Description of the Microchannels Flow, an Application of the Theory of the Micropolar Fluid

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Abstract

In this paper we concentrate on micro channel flows. The problem arises because some experiments and theoretical explanations indicate that for micro channel flows of real fluids the micropolar effects become important when the width of the channel is comparable to the dimensions of the fluid molecules. In such a case, it can be expected that the assumption of continuum medium model is no longer justified and flows should be modelled on the molecular level. On the other hand, when the geometrical dimensions of the flow become sufficiently large, the experimental results agree well with the Navier-stokes hydrodynamic predictions.

KEYWORDS: Microfluids, Microscale fluid flow, Microchannels Navier- stokes equations.

1. INTRODUCTION:

Microfluidics plays a major role in the development of many innovative research activities aimed at the development of miniaturized devices and systems, and new applications related to micro scale handling of fluids. As the field of microfluidics continues to grow, it is becoming increasingly important to understand the mechanism and fundamental differences involved in microscale fluid flow.

Over the past decade, micromachining technology has been used to develop a number of microfluidic systems in silicon glass, quartz and plastics. Microchannels and Chambers are essential components of any such system.In addition to connecting different devices, microchannels are also used for reactant delivery, as bio-chemical reaction Chambers [32, 34] in physical particle separation [9, 15, 30], in inkjet print heads [23], or as heat exchangers for cooling computer chips [7, 33]. Currently, fluid flows in microchannels and micromachined fluid systems (e.g., pumps and valves) are analyzed using the Navier-Stokes equations [8, 9]. However, a number of publications have shown that

(a) Flows on the microscale are different from that on the Macroscale and that
(b) The Navier-Stokes equations alone are incapable of modeling the occurring phenomena [9, 25, 27].

At present, the smallest, standardized, mass-produced microchannels have with h= 0.1 µm. It is expected that they will soon reach a dimension ofh=0.02 µm [13]. Successful design of microfluidic devices involving microchannels with such small dimensions requires new methods for predicting the characteristics of a flow in them.

Numerous experimental results indicate that the classical continuum approach is not applicable to describe micro-and nano-flows. However there exists and extension of Navier-stokes approach, the micro polar fluid theory [4, 5] which augments the classical continuum fluid mechanics by incorporating the effects of micro rotation of fluid molecules, and whose hydro dynamic predictions for micro-flow agree quite well with some experimental results [17,18,28]. Moreover, molecular dynamic simulations results show that during the Poiseuille flow in very narrow channels the micro rotation velocity (missing in the classical Navier-stokes theory)exists and those results agree well enough with that based on the analytical solution of the micropolar fluid flow.

From that reason it seems that the micropolar fluid Theory could be a useful tool in modeling micro flows, but first we should establish precisely dimension of the flow field for which the micropolar modelling can be addressed to. In this paper we concentrate on micro channel flows. The problem arises because some experiments and theoretical explanations indicate that for micro channel flows of real fluids the micropolar effects become important when the width of the channel is comparable to the dimensions of the fluid molecules. In such a case, it can be expected that the assumption of continuum medium model is no longer justified and flows should be modelled on the molecular level. On the other hand, when the geometrical dimensions of the flow become sufficiently large, the experimental results agree well with the Navier-stokes hydrodynamic predictions.

2. EQUATIONS OF MICROPOLAR FLUID FLOW:

Eringen formulated the micropolar fluid theory in 1966 [2] as an extension of the Navier-stokes model of classical hydrodynamics to facilitate the description of the fluids with Complex molecules. The micropolar fluids are usually defined as isotropic, polar fluids in which deformation of molecules is neglected. Physically, a micropolar model can represent fluids whose molecules can rotate independently of the fluid stream flow and its local vorticity. In other words, micropolar fluids are the medium whose behavior during their flows is affected by the microrotation, i.e. the local rotational motion of fluid molecules contained in a given fluid volume element. The occurrence of the microrotation vector, which differs from the stream flow vorticity vector $\omega \neq \mathrm{rot}V$and from the angular velocity $\omega \neq 1/2\,\mathrm{rot}V$, results in the formation of anti-symmetric stresses and coupled stresses which consequently result in an increase in the energy dissipation in the energy dissipation. In the equations describing the flow field, there occur two independent kinematic variables: the velocity V and the microrotationω'.

