Fuzzy Approach; Interval Transportation Problem

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Abstract

In this paper, we focus on the solution procedure of the Transportation Problem (TP) where cost coefficients of the objective function, has been expressed as interval values by the decision maker. This problem has been transformed into a classical TP where to minimize the interval objective function, the order relations that represent the decision maker's preference between interval profits have been defined by the right limit, left limit, center, and half- width of an interval. Finally, the equivalent transformed problem has been solved by fuzzy programming technique.

KEYWORDS :- Transportation problem, interval co-efficient, nonlinear membership function, linear membership function.

1 Introduction

The conventional single objective TP is a special type of linear programming problem and the constraints follow a particular mathematical structure. The source parameters (a_i) may be production facilities, warehouse, etc., whereas the destination parameters (b_j) may be warehouse, sales outlet, etc. The penalty (c_{ij}) that is, the co-efficient of the objective function, could represent transportation cost, delivery time, number of goods transported, unfulfilled demand, and many others. Thus multiple penalty criteria may exist concurrently which lead to the research work on multiobjective transportation problems. In 1979, Isermann [9] presented an algorithm, for solving a linear multiobjective transportation problem, by which the set of all efficient solutions was enumerated. Ringuest and Rinks [13] developed two interactive algorithms to obtain the solution of linear multiobjective transportation problems. Bit et al. [3] developed a procedure using fuzzy programming technique for solution of the multicriteria decision-making transportation problems. Leberling [11] used a special- type nonlinear (hyperbolic) membership function for the vector maximum linear programming problem. He showed that solutions obtained by fuzzy linear programming with this type of non-linear membership function are always efficient.

Various effective algorithms were developed for solving transportation problems with the assumption that the co-efficients of the objective function, and the supply and demand values are specified in a precise way, namely, crisp. However, these conditions may not be satisfied always. For example, the unit transportation costs are rarely constant. To deal with ambiguous co-efficients in mathematical programming, inexact, fuzzy and interval programming techniques have been proposed [1,2,4,7,12]. Chanas and kuchta [5] defined the transportation problem with fuzzy cost co-efficients and developed an algorithm for the solution. Tong [14] has proposed linear programming models with interval objective functions. Ishibuchi and Tanaka [10] developed a concept for optimization of multi-objective programming problems with interval objective functions. A new treatment of the interval objective in linear programming problems was developed by Inuiguchi and Kume [8] by introducing the minimax regret criterion as used in decision theory. Chanas and Kuchtra [4] have generalized the known concept of the solution of linear programming problem with interval co-efficients in the objective function based on preference relations between intervals. Das, Goswami and Alam [6] have proposed a method to solve the multiobjective transportation problem in which the co-efficients of the objective functions as well as the source and destination parameters are in the form of interval.

2. Interval Transportation Problem

$$\text{Minimize} \quad Z(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{Lij}, c_{Rij}]x_{ij} \qquad (1)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i=1,2,...m. \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j=1,2,...n. \qquad (3)$$

$$x_{ij} \geq 0 \quad \text{for all } i, j \qquad (4)$$

with $[c_{Lij}, c_{Rij}]$ is an interval representing the uncertain cost for the transportation problem.

Setting M = 1,2,…,m, N = 1,2,…,n and J = {(i, j) i∈ M, j∈ N}, the problem may be restated as

$$\text{Minimize} \left\{ Z = \sum_{(i,j)\in J} c_{ij}x_{ij} \;\middle|\; \begin{array}{l} \sum_{j} x_{ij} = a_i \\ \sum_{i} x_{ij} = b_j \\ x_{ij} \geq 0 \\ \sum_{i} a_i = \sum_{j} b_j \text{ for all } i \in M \text{ and } j \in N \end{array} \right\} \qquad (5)$$

where Z ∈ R and $c_{ij} = [c_{Lij}, c_{Rij}]$ are left bound and right bound of c_{ij}. The set of all feasible solutions of the problem will be denoted by S

3. Fuzzy programming technique for the solution of the problem

The first step to solve the problem is to assign, for each objective, two values U_p and L_p as upper and lower bounds respectively, for the Z_R and Z_C objectives, where lower bound indicates aspiration level of achievement and upper bound indicates highest acceptable level of achievement for the Z_R and Z_C objectives. Let $d_p = (U_p - L_p)$ is the degradation allowance for the Z_R and Z_C objectives.

