Game Theory in Inventory Management

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Abstract

Production and the sales of spare parts is a very profitable business for many companies. They earn more from the production of these spare parts than from their basic products. We will see how the use of game theory in Management of inventories will help in profit maximization. We will study this inventory through cooperative noon game with an N-person non-zero-sum single-stage (shot) game where players play one after another in a simultaneous way. The game is played between two players only, market and manufacturing companies.

The market is a subject where its strategic choices have an impact payoff of manufacturing companies and have no interest on the outcome of the game. This is a game against nature, which means that manufacturing companies play against the market. Manufacturing companies decide on a pricing strategy in a competitive manner with low production costs, and the message of the stock level and strategic level of inventory is not dominated. The game has a mixed strategy solution. This solution maximizes payoff for manufacturing companies setting inventory levels and prices based on assumptions about the limits of low and high distribution of the request parameters.

KEYWORDS: Game theory, inventory, spare parts price, stochastic demand

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1. Introdution

The spare parts business includes the acquisition, storage and their sale. In some cases we have and after sale services and warranties (Van Der Heijden, Alvarez, & Schutten, 2013). For some companies which produce durable products, production of spare parts results more profitable than producing the base product (Aronis, Magou, Dekker, & Tagaras, 2004). The spare parts business on the automobile industry is four to five times bigger than the business of the original business. In 2006, the business of spare parts and after sales services in America amounted to 8% of GDP (Cohen, Agrawal, & Agrawal, 2006).

In Albania, companies representative of automobile companies have a bigger profit on the sale of spare parts and after sale services than from selling cars. This shows the importance of the spare parts business. When a part of the basic device fails, it should be replaced. The rate of failure is not deterministic and it has a link to the quality of maintaining and aging. This in turn causes an unexpected request for spare parts. Maintenance for each machine can be categorized in Preventive and Corrective group. From the perspective of spare parts manufacturing companies, preventive maintenance may result in periodic request but stochastic. On the other hand the demand for corrective maintenance is deterministic under the assumption that only one failure can happen at any moment but stochastic in the time of arrival. Thus, in both cases the nature of the demand is terminated and forecasting methods may predict the demand. Several factors distinguish inventories of spare parts from other types of inventories. The main factor is the expectation of quality customer service. We will propose the game of spare parts inventory, application market, the function of the expected costs of manufacturing companies, and the solution of a mixed strategy which are subsequently derived. Also we will present results of the study followed by the outcome. And we end this paper with conclusions.

2. LITERATURE REVIEW

2.1 Inventory Game

Game theory can be used for decision making when multiple entities are part of the subject for decision and when between them there are conflicts. This is based on the existence of the equilibrium on non cooperative game. We going to use the game as an N person-non zero sum- single shot game where players play in a simultaneous way. To archive this, we simplify the game to two persons, non cooperative game setup. The game has two players: (1) market, (2) manufacturing companies. The game has been set up from the perspective of the manufacturing companies, which means that the game gives solution in maximum payoff or minimum loss for the manufacturing companies. Game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers (Straffin, 1993). Originally, it addressed zero-sum games, in which one person's gains result in losses for the other participants. Today, game theory applies to a wide range of behavioral relations, and is now an umbrella term for the science of logical decision making in humans, animals, and computers (Roughgarden, 2010).

Modern game theory began with the idea regarding the existence of mixed-strategy equilibria in two-person zero-sum games and its proof by John von Neumann. (Turocy & Stengel, 2003) Von Neumann's original proof used Brouwer fixed-point theorem on continuous mappings into compact convex sets, which became a standard method in game theory and mathematical economics. His paper was followed by the 1944 book Theory of Games and Economic Behavior, co-written with Oskar Morgenstern, which considered cooperative games of several players. The second edition of this book provided an axiomatic theory of expected utility, which allowed mathematical statisticians and economists to treat decision-making under uncertainty.

This theory was developed extensively in the 1950s by many scholars. Game theory was later explicitly applied to biology in the 1970s, although similar developments go back at least as far as the 1930s. Game theory has been widely recognized as an important tool in many fields. With the Nobel Memorial Prize in Economic Sciences going to game theorist Jean Tirole in 2014, eleven game-theorists have now won the economics Nobel Prize. John Maynard Smith was awarded the Crafoord Prize for his application of game theory to biology. Game theory is the logical analysis of a situation where there are at least two players and where each of the players has a number of possible strategies and the strategies chosen by each player determine the outcome of the game. Each possible outcome of the game is set of numerical payoffs for each player. Game theory shows how the player should play, what strategy they use in order to reach the maximum profit

(Osborne, 2000). This is called Nash Equilibria (Holt & Roth, 2004). A game may have multiple Nash equilibia or none at all.

In this paper we assume that the game is a non cooperative game and the market is unkind and chooses hostile strategies. The manufacturing companies knows the demand for spare parts and this comes as a non stationary Poisson process but does not know the exact distribution and can only forecast the bounds of the intensity factors. Also, the sale price of the manufacture company in comparison to the will-fitters sale prices has an influence on the demand intensity factors which are estimated by the manufacture company. We consider the second player, the market, as an unreasoning entity whose strategic choices affect the payoff the manufacturers, but which has no interest in the outcome of the game. The aforementioned characteristic of the market enables us to consider the spare parts inventory game as a game against nature.