The constitutive equations for micropolar fluid[2] define the stress tensor $T = \{T_{ij}\}$, which is a nonsymmetric tensor and the couple stress tensor $C = \{C_{ij}\}$as follows:

$$T_{ij}=(-p+\lambda V_{k,k})\delta_{ij}+\mu(V_{i,j}+V_{j,i})+K(V_{j,i}-\varepsilon_{ijk}\omega_k) \quad \text{---------------}(1)$$

$$C_{ij}=\alpha\omega_{k,k}\delta_{ij}+\beta\omega_{i,j}+\Upsilon\omega_{j,I} \quad \text{---------------}\ (2)$$

Where the symbols denote: p – pressure, λ,μ,k – coefficient of bulk, shear and vortex viscosities, α,β,Υ – the respective coefficients of coupled viscosities, ε_{ijk} – the Levi-Civita tensor, δ_{ik}– the Kronecker delta.

Relation exists as follows in coefficients,

$\kappa > 0$, $3\lambda + K + 2\mu > 0$, $2\mu + \kappa > 0$

$3\alpha + 2\Upsilon > 0$, $-\Upsilon < \beta < \Upsilon$, $\Upsilon > 0$.

If we assume that the specific internal energy of the fluid is proportional to its temperature and that Fourier law holds, then, for the flow of a micropolar incompressible fluid with constant material coefficients, the flow equations can be uncoupled from the energy conservation equation [1,2,20] and becomes:

$$\text{div } V = 0 \qquad \text{--------------(3)}$$

$$\rho\frac{dV}{dt} = \rho f - \text{grad } p - (\mu + \kappa)\text{rotrot}V + K\text{rot}\omega \text{--------------(4)}$$

$$\rho I\frac{dw}{dt} = (\alpha + \beta + \gamma)\ \text{graddiv}\omega - \gamma\text{rotrot}\omega + \kappa\text{rot}V - 2\kappa\omega + \rho g \qquad \text{------------(5)}$$

where: ρ – fluid density

ω – microrotation, $\omega = (\omega1, \omega2, \omega3)$

V – velocity, V = (V1, V2, V3)

f – body force per unit mass, f = (f1, f2, f3)

g – body torque per unit mass, g = (g1, g2, g3).

3. DIMENSIONAL ANALYSIS OF THE MICROPOLAR FLUID FLOW EQUATIONS:

Our main target in this section is to show that if the characteristic geometrical linear dimension of the fluid flow field becomes appropriately large, the equations describing the micropolar fluid flows can be transformed into Navier-stokes equations. We shall confine ourselves to studying a particular form of equations (4) - (5) describing micropolar fluid flows for which the body torque is g=0, and the force f=0, We shall present their non-dimensional form and investigate the effect of the new non-dimensional microstructure parameters, following from a micropolar fluid model, on the form of the flow equations.Let us assume, that for a particular flow relevant to the physical problem to be studied, characteristic - or reference - quantities of: linear dimension, time, velocity and density are the quantities denoted with L,T,U and p , respectively. Symbols with a prime are used to denote non dimensional quantities:

$$V' = V/U,\ t' = t/Tc,\ \nabla' = Lc\nabla \qquad \text{---------------(6)}$$

$$\omega' = \omega Lc/U,\ x' = x/Lc,\ p' = \frac{p}{\frac{(\mu+\kappa)U}{Lc}} \qquad \text{-------------(7)}$$

By using standard method of dimensional analysis to the fluid flow equations(4) - (5) we can write them in the non-dimensional form:

$$R\left(\frac{1}{S}\frac{dV'}{dt'} + V'\nabla'V'\right) = \nabla'p' - \nabla' \times \nabla' \times V' + 2N^2\nabla' \times \omega'$$

$$R\frac{I}{l^2}\left(\frac{1}{S}\frac{\partial\omega'}{\partial t'} + V'\nabla'\omega'\right) = 2L^2N^2(-2\omega' + \nabla' \times V')$$

$$+2(1-N^2)\left(\frac{\alpha+\beta+\lambda}{\lambda}\nabla'\nabla'\omega' - \nabla'\times\nabla'\times\omega'\right) \text{-------------(8)}$$

In the equations, besides classical counterparts of the non-dimensional numbers: R - Reynolds and S – Strouhal:

$$R= \frac{U\mathrm{Lc}\rho}{\mu+\kappa} \qquad S = \frac{U\mathrm{Tc}}{\mathrm{Lc}} \qquad \text{-------------(9)}$$

