Some Considerations of Five Dimensional P- Space-Time

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Abstract

In this paper the exact plane wave like solutions of field equations in the case where an electromagnetic null field present are obtained in higher five dimensional P-space-time.

KEYWORDS : Five dimensional P- space-time.

1. Introduction

On the basis of Takeno's (1961) definition of plane gravitational wave the theory of plane gravitational and electromagnetic waves is developed. Takeno (1961) has solved field equations in general theory of relativity and obtained plane wave solutions in the form of $(z-t)$-type and (t/z)-type waves. On the lines of Takeno (1961) various attempts have been made to investigate the gravitational and an electromagnetic waves in four as well as in higher five dimensional space-time and obtained different types of plane wave solutions [Adhao K S and Karade T M (1994), Ambatkar B G (2002), Zade V T (2002), Kadhao S R (2003), Ladke L S (2003)]. If we adopt some other definitions we may obtain some different results. A Peres has found an exact plane wave like solutions of Einstein's field equation for empty region

$$R_{ij} = 0$$

where R_{ij} is the Ricci tensor of the space-time.

Peres has considered the line element in V_4 as follows:

$$ds^2 = -dx^2 - dy^2 - dz^2 + dt^2 - 2f(x,y,Z)(dz-dt)^2 \tag{1.1}$$

where f is function of x, y and $Z = (z-t)$ satisfying

$$(\partial_{11} + \partial_{22})f = 0 \quad \text{i.e.,} \quad \Delta f = 0, \quad (\Delta = \partial_{11} + \partial_{22},\ \partial_{11} = \partial^2/\partial x\partial x, \text{ etc}) \tag{1.2}$$

According to Takeno (1960), the space-time defined by (1.1) in which $f = f(x,y,Z)$ is an
arbitrary function of its arguments and accordingly does not satisfy (1.2) in general is called a P-space-time or simply P.

The original coordinate system in which the metric takes the form (1.1) is called the (**O**)-system and a co-ordinate system in which the metric is of the same form as (1.1) is known as **O**-system.

Considering the space-time of Peres (1.1), Takeno (1961) has further obtained some exact plane wave like solutions of the field equations in the case where an electromagnetic null field is present. And also he has obtained some results regarding phase velocity, the coordinate condition of dedonder, the energy momentum pseudo-tensor, etc. in four dimensional space-time V_4 due to which he came to the conclusion that, it is very difficult to give any definite conclusion to the nature of the gravitational waves at the present stage of the investigation because no experimental work is available in large amount. We have observed that the study regarding plane wave solutions in P-space-time in four dimensional space-time can further be extended to higher five dimensional P-space-time and hence finds an attempt in the present paper.

§ 2 P-Space-time in V_5

We consider the five dimensional P-space-time as follows:

$$ds^2 = -du^2 - dx^2 - dy^2 - dz^2 + dt^2 - 2f(u,x,y,Z)(dz-dt)^2 \tag{2.1}$$

in which f is arbitrary function of (u,x,y) and $(z-t)$, $(Z \equiv z-t)$.

We denote **(O)**-system and the **O**-system such that the original coordinate system in which the metric is (2.1) and coordinate system in which the metric is of the same form as (2.1) in V_5 respectively.

We have form (2.1)

$$g = \det(g_{ij}) = 1, \qquad g^{11} = g^{22} = g^{33} = -1, \qquad g^{44} = -1 + 2f,$$

$$g^{45} = g^{54} = -2f, \qquad g^{55} = 1 + 2f, \qquad \text{other } g^{ij} = 0. \tag{2.2}$$

The non-vanishing independent components of $\left\{ {i \atop jk} \right\}$ are as follows:

$$\left\{ {4 \atop 14} \right\} = \left\{ {5 \atop 14} \right\} = -\left\{ {4 \atop 15} \right\} = -\left\{ {5 \atop 15} \right\} = -\left\{ {1 \atop 44} \right\} = \left\{ {1 \atop 45} \right\} = -\left\{ {1 \atop 55} \right\} = \partial_1 f,$$

$$\left\{ {4 \atop 24} \right\} = \left\{ {5 \atop 24} \right\} = -\left\{ {4 \atop 25} \right\} = -\left\{ {5 \atop 25} \right\} = -\left\{ {2 \atop 44} \right\} = \left\{ {2 \atop 45} \right\} = -\left\{ {2 \atop 55} \right\} = \partial_2 f,$$

$$\begin{Bmatrix}4\\34\end{Bmatrix}=\begin{Bmatrix}5\\34\end{Bmatrix}=-\begin{Bmatrix}4\\35\end{Bmatrix}=-\begin{Bmatrix}5\\35\end{Bmatrix}=-\begin{Bmatrix}3\\44\end{Bmatrix}=\begin{Bmatrix}3\\45\end{Bmatrix}=-\begin{Bmatrix}3\\55\end{Bmatrix}=\partial_3 f\,,$$

$$\begin{Bmatrix}4\\44\end{Bmatrix}=\begin{Bmatrix}5\\44\end{Bmatrix}=-\begin{Bmatrix}4\\45\end{Bmatrix}=-\begin{Bmatrix}5\\45\end{Bmatrix}=\begin{Bmatrix}4\\55\end{Bmatrix}=\begin{Bmatrix}5\\55\end{Bmatrix}=\bar{f}\,.$$

(2.3)

