A Note on Random Process

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Abstract

This note is dealt with discrete and continuous random processes. It also take discussion on different examples on random process.

KEYWORDS: Discrete random variables; Continuous random variables; Probability.

INTRODUCTION

Random variable:

A random variable is a real valued function whose numerical value is determined by the outcome of a random experiment. In other words the random variable X is a function that associated each element in the sample space Ω from with the real numbers (i.e. $X: \Omega \rightarrow \mathbb{R}$).

Discrete random variable:

A random variable X is called a discrete random variable if its set of possible values is countable (integer).

Continuous random variable:

A random variable X is called a continuous random variable if it can take values on a continuous scales.

RANDOM PROCESS

Random process is a family of random variables, $\{X(t), t \in T\}$,or $\{X_t, t \in T\}$That is, for each t in the index set$T, X(t)$ is a random variable.

Random process also defined as a random variable which a function of time t,that means, $X(t)$) is a random variable for every time instant $t$or it's a random variable indexed by time.

We know that a random variable is a function defined on the sample space Ω. Thus a random process $\{X_t, t \in T\}$is a real function of two arguments $\{X(t, \omega),\ t \in T, \omega \in \Omega\}$. For fixed $t(= t_k), X(t_k,\ \omega) = X_k(\omega)$is a random variable denoted by $X(t_k)$, as ω varies over the sample space Ω. On the other hand for fixed sample space $\omega_h \in \Omega, X(t,\ \omega_h) = X_h(t)$is a single function of time t, called a sample function or a *realization* of the process. The totality of all sample functions is called an *ensemble*.

If both ω and t are fixed, $X(t_k,\ \omega_h)$ is a real number. We used the notation $X(t)$to represent $X(t, \omega)$.

Description of a Random Process:
In a random process $\{X(t), t \in T\}$ the index t called the *time-parameter* (or simply the time) and $T \in \mathbb{R}$ called the parameter set of the random process. Each $X(t)$ takes values in some set $S \in \mathbb{R}$ called the *state space*; then $X(t)$ is the state of the process at time t, and if $X(t) = i$ we said the process in state i at time t.

Definition:
$\{X(t), t \in T\}$ is a discrete-time (discrete parameter) process, if the index set T of the random process is discrete. A discrete-parameter process is also called a random sequence and is denoted by $\{X(n),\ n = 1, 2, 3, \ldots\}$ or$\{X_n,\ n = 1,\ 2,\ 3, \ldots\}$.
In practical this generally means $T = \{1, 2, 3, \ldots\}$.
Thus a discrete-time process is $\{X(0), X(1), X(2), \ldots\}$: a new random number recorded at every time$0, 1, 2, \ldots$.

Definition:
$\{X(t), t \in T\}$ is continuous-time (continuous parameter) process if the index set T is continuous.
In practical this generally means $T = [0, \infty)$, or $T = [0,\ K]$ for some K.

Thus a continuous-time process $\{X(t), t \in T\}$ has a random number $X(t)$ recorded at every instant in time.
(Note that $X(t)$ needs not change at every instant in time, but it is allowed to change at any time; i.e. not just at $t = 0, 1, 2, \ldots$ like a discrete-time process).

Definition:
The state space, S: is the set of real values that $X(t)$ can take.
Every $X(t)$ takes a value in $\mathbb{R}$, but S will often be a smaller set: $S \subset \mathbb{R}$.$ For example, if$X(t)$is the outcome of a coin tossed at time t, then the state space is $S = \{0,\ 1\}$.

Definition:
The state space S is called a discrete-state process if it is discrete, often referred to as a chain. In this case, the state space S is often assumed to be $\{0,\ 1,\ 2, \ldots\}$.If the state space S is continuous then we have a continuous-state process.

Examples:
Discrete-time, discrete-state processes

1. Tossing a balanced die more than once, if we interest on the number on the uppermost face at toss n, say $X(1)$ the number appears on the first toss, $X(2)$ number appears in the second one, …, etc., then $\{X(n),\ n \in T\}$$ is the random process, and the random variable $X(n)$ denotes the number appears at toss n. where n is the parameter. □ $= \{1,\ 2,\ 3, \ldots\}$ and □ $= \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$.
2. The number ofemails in your inbox attime □. □ $= \{1,\ 2,\ 3, \ldots\}$ and □ $= \{0,\ 1,\ 2, \ldots\}$.
3. Your bank balance on day □.
4. The number of occupied channels in a telephone link at the arrival time of the n[th] customer, □ $= 1, 2, 3, \ldots$.

Continuous-time, discrete-state processes

5. The number of occupied channels in a telephone link at time □ > 0.

6. The number of packets in the buffer of a statistical multiplexer at time □ > *0.*

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