There occur new non-dimensional parameters N and L:

$$N = \frac{\sqrt{\kappa}}{\sqrt{2}\mu N+\kappa}, \ L = \frac{\mathrm{Lc}}{l}, \ l = \sqrt{\frac{\lambda}{4\mu N}} \qquad \text{-----------(10)}$$

Parameter L,L>0 characterizes the relationship between the geometric dimension of the flow L_c and the rheologic properties of fluid and is also called measure of the relative length of the fluid microstructure. The value of ℓ reflects the microscopic properties of the fluid [1,5,21] (the bigger the molecules the greater thevalue of the parameter).Parameter N, $0 \leq N \leq 1$ characterize the coupling between the vortex viscosity Coefficientκ and the Shear viscosity Coefficient μ. Letus observe that the value of the parameter N and for a given fluid is constant, whereas the value of the parameter L depends explicitly on the characteristic linear dimension of the flow.We shall now investigate what is the effect of the boundary values of the parameters on the form of the flow equations (7) – (8).

If the value of the parameter N → 0, then the equations ofmomentum (7) and angular momentum (8) become independent of each other and the first one transforms into a classical Navier Stokes equation for Newtonian fluid.

It is in agreement with the result obtained [10] in which, in aspect of longtime behaviour micropolar fluid flows, it was shown that when κ→0, the velocity field of the micropolar fluid model converges to the velocity field of the classical Navier-stokes model.

If the value of the parameter L→0, then the stream velocity rotation Δ' x V' is removed from the right side of equation (8). Thissituation can be interpreted asa description of a fluid flow whose angular acceleration is not affected by the fluid stream vorticity.In the other limiting case, for L→ □,from the angular momentum equation (8), we obtain the relationship defining the microrotation vector as equal to one half of the vector of the fluid stream vorticity

$$2\omega' = \nabla' \times V' \qquad \text{----------------(11)}$$

That is to say, to the angular velocity of the fluid volume element if the element moved as a rigid body.Formula (11) allows eliminating the microrotation vector from the angular momentum equation (7), which now, in dimensional variables takes the form:

$$\rho\frac{dV}{dt} = -\nabla \rho - \mu N\nabla \times\nabla \times V \qquad \text{----------------(12)}$$

This is the Navier-stokesequation for Newtonian fluids.

The way we obtained this asymptotic form of micropolar fluid flow equations unmistakably indicates that the micropolar effects in the fluid flow distribution can be neglected only if the value of Lc, that is, the characteristics linear dimension of flow L, is sufficiently large, as ℓ = const for Lc.Therefore, together with a decrease in the value of Lc, there will appear discrepancies between the solutions obtained through application of the micropolar and the classical fluid models.

An analysis of the effect of non-dimensional parameters on the form of the equations also indicate to what domain a micropolar fluid theory is suitable to be applied: to flows which occur in microdevices, defectoscopy, tribology and living organisms.

4. ANALYSIS OF POISEUILLE FLOW MODELLING IN MICROCHANNELS:

Our target now is to predict the value of the characteristic linear dimension Lc of the microchannel flow below which the micropolar effects of the fluid during the flow appear and thus the micropolar modeling for Poiseuille flows is more reliable than when using classical Navier-stokes dynamics.It will be done by the comparison of the volume flow rate calculations based on exact stationary solutions of the micropolar fluid and the Navier-stokes equations of the Poiseuille flow in the circular microchannels.The comparison will be performed for real fluids and in terms of dimensionless microstructural parameters.For the flows in circular microchannels the linear dimension Lcis defined as Lc=d, d-diameter of the pipe.

Analytical solution of the equation describing the flows of micropolar fluid in the circular channel of radius R,

in a cylindrical co-ordinates system (r,**θ**,z)is given by the formulae [25]

$$\frac{V_z(\tilde{r})}{V_0} = 1 - \tilde{r}^2 + \frac{2\delta}{2+\delta}\frac{I_0(kR)}{kRI_1(kR)}\left[\frac{I_0(kR\tilde{r})}{I_0(kR)} - 1\right] \qquad \text{------------(13)}$$

$$\frac{\omega((\tilde{r})R}{V_0} = \tilde{r} - \frac{2\xi+\delta}{2+\delta}\frac{I_1(kR\tilde{r})}{I_1(kR)} \qquad \text{----------(14)}$$