The non-vanishing independent components of R_{ijkl} are calculated as under:

$$R_{1414}=-R_{1415}=R_{1515}=\partial_{11} f\,, \qquad R_{2424}=-R_{2425}=R_{2525}=\partial_{22} f\,,$$

$$R_{3434}=-R_{3435}=R_{3535}=\partial_{33} f\,, \qquad R_{1424}=-R_{1425}=R_{1525}=\partial_{12} f\,,$$

$$R_{1434}=-R_{1435}=R_{1535}=\partial_{13} f\,, \qquad R_{2434}=-R_{2435}=R_{2535}=\partial_{23} f\,,$$

$$R_{a4b4}=-R_{a4b5}=R_{a5b5}=\partial_{ab} f\,, \qquad (a,b=1,2,3)\,.$$

(2.4)

The non-vanishing independent components of Ricci tensor assume the form as follows:

$$R_{44}=-R_{45}=R_{55}=\Delta f \quad \text{i.e.,}\ R_{\alpha\beta}=(\partial_{11}+\partial_{22}+\partial_{33}) f\,,$$

(2.5)

where and throughout this paper the indices $a,b,\cdots$ and $\alpha,\beta,\cdots$ stand for 1 or 2 or 3 and 4 or 5 respectively.

From (2.1) and (2.4), we have

$$R=g^{ij}R_{ij}=0\,.$$

(2.6)

From (2.4) we have

Conclusion A necessary and sufficient condition that a higher dimensional P-space-time be Minkowskian is

$$\partial_{ab} f=0$$

(2.7)

in an (**O)**-system.

Recently Adhao and Karade (1994) have obtained some five dimensional plane wave solutions in purely gravitational field and established the existence of $(z-t)$-type and (t/z)-type plane waves in V_5, investigating the line element

$$ds^2 = -Adx^2 - 2Ddxdy^2 - Bdy^2 - 2Fdxdu - 2Gdydu - Hdu^2$$

$$-(C-E)dz^2 - Edzdt + (C+E)dt^2$$

(2.8)

where A, B, D, E, F, G and H are functions of Z.

Remarks If we put $A = B = H = 1$, $C = 1$, $D = 0$, $F = 0$, $G = 0$ and $E = -2f$ in (2.8) then we have (2.1) in which f is a function of Z.

Thus a P-space-time with $f = f(Z)$ is an H-space-time. But form the above conclusion (2.7) this H-space-time is flat. Science f contains u, x, y in general then P-space-time is not flat in general and moreover we can prove that a P-space-time can not be an H-space-time in general.

§3.Exact solution of the field equations in V_5 where an electromagnetic null field is present

We consider the following field equations for a system in which an electromagnetic field F_{ij} coexists with the gravitational field:

$$R_{ij} = -8\pi E_{ij}, \qquad (i, j = 1,2,3,4,5)$$

(3.1)

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \qquad \text{and}$$

(3.2)

$$F^{ij}_{;j} = 0$$

(3.3)

where R_{ij} is the Ricci tensor of the space-time, F_{ij} anti-symmetric tensor describing the electromagnetic field, and a semicolon denotes the covariant derivative

$$E_{ij} = \frac{1}{4} g_{ij} F_{kl} F^{kl} - F_{ik} F_{jl} g^{kl}$$

(3.4)

is the electromagnetic energy tensor and g_{ij} is the fundamental tensor of the space-time,

The plane wave solutions obtained by Adhao and Karade (1994) have been generalized by Ambatkar (2002) considering the case where an electromagnetic field co-exists with gravitational field for a H-space-time in V_5.

H-space-time : A space-time satisfying the equations (3.1), (3.2) and (3.3) is called the H-space-time or simply H.

By the analogy of H-space-time studied by Ambatkar (2002), we consider in P- space-time a transverse electromagnetic field whose components in the (**O**)-system are

$$F_{12} = F_{13} = F_{23} = F_{45} = 0, \qquad F_{14} = -F_{15} = -\sigma = \rho_1,$$

$$F_{24} = -F_{25} = \rho = \rho_2, \qquad F_{34} = -F_{35} = \eta = \rho_3$$

(3.5)

Where ρ_1, ρ_2 and ρ_3 are functions of u, x, y and Z.

Then we can show that F_{ij} is null that is it satisfies

$$F_{ij}F^{ij} = F_{ij}^{*}F = 0.$$

(3.6)

The non-vanishing components of E_{ij} are

$$E_{44} = -E_{45} = -E_{54} = E_{55} = \rho_1^2 + \rho_2^2 + \rho_3^2$$

(3.7)

and the first field equation (3.1) becomes

$$\Delta f = (\partial_{11} + \partial_{22} + \partial_{33}) f = -8\pi(\rho_1^2 + \rho_2^2 + \rho_3^2).$$

(3.8)

The generalized Maxwell equations (3.2) and (3.3) are equivalent to the following Cauchy-Riemann type equations:

$$\partial_1\rho_2 - \partial_2\rho_1 = 0, \qquad \partial_1\rho_3 - \partial_3\rho_1 = 0, \qquad \partial_2\rho_3 - \partial_3\rho_2 = 0$$

(3.9)

Hence we have

Conclusion A necessary and sufficient conditions that g_{ij} given by (1.1) and F_{ij} given by (3.5), where f, ρ_1, ρ_2 and ρ_3 are functions of u, x, y and Z satisfy the field equations (3.1), (3.2) and (3.3) is that f, ρ_1, ρ_2 and ρ_3 satisfy (3.8) and (3.9).

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