displaying the interior angles and their values.
m) Calculate the angle sum by entering sum = $\alpha + \beta + \gamma$ into the Input Bar and hitting the *Enter*-key.
n) ABC Insert the angle sum as a dynamic text: $\alpha + \beta + \gamma =$
o) Calculate the angle sum by entering sum = $\alpha + \beta + \gamma$ into the Input Bar and hitting the *Enter*-key.
p) Match colours of corresponding angles and text. Fix text that is not supposed to be moved. The GeoGebra view will be as:

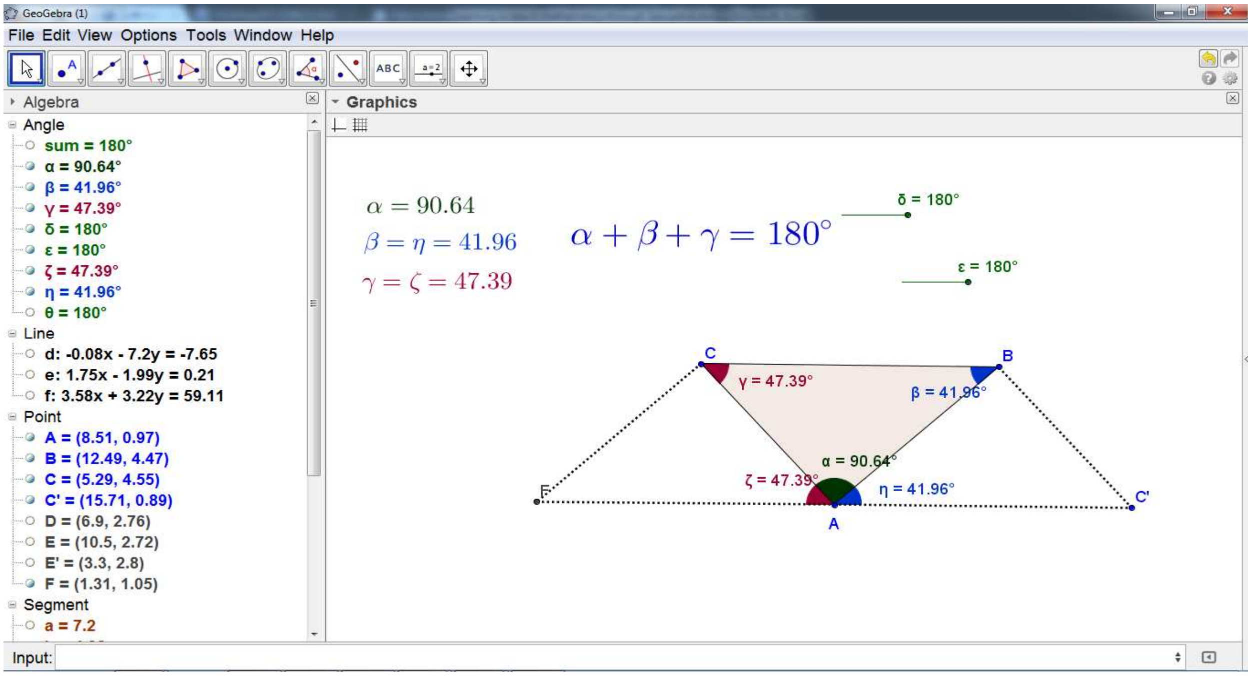


Figure 7

Discussion: It is important for teachers to understand and know basic geometric proofs. Of course, knowing more complicated proofs is also important. However, too many times mathematics teachers are not able to recall this situation's of basic geometrical proof. The proof itself is not complicated, but the additional information is necessary to broaden understanding and help trigger recall. In order to answer the fundamental postulate, the teacher should be aware of the possible correlated mathematics. Students must aware of the theorem about parallel lines and transversal. Teacher should be able to clarify the student's misconceptions and confusions with the summation of the measures of interior triangular angles. So why do we as teachers need to know this property of triangles? The bottom line is triangles are a fundamental shape in geometry. Any polygonal shape can be triangulated, i.e. the polygonal shape can be broken down into a set of triangles. Triangulation breaks down complex shapes into a set of simple shapes thus theorems involving triangles can be used to prove conjectures about complex shapes. In addition, without triangles we would not have Trigonometry. Triangles are a staple to geometry and mathematics as a whole. Without triangles our explorations and knowledge of mathematics would be limited.