Where :V =(0,0,V_z(r)) – Velocity vector, ω= (0,ωI(r),0)-microrotation vector and α_0 - constant describing the fluid-wall interaction which appears in the equation defining boundary conditions for microrotation on the walls, $0 \le \alpha_0 \le 1$ [21, 25, 27], p = p(z) pressure I_0,I_1 - are Bessel functions of first kind.The remaining symbols denote:

$$\delta = \frac{\kappa(1-\alpha_0)}{\mu_N}, \kappa = \sqrt{\frac{(2\mu+\kappa)\kappa}{(\mu+\kappa)\gamma}} \qquad \text{---------(15)}$$

$$\xi = 1 - \alpha_0, V_0 = \frac{R^2(-dp/dz)}{4\mu_N}, \tilde{r} = \frac{r}{R} \qquad \text{----------(16)}$$

Integrating velocity (13) over the cross section of the channel we obtain the formulae for the volume flow rate, which depends on parametersδ,k and R:

$$Q_m(R) = Q_N(R)\left\{1 + \frac{2\delta}{2+\delta}\frac{4}{(kR)^2}\left[1 - \frac{kRI_0(kR)}{2I_1(kR)}\right]\right\} \qquad \text{-----------(17)}$$

Where Q_N(R) denotes the volume flow rate of the classical, Newtonian fluid with viscosity μ_N which flows through the microchannel of radius R and stands:

$$Q_N(R) = \frac{\pi R^4(-dp/dz)}{8\mu_N} \qquad \text{--------------(18)}$$

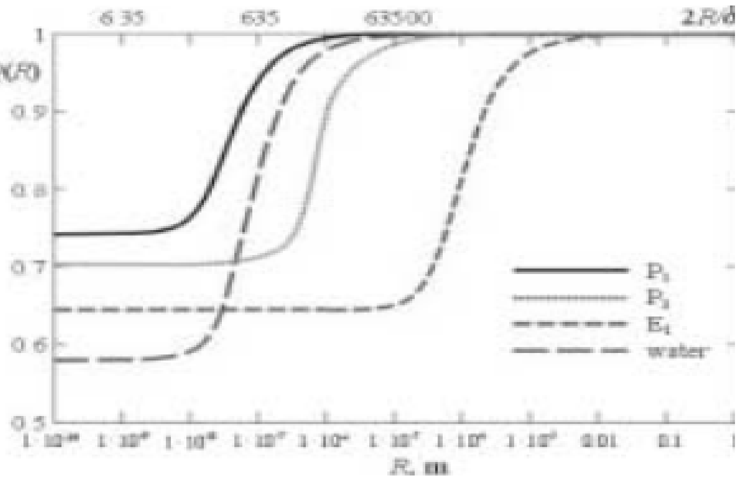
The values of the parameters δ and k can be determined experimentally [20,25,27] for instance, Table1 gives the valuesof the parameters for exemplifying fluids determined for the fluid flows in quartz channels [27]. The chemical composition of the fluids was given in the detail in [25,27].

To make visible the usability of micropolar fluid flow model to the flow calculations in term of the micro channel radius R, we define the relative volume flow rate

Q(R) – "Comparison parameter" as ac quotient:

$$Q(R) = Q_m(R)/Q_N(R) \qquad \text{----------------(19)}$$

If the value of the relative volume flow rate Q(R) = 1, it means that values of volume flow rate calculated by use Navier-Stokes equation and micropolar fluid dynamic equations are the same.In such situation the flow in the microchannel of radius R is well described by the classical model of the fluid, and it pays off to carry on the calculation on the basis of the classical dynamics,Navier-Stokes equations which are simpler than those of micropolar fluid dynamic.



Relative volume flow rate Q(R) for water, fluids P_1, P_2, E_1,δ – linear dimension of water molecule, $\delta = 3.15*10^{-10}$[20]

Table – [1]Values of parameters δ and κ of some fluids,

Fluids	$k.10^{-7},m^{-1}$	δ
P_1	15.20	0.695
P_2	8.75	.800
Water	7.03	1.45
E_1	5470.00	1.356

For some real fluids the curves illustrating the dependence of the relative fluid flow rate Q(R) on radius R of the channel are plotted in fig [1]. Experimental data from Table [1]were used for the calculations. Results from Fig. [1] show that, for every fluid beginning from a given channel diameter, d=2R, the volume flow rate calculated using the micropolar fluid model is smaller than that calculated with the use of the classical Newtonian model of the fluid. The difference does increase with the decrease in the channel diameter.