Once the aspiration levels and degradation allowance for each objective have been specified, we have formed the fuzzy model. Our next step is to transform the fuzzy model into a "Crisp" model. The steps of the fuzzy programming technique are as follows:

Step 1: Solve the interval transportation problem as a single objective transportation problem using, each time, only one objective and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From Step 2 we may find, each objective, the worst (U_p) and the best (L_p) values corresponding to the set of solutions. The initial fuzzy model can then be stated in terms the aspiration levels of each objective, as follows:

Find { x_{ij}, i =1, 2, …,m; j = 1, 2, …,n; } (6)

so as to satisfy $Z \lessapprox L$, and constraints

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\dots,m \tag{7}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\dots,n \tag{8}$$

$$x_{ij} \geq 0 \quad \text{for all } i, j \tag{9}$$

$\lessapprox$(fuzzification symbol)) indicates nearly less than equal to

Step 4: Defining membership functions

C**ase (i)**: A hyperbolic membership function is defined by

$$\mu^H Z_{[R,C]}(x)=\begin{cases} 1, & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij} \leq L_p \\ \frac{1}{2}\tanh\left(\left(\frac{U_p+L_p}{2}-\sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij}\right)\alpha_p\right)+\frac{1}{2}, & \text{if } L_p < \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij} < U_p \\ 0, & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij} \geq U_p \end{cases} \tag{10}$$

Where α_p is a parameter given by $\alpha_p = \frac{3}{(U_p-L_p)/2} = \frac{6}{(U_p-L_p)}$

Step 5: Find an equivalent crisp model by using a hyperbolic membership function for the initial fuzzy model.

Step 6: solve the the crisp model by an appropriate Mathematical programming algorithm. The solution obtained in step 6 will be the optimal compromise solution of the transportation problem. If we use the hyperbolic membership function, then an equivalent crisp model for the fuzzy model can be formulated as:

$$\text{Maximize} \quad \lambda \tag{11}$$

subject to

$$\lambda \leq \frac{1}{2}\tanh\left(\left(\frac{U_p+L_p}{2}-\sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij}\right)\alpha_p\right)+\frac{1}{2}, \tag{12}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\dots,m \tag{13}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\dots,n \tag{14}$$

$$x_{ij} \geq 0 \quad \text{for all } i, j \tag{15}$$

The above problem (11-15) can be further simplified as:

$$\text{Maximize } \; X_{mn+1} \tag{16}$$

subject to

$$\alpha_p \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij} + X_{mn+1} \leq \alpha_p(U_p + L_p)/2, \tag{17}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\ldots,m \tag{18}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\ldots,n \tag{19}$$

$$x_{ij} \geq 0 \quad \text{for all i, j and } X_{mn+1} \geq 0 \tag{20}$$

where $X_{mn+1} = \tanh^{-1}(2\lambda-1)$

Case (ii): Linear membership function is defined as follows:

$$\mu_p(x) = \begin{cases} 1, & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij} \leq L_p \\ \dfrac{U_p - \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij}}{L_p - U_p}, & \text{if } L_p < \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij} < U_p \\ 0, & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R, Z_C]x_{ij} \geq U_p \end{cases} \tag{21}$$

Then the crisp model can be simplified as:

$$\text{Maximize } \; \lambda \tag{22}$$

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n}[(Z_R, Z_C)]x_{ij} + \lambda(U_p - L_p) \leq U_p, \tag{23}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\ldots,m \tag{24}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\ldots,n \tag{25}$$

$$x_{ij} \geq 0 \quad \text{for all i, j and } \lambda \geq 0 \tag{26}$$

4. Numerical Example

$$\text{Minimize } Z(x) = \sum_{i=1}^{3}\sum_{j=1}^{3}[c_{Lij}, c_{Rij}]x_{ij} \tag{27}$$

subject to

$$\sum_{j=1}^{3} x_{1j} = 14, \quad \sum_{j=1}^{3} x_{2j} = 16, \quad \sum_{j=1}^{3} x_{3j} = 12$$

$$\sum_{i=1}^{3} x_{i1} = 10, \quad \sum_{i=1}^{3} x_{i2} = 15, \quad \sum_{i=1}^{3} x_{i3} = 17, \tag{28}$$

$$x_{ij} \geq 0, \; i = 1,2,3. \quad j = 1,2,3.$$

where

$$C = \begin{bmatrix} [2,16] & [9,19] & [12,12] \\ [10,22] & [7,13] & [9,19] \\ [2,14] & [12,28] & [4,8] \end{bmatrix}$$

The equivalent deterministic problem becomes:

$$\text{Minimize } Z_R(x) = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{Cij}x_{ij} + \sum_{i=1}^{3}\sum_{j=1}^{3} c_{Wij}\, x_{ij}$$

$$\text{Minimize } Z_C(x) = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{Cij}x_{ij}$$

$$Z_R(x) = \begin{bmatrix} 9 & 14 & 12 \\ 16 & 10 & 14 \\ 8 & 20 & 6 \end{bmatrix} + \begin{bmatrix} 7 & 5 & 0 \\ 6 & 3 & 5 \\ 6 & 8 & 2 \end{bmatrix}$$

$$Z_R(x) = \begin{bmatrix} 16 & 19 & 12 \\ 22 & 13 & 19 \\ 14 & 28 & 8 \end{bmatrix}$$

$$Z_C(x) = \begin{bmatrix} 9 & 14 & 12 \\ 16 & 10 & 14 \\ 8 & 20 & 6 \end{bmatrix}$$