Analysis of the training programme

Before we start the training, pre-questionnaire containing thirteen items was set up to investigate their job satisfaction, technique use to teach mathematics and their opinion about the existing curricula and evaluation system practiced in our board of education (MBSE). The pre and post questionnaire has for options to choose: Strongly Agree, Agree, Disagree a little and disagree completely. The following points were assigned:
Strongly Agree – 4, Agree – 3, Disagree a little – 2 and Disagree completely – 1.
Participant's level of satisfaction is categorized as High, Medium and Low as follows:

If 1<Mean<2, Low satisfaction
If 2<Mean < 3, Medium satisfaction
If 3<Mean < 4, High satisfaction

From the table below we find that almost all participants had a job satisfaction and they have medium satisfaction about their teaching style in the class, textbook for different class, time allot for the subject and the system of evaluation practiced under Mizoram Board of Education. They have a low response in forming a group of students in the class, conducting seminar/workshop in their school and use of technology for teaching and learning material.

Table 2: Pre-questionnaire report

Sl.No	Items	Number	Mean	S.D
1	I think that technology based learning Mathematics will be useful for teaching and learning of mathematics at High School level.	20	3.20	0.83
2	I enjoy teaching Mathematics subject in our school	20	3.30	0.73
3	I have job satisfaction in my profession which is a “Mathematics teacher”	20	3.05	0.75
4	I believe that students are satisfied with my teaching.	20	2.75	0.55
5	I take extra steps for the improvement of poor students in our school.	20	2.70	0.92
6	I use software for teaching mathematics in our school.	20	1.85	0.67
7	I have enough time to complete our syllabus in the academic calendar	20	2.25	0.91
8	I think that mathematics subject should not be a compulsory subject at class X	20	1.90	1.07
9	The teaching style we practice in school level is good enough for teaching mathematics subject.	20	2.55	0.60
10	The textbook we use in High school is sufficient for students to understand the concept on mathematics.	20	2.40	0.88
11	The question pattern set by MBSE is satisfactory for testing their understanding level on mathematics exercises.	20	2.30	0.97
12	We usually conduct seminar/workshop in our school.	20	1.75	0.55
13	We form a group of student in our school to work together on mathematics exercise.	20	1.90	0.78

On the last part of the programme, participants were asked to fill up post-questionnaire. The post questionnaire was designed to find out the impact of the training programme. There are eight items in the questionnaire which focus mainly on whether the use of software (GeoGebra) had influenced their attitude towards teaching and learning mathematics. The table below highlights participant’s response about the training programme.

Table 3:Post-questionnaire report

Sl.No	Items	Number	Mean	S.D
1	The workshop shed light upon me on the importance of technology as a teaching tool at high school level	20	3.70	0.47
2	Use of physical objects and real life examples to introduce mathematical ideas are essential components of learning mathematics.	20	3.25	0.44
3	Doing mathematics involves creativity, thinking, and trial and error.	20	3.40	0.50
4	The workshop gave me a great challenge.	20	3.70	0.47
5	Technology is now an essential tool to upgrade our education system.	20	3.30	0.47
6	I learn that visual aid helps me to understand the concept on mathematics.	20	3.50	0.51
7	I want to practice technology base learning in our school.	20	3.30	0.47
8	I want to attend such kind of workshop in my future.	20	3.60	0.50

Conclusions

The process of successful integration of technology into mathematics teaching and learning is progressing slowly and turned out to be rather complex. Today many teachers and students have access to computers and although appropriate software is available both in schools and at home, technology is rarely integrated substantially into everyday teaching. Being aware of the vital role that teachers play in technology-supported mathematics classrooms, professional development opportunities need to be adapted in order to better prepare teachers for new challenges of integrating technology into their teaching practices. The study described in this paper represents an initial step towards the goal of providing more successful introductory materials for professional development with dynamic mathematics software through identifying impediments that teachers face when being introduced to this new technological tool.

From table 3 above one can see that participants gave the highest score on 1st and 4th item which has mean 3.70 and Standard deviation 0.47 each. The lowest response was given on 2nd item which has mean 3.25 and standard deviation 0.44 which is also a high response in the categorized measurement. On all items their mean response are higher than three. This proves that the majority of participants attending the training programme gain not only knowledge but also acquired better concepts of what they had practiced in the classroom.

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