The calculation of Q(R) performed for a real fluid matches the result obtained in the previous section. The formulae describing the interrelations between the parameter group δ and k, the parameters ℓ and N, discussed in the previous section are as follows:

$l^2 = \frac{\delta}{(2\xi+\delta)\kappa^2}$ ------------(20)

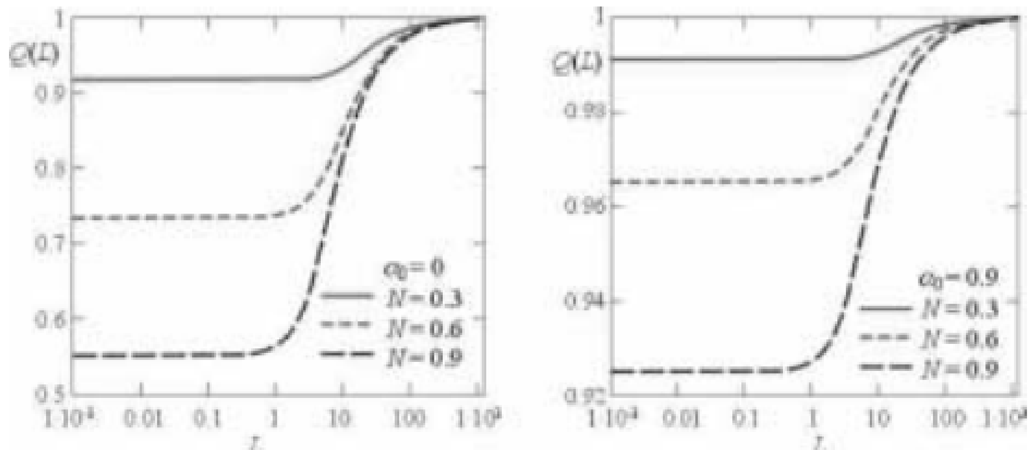
$N^2 = \frac{\delta}{(2\xi+\delta)}$ ------------(21)

where $\xi = 1 - \alpha_0$

Table [2]Values of parameters ℓ and N of some fluids

Fluid	ℓ,m.		N	
	$\alpha_0 = 0$	$\alpha_0 = 0.9$	$\alpha_0 = 0$	$\alpha_0 = 0.9$
P_1	$3.34095*10^{-9}$	$5.79745*10^{-9}$	0.50728	0.88921
P_2	$6.1088*10^{-9}$	$1.022*10^{-8}$	0.53432	0.89443
Water	$9.2614*10^{-9}$	$1.3392*10^{-8}$	0.6483	0.93744
E_1	$1.2713*10^{-9}$	$1.867*10^{-5}$	0.63565	0.93352

Making use of above formulae (20) and (21) it is possible to calculate the range of the changes of □ and N values with regard to α_0 for the fluids from Table[1]. The calculated values are listed in Table[2]. Considering the □values given in table[2]and results depicted in Fig [1], it can be concluded that the greatest differences between the two fluid models occur when the value of the parameter L = d/□satisfies the inequality L < 10.However, one can also observe that Q(R) = 1 if the value of parameter satisfies the inequality L<1000.



Relative volume fluid flow in circular channel as a function of parameters

L and N for α_0 =0 to 0.9

Let us now examine the effect of the microstructural parameters L and N on the relative volume flow rate detail. Using formula (21) and (20) to (17) and (19) we obtain:

:

$$\left\{\frac{1+8N^2 2\zeta}{(2+2N^2\zeta)(LN)^2}\left[1 - \frac{1-LNI_0(LN)}{2I_1(LN)}\right]\right\}$$ -----------(22)

Results of calculations of relative volume fluid flow rate $Q(L,N,\alpha_0)$ are depicted in fig.[2]. It can be observed that the Q value is strongly affected by the parameters N and α_0. An increase in the value of parameter N (at constant L and α_0 values) brings about a decrease in the value of Q.Next, an increase in the value of parameter α_0 (at constant L and N) increases the Q value.Analyzing the resultsit can be concluded that for $L > 1000$ the value of $Q = 1$, otherwise $Q < 1$, which means that there occur discrepancies in calculations, obtained making use of the two fluid models.For $L = 0(1)$ and smaller values, $Q(L, N, \alpha_0)$ value is the smallest.Obtained results match the estimate for real fluids.