2.2 The market demand

Spare parts demand is often intermittent or lumpy which means long variable periods without any demand are frequent. When demand occurs and cannot be met, high losses may occur. Demand forecasting methods can be used to plan inventory. (Sideratos & Hatziargyriou, 2007) Introduced a classical method for demand forecasting and (Acar & Gardner, 2012)has provided a related literature review covering work over the last fifty years. In this paper we assume that the manufacture company has limited information about the market's expected demand. The manufacture company knows that demands for spare parts arrive as a non-stationary Poisson process , i.e. the rate of the process changes with time,but the exact distribution of these factors over the time is not observed and only the upper and lower bounds of the intensity factors are forecasted. In the aftermarket business, other than the manufacture company as an original manufacturer, there are other low cost manufacturers, known as will-fitters, who can manufacture the same parts and deliver them to the market. Based on the sale price of the manufacturers, the market share for spare parts will be allocated among suppliers. In other words, manufacturers compete with each other on their sale prices to absorb more customers, so the sale price is a decision variable for the manufacture company tooptimize payoff in the aftermarket. Because of the intermittent and slow moving characteristics of spare parts demand, we consider it to be a Poisson process. A Poisson process with an intensity factor or rate of _λ_ is a stochastic process in which the inter-arrival time distribution is exponential with mean time of _μ = 1/λ_ and the arrival distribution is Poisson with the rate of_λ_. If _λ_ is constant over time, the process is a stationary Poisson process and when _λ_ changes over time, the process is a non-stationary Poisson process. In the case of spare parts management, the rate of demand depends on three factors: Quality; Usage; Maintenance;

2.3 Cost function of manufacture company

The manufacture company must determine the sale price and the spare parts stock level in the order-up-to level inventory policy. The payoff for manufacture company is the profit of the manifactue company K_b which is the difference between the cost of production and inventory

and the revenue attained by selling spare parts. Let (X) be a random variable and Pr{X=x} determines the probability that the random variable (X) takes on a specific value x from some unspecified probability distribution. The goal is to reach a low level of the backorder or a high level of fill rate with minimum investment on inventory. We must calculate the cost of production and inventory, as well as the revenue from selling the products. Gives the cost of production of inventory is:

$$K_M = c_p \times S + p \times EBO(S) + h \times EI(S)$$

The revenue for selling products:

$$K_R = c_s \times D \times EFR(S)$$

The payoff of company is given by:

$$K_B = K_R - K_M$$

3. The Game

We have two players, manufacture company and market. Each of the players choose their strategies simultaneously. The game is a static game that can be modeled and solved by finding Nash Equilibrium. The game goes as follow:

- Players play simultaneously;
- Manufacture company possesses the information of the original parts failure rates and can predict the allocated demand rates including: the upper bound intensity factor λ_{upper} and the lower bound intensity factor λ_{lower} with respect to its selected sale price;
- The market as a nature has two choices of Poisson process demand types with upper and lower bounds intensity factors;
- The probability that the market plays with lower bound demand is (P);
- Respectively the probability that the market plays with upper bound demand would be (1-P);
- The manufacture company has several strategies which are order-up-to inventory levels (as discrete numbers) that varies from1 to N;

The game set is shown as a matrix form known as a matrix payoff. The investigation of the payoff matrix declares that there is no dominant strategy in this game. In other words, the lack of dominant strategy leads to the lack of pure strategy, implying the optimal solution depends on the probability of the market's demand.

4. Conclusion

The spare parts inventory management is studied as an inventory game in case of an N-person non-zero sum single-shot game where players play simultaneously. In order to achieve this goal, the problem is simplified in a two person, non-cooperative game setup. The game has two players: the manufacture company and the market, and the game has been set up from the perspective of manufacture company, which results in the maximum payoff or minimum loss. It has been assumed that the game is a non-cooperative game and the market is unkind and chooses hostile strategies. The manufacture company can only forecast the upper and lower bounds of the market's intensity factors. In the aftermarket business, other than the manufacture company as an original manufacturer, there are other low cost manufacturers, known as will-fitters, who can manufacture the same parts and deliver them to the market. Based on the sale price of the manufacturers, the total demand for the spare parts will be allocated among suppliers. In other words, manufacturers compete with each other on their sale prices to absorb more customers, so the sale price is a decision variable for the manufacture company to optimize its payoff in the aftermarket. The manufacture company predict the allocated demand rates including:

the upper bound intensity factor and the lower bound intensity factor with respect to its selected sale price. The market is considered as an unreasoning entity whose strategic choices affect the payoff the manufacture company, but which has no interest in the outcome of the game. This is modeled as the game against nature which means the manufacture company plays against the market. In our game there is no dominant level of inventory for the OEM i.e. the game has the mixed strategy solution. The solution of the mixed strategy, determines the strategies of the manufacture company to maximize the payoff in the aftermarket business. The company chooses the optimal level of inventory with respect to the probability of intensity factors that the market can produce. A comparison of the maximum attainable payoff and guaranteed payoff in the uncertain situation would justify the company's extra investment on improving the demand forecasting efforts.

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