Step 1 and step 2:

Optimal solution which minimizes Z_R subject to constraints (27) is as follows:

$x_{11} = 9$, $x_{13} = 5$, $x_{21} = 1$, $x_{22} = 15$, $x_{33} = 12$,

with $Z_R(x_1) = 517$, $Z_R(x_2) = 518$

Optimal solution which minimizes Z_C subject to constraints (27) is as follows:

$x_{11} = 10$, $x_{13} = 4$, $x_{21} = 15$, $x_{23} = 1$, $x_{33} = 12$,

with $Z_C(x_1) = 374$, $Z_C(x_2) = 379$

Step 3:

$U_1 = 518$, $L_1 = 517$, $U_2 = 379$ $L_2 = 374$

Find { x_{ij}, i = 1,2,3; j = 1,2,3.} so as to satisfy

$Z_R \leq 517$, $Z_C \leq 374$ and constraints (27)

Step 4:

If we use hyperbolic membership function,

The problem was solved by the Linear Interactive and Discrete Optimization (LINDO) Software The optimal solution is presented as follows:

$$X^* = \begin{cases} x_{11} = 9.5,\ x_{13} = 4.5,\ x_{21} = 0.5 \\ x_{22} = 15,\ x_{23} = 0.5,\ x_{33} = 12 \\ \text{rest all } x_{ij} \text{ are zeros.} \end{cases}$$

$X_{mn+1} = 0.00$. Therefore $Z_R = 517.5$, $Z_C = 376.5$ and $\lambda = 0.50$

If we use the linear membership function, an equivalent crisp model can be presented as follows:

The optimal solution is presented as follows:

$$X^* = \begin{cases} x_{11} = 9.5,\ x_{13} = 4.5,\ x_{21} = 0.5 \\ x_{22} = 15,\ x_{23} = 0.5,\ x_{33} = 12 \\ \text{rest all } x_{ij}\text{'s are zeros.} \end{cases}$$

$$Z_R = 517.5,\ Z_C = 376.5 \text{ and } \lambda = 0.50$$

8 Conclusion

The present Chapter proposes a solution procedure of the interval transportation problem, where the co-efficient of the objective functions have been considered as interval. Initially, the problem has been converted into a classical transportation problem where the objectives, which are right limit and center of the interval objective functions, are minimized. These objective functions can be considered as the minimization of the worst case and the average case. To obtain the solution of the transformed classical transportation problem, the fuzzy linear and non-linear programming techniques has been used. If we use the hyperbolic membership function, then the crisp model becomes linear. The optimal compromise solution does not change if we compare with the solution obtained by the linear membership function.

Reference:

1) Alefeld, G., Herzberger, J., (1983) Introduction to interval computations, Academic Press, New York,
2) Asi, K., Tanaka, H., (1984) Fuzzy linear programming with fuzzy number, Fuzzy Sets and systems. **13,** pp.1-10
3) Bit, A. K., (2004) Fuzzy programming with hyperbolic membership functions for Multi-objective capacitated transportation problem, OPSEARCH. 41,pp. 106-120
4) Chanas, S., D. Kuchta, (1996) Multiobjective programming in optimization of interval objective functions a generalized approach, European J.Oper.Res.94,pp. 594-598.
5) Chanas, S., D. Kuchta, (1996) A concept of solution of the Transportation Problem with fuzzy cost co-efficient, Fuzzy Sets and Systems. 82, pp.299-305.
6) Das, S.K., Goswami, A., Alam, S. S., (1999) Multiobjective transportation problem with interval cost, source and destination parameters, European J. Oper. Res. 117,pp. 100-112.
7) Inuiguchi, M., Kume, Y., (1991) Goal programming with interval co-efficient and target intervals, European J.Oper.Res. 52,pp. 345-360.
8) Inuiguchi, M., Kume, Y., (1991) Efficient solutions versus non-dominated solution in linear programming with multiple interval objective functions,

Bulletin of University of Osaka Preference Series A: Engineering and Natural Science. 40,pp. 293-298.

9) Isermann, H., (1979) The enumeration of all efficient solutions for a linear multiobjective transportation problem, Naval Research Logistics Quarterly. 26,pp. 123-139.
10) Ishibuchi, H., Tanaka, H. (1990) Multi-objective programming in optimization of the interval objective function, European Journal of Operations Research. 48,pp. 219-225.
11) Leberling, H, (1981) On finding compromise solution for multicriteria problems using the fuzzy min-operator, Fuzzy sets and systems. 6,pp. 105-118.
12) Moore, R. E., (1979)Method and Applications of interval Analysis, SLAM, Philadelphia, PA,
13) Ringuest, J. L., and Rinks, D. B., (1987) Interactive solutions for the linear multiobjective transportation problem, European Journal of Operational Research. 32,pp. 96-106
14) Tong, S., (1994) Interval number and fuzzy number linear programming, Fuzzy Sets and Systems. 66,pp. 301-306.