To summaries the results of obtained, we can state that the channel diameter at which micropolar effects are small enough to justify carrying out calculations based on the classical fluid dynamics crucially depends on the rheological parameters of the fluid and parameters α_0. An effective application of the micropolar fluid model for calculations depends of on the values of parameter L,N and α_0. For $L=0(1)$, thereoccur maximum differences between the results of the calculations based on the two models. For $L > 100$ there are hardly any differences.

It should be pointed out that the analyses of an effective application of a micropolar fluid theory to modeling other micro flows (between squeezing plates, between converging spheres) shows exactly the same dependence of the geometrical range of its applicability on microstructure parametersL,N [20] as presented here, for flow in microchannels.

5. RESULTS AND DISCUSSION:

The above results show that the geometric size of flow field plays a crucial role in the useful applicability micropolar fluid theory to modeling microflows. The general result was established theoretically, which shows that when the characteristic linear dimension of the flow field is large enough, the micropolar model can be reduced to the classical Navier-Stokes equation. This result was obtained through an application of dimensional analysis to the set of equations describing the micropolar fluid flow. It should be pointed out that reasoning of that type could sometimes lead to an error. This is not the case here since the classical model is not a singular perturbation of the micropolar model. For flow in microchannels the upper limit of usability of micropolar fluid theory have been established as a result of comparing volume flow rate of Poiseuille flow based on the classical and the micropolar fluid mechanics.

The experimentally determined values of rheological constants of the fluid have been used in calculations. It was confirmed that the micropolar model is applicable for small characteristics geometrical dimension of the flow. Furthermore, the particular value of the diameter of the channel for which the flow should be modelled using the micropolar approach was established. Results indicate that this “limiting dimension” depends on the rheological properties of the fluid that can be expressed through define here non-dimensional microstructure parameters.

For wider microchannels, the flow is quite well described by the classical model of the fluid, and it pays off to carry on the calculations on the basis of the classical dynamics, Navier-Stokes equations, which are simpler than those of micropolar fluid dynamics equations. The obtained results Match some of the earlier obtained experimental estimate. The lower limit of applicability of the micropolar fluid theory to modelling micro flows as was mentioned before results from fundamental questions for how small dimensions of the flow field the micropolar fluid model can be treated as continuum medium.

Based on the results it may be concluded that the micropolar theory is applicable to modelling fluid flows in channels of width not smaller than 10 diameters of the fluid molecule.

REFERENCES:

[1] A.C. Eringen, Simple microfluids, Int. J. Engg. Sci. 2(1964) 205.

[2] A.C. Eringen, Theory of micropolar fluids, J. Math. Mac. 16(1966).

[3] A.C. Eringen, Int. J. Engg. Sci., 2, 205(1964).

[4] Agarwal, R.S. Bhargava, R. and Balaji , A.V.S. Finite Element Solution of non – steady three dimensional micropolar fluid flow at a stagnation point. Int. J. Engg. Sci., 28, 851 (1990).

[5] A. J. Willson, "Basic flows of micropolar liquid", Appl. Sci. Res. 20, 338-335(1969).

[6]A. Kucaba-Pietal, Z. A. Walenta and Z. Peradzynski, "Flows in Microchannels", TASK Quaterly 5(2), 179-189 (2001).

[7] Bowers, M. B. and Mudawar, I., 1994, "Two – Phase electronic cooling using mini – channel and micro – channel heat sinks," Trans. ASME, vol. 116, p. 290.

[8] Brody J. and Yager, P., 1996, "Low Reynolds number micro – fluidic devices," Solid – State Sensor and Actuator Workshop, Hilton Head, SC, June 2 – 6, pp. 105 – 108.

[9] Effenhauser , C., Manz, A., and Widmer, M., 1993, "Glass chips for high – speed capillary electrophoresis separation with submicrometer plate heights," Anal.Chem., vol. 65, pp. 2637 – 2642.

[10] G. Lukaszewicz, Micropolar Fluids. Theory and Application, Birkhouser, Berlin, 1999.

[11] G. Wilks, Combined forced and free convenction flow on vertical surfaces, Int. J. Heat Mass Transfer, 16(10), pp. 1958-1964, 1973.

[12] G. Ahmadi, Self-similar solution of incompressible micropolar boundary layer flow over a semiinfinite plate, Int. J. Engin. Sci. 14(1976), 639-646.

[13] Gad-El-Hak, "The fluid mechanics of microdevices", The Freeman scholar lecture", J. Fluids Engng., 1215 – 1233 (1999).

[14] Garg, V.K., "Heat Transfer due to Stagnation Point Flow of a Non-Newtonian Fluid", Acta Mech., 104, 159-171, 1994.

[15] Harrison, D. J., Fan, Z., Fluri, K., and Seiler, K., 1994, “Integrated electrophoresis systems for biochemical analysis,”IEEE Solid – State Sensor and Actuator Workshop, Hilton Head, SC, June 13 – 16.

[16] Hiemenz, K., “Die Grenzschicht an Einem in den GleichformingenFlussigkeitsstromEingetauchtenGeradenKreiszylinder”, Dinglem Polytech. J., 326, 321 – 410, 1911.

[17] I. Papautsky, J. Brazzle, T. Amel and A.B. Frazier, “Microchannel fluid behavior using micropolar fluid theory”, Proc.EMBS 97, Microelectromech. Sys., Chicago, 2285-2291 1997.

[18] I. Papautsky, J. Brazzle, T. Ameel and A. B. Frazier, “Laminar fluid behavior in microchannels using micropolar fluid theory”, Sensors and Actuators A 73(2), 101 – 108 (1999).

[19] J. Delhommelle and D.J. Evans, “Poiseille flow of micropolar fluid”, Molecular Physics 100(17), 2857-65 (2002).

[20] K. A. Kline and S. J. Allen, “Nonsteady flows of fluids with microstructure”, Physics of Fluids 13,236-283 (1970).

[21] K. Ogata, State Space Analysis of Control Systems, Prentice-Hall, New Jersey, 1967.

[22] K. P. Travis, B. D. Todd and D. J. Evans, “Departure from Navier-Stokes hydrody-namics in confined liquids”, Phys. Rev. E55(4), 4288-4295 (1997).

[23] Krause, P., Obermeier, E., and wehl, W., 1995, “Backshooter – A new smart Micromachined single – chip inkjet printhead,” Transducers’95, Stockholm, Sweden, June 25 – 29, pp. 325 – 328.

[24] K. Tewari, P. Sing. Natural convection in a thermally stratified fluid saturated porous medium, Int. J. Engng. Sci., 30(8), pp. 1003 – 1007, 1992.

[25] Migoun, N., 1996, Kapilarnawiskozymetriadlamikroobjemowzydkostjej, Sankt Petersburg, Russia.

[26] N.A. Kalson and T.W. Farrell, “Micropolar flow over a porous stretching sheet with strong suction or injection. International Communication in Heat and Mass Transfer”, V-28, 1-4, Mass 2001, Pg. 479-489.

[27] Peng, X. F., and Wang, B. X., 1994, Proc. 10th International Heat Transfer Conference, Brighton, UK, Aug. 14 – 18, pp. 159 – 177.

[28] Preston, J.H., “The Boundary Layer Flow over a Permeable Surface through which Suction is Applied”, Reports and Memoirs. British Aerospace Research Council, London, No. 2244, 1946.

[29] Rudra Pratap, “Getting Started with MATLAB 7” Oxford University Press.

[30] Reston R. and Kolesar, E., 1994, “Silicon – Micromachined gas chromatography system used to separate and detect ammonia and nitrogen dioxide,” Microelectromech. Sys., vol. 3, pp. 134 – 146.

[31] S.D. Harris, D. B. Ingham, I. Pop, Unsteady mixed convection boundary layer flow on a vertical surface in a porous medium, Int. I Heat Mass Transfer, 42(2), pp.357 – 372, 1999.

[32] Srinevasan, R., Hsing, I.M., J. Harold, M.P., Jensen, K.F., and Schidt, M.A.., 1996, “Micromachined chemical reactors for surface catalyzed oxidation reactions,” IEEE Solid – State Sensor and Actuator Workshop, Hilton Head, SC, June 2 – 6.

[33] Tuckerman, D. B. and Pease, R. F. W., 1981, “High – performance heat sinking for VLSI,” IEEE Electron Dev. Lett., vol. EDL -, No. 5, pp. 126 – 129.

[34] Van Den Berg, A., Olthuis, W., and Bergveld, P., eds., 2000 Micro Total Analysis Syatems 2000, Kluwr Academic Publishers, Boston, MA.

[35] V. Girault and P. A. Raviart. Finite Element Methods for Navier-Stokes Equations. Springer, 1986.