# Quasi ξ- Normal Spaces and □Gξ-Closed Sets

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## Abstract

In this paper, we introduce the concept of □gξ-closed sets as a weak form of □g-closed sets. By utilizing □gξ-closed sets, we define □gξ-closed, almost □gξ-closed, □gξ-continuous and almost □gξ-continuous functions. We also introduce the notion of quasi ξ-normal spaces and obtain a characterization and some preservation theorems for quasi ξ-normal spaces. Further we show that this property is a topological property and it is a hereditary property only with respect to closed domain subspaces. The relationships among normal, π-normal, quasi normal, softly normal, mildly normal, α-normal, πα-normal, quasi α-normal, softly α-normal, mildly α-normal, ξ-normal, πξ-normal, quasi ξ-normal, softly ξ-normal and mildly ξ-normal are investigated.

## 1. Introduction

The concept of α-open sets were introduced by Njastad [**10**]. The notion of quasi normal space was introduced by Zaitsev [**14**]. Levine [**8**] initiated the study of so called g-closed sets in order to extend many of the most important properties of closed sets to a large family. This concept was found to be useful and many results in general topology were improved. Singal and Singal [**13**] have introduced the notion of mildly normal spaces which are weaker form of quasi normal spaces due to Zaitsev [**14**]. Lal and Rahman [**7**] have further studied notions of quasi normal and mildly normal spaces. Dontchev and Noiri [**4**] introduced the notion of □g-closed sets as a weak form of g-closed sets due to Levine [**8**]. By using □g-closed sets, Dontchev and Noiri [**4**] obtained a new characterization and some preservation theorems for quasi normal spaces. Devi et al. [**3**] introduced the concept of ξ-closed sets. Arockiarani and Janaki [**1**] introduced the concept of πgα-closed sets as a weak form of πg-closed sets due to Dontchev and Noiri [**4**] and by using πgα-closed sets, obtained a characterization and some preservation theorems for quasi α-normal spaces. Kalantan [**6**] introduced a weaker version of normality called □-normality and proved that □-normality is a property which lies between normality and almost normality. The notion of α-normal space was introduced by Benchalli and Patil [**2**]. Hamant Kumar and M. C. Sharma [**12**] introduced the concept of softly normal spaces and obtained its properties. Hamant Kumar [**5**] introduced the concept of gξ-closed sets and by using gξ-closed sets, obtained a characterization and some preservation theorems for ξ-normal and ξ-regular spaces.

**2010 AMS Subject classification**: 54D15, 54D10, 54A05, 54C08.
**Keywords**: □-open, ξ-open, □gξ-open, □-closed, ξ-closed, and □gξ-closed sets; □gξ-closed, almost □gξ-closed, □gξ-continuous and almost □gξ-continuous functions; quasi ξ-normal spaces.

## 2. Preliminaries

Throughout this paper, spaces (X, □), (Y, □), and (Z, □) (or simply X, Y and Z) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and interior of A are denoted by cl(A) and int(A) respectively. A subset A is said to be **regular open** (resp. **regular closed**) if A □ int(cl(A)) (resp. A □ cl(int(A)). The finite union of regular open sets is said to be □**-open**. The complement of a □-open set is said to be □**-closed.** A is said to **α–open [10]** if A □ int(cl(int(A))). The complement of a α-open set is said to be **α-closed**. The intersection of all α-closed sets containing A is called **α-closure** of A, and is denoted by **αcl(A)**. The **α-interior** of A, denoted by **αint(A)**, is defined as union of all α-open sets contained in A.

**2.1 Definition.** A subset A of a space X is said to be
**(1) generalized closed** (briefly **g-closed**) **[8]** if cl(A) □□U whenever A □□U and U □ □□
□□□□□**g-closed** [**4**] if cl(A) □□U whenever A □□U and U is □□open in X.
**(3) α-generalized closed** **[9]** if αcl(A) □□U whenever A □□U and U □ □□
□□□□□**gα-closed [1]** if αcl(A) □□U whenever A □□U and U is □□open in X.
**(5) ξ-closed** [**3**] if αcl(A) □□U whenever A □□U and U is gα-open in X.
**(6) g-open** (resp**.** □**g-open**, α**g-open**, □**gα-open, ξ-open**) if the complement of A is g□closed (resp. □g-closed, αg-closed, □gα□closed, ξ-closed).

The intersection of all ξ-closed sets containing A is called **ξ-closure** [**5**] of A, and is denoted by ξcl(A). The **ξ-interior** [**5**] of A, denoted by ξint(A), is defined as union of all ξ-open sets contained in A.

**2.2 Definition.** A subset A of a space X is said to be
**(1) generalized ξ-closed** (briefly **gξ-closed**) **[5]** if ξcl(A) □□U whenever A □□U and U □ □□
□□□□□**gξ-closed** if ξcl(A) □□U whenever A □□U and U is □□open in X.

**2.3 Remark.** We have the following implications for the properties of subsets:

closed □ g-closed □□□□□□ □g-closed

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α-closed □ αg-closed □□□□□□ □gα-closed

□□ □□□□□□□□□□□ □□□□

ξ-closed □ gξ-closed □□□□□□ □gξ-closed

Where none of the implications is reversible as can be seen from the following examples:

**2.4 Example.** Let X = {a, b, c} and □ = {□, {a}, X}. Then A = {b} is g-closed as well as αg-closed but not closed.

**2.5 Example.** Let X = {a, b, c} and □ = {□, {a}, X}. Then A = {a, b} is g-closed as well as αg-closed. It is also gξ-closed but not closed.

**2.6 Example.** Let X = {a, b, c, d} and □ = {□, {c}, {d}, {a, c}, {c, d}, {a, c, d}, X}. Then A = {a} is α-closed as well as ξ-closed but not closed.

**2.7 Example.** Let X = {a, b, c, d} and □ = {□, {c, d}, X}. Then A = {a, b, c} is ξ-closed as well as gξ-closed but not closed.

**2.8 Example.** Let X = {a, b, c, d} and □ = {□, {a}, {c}, {a, c}, {a, d}, {a, b}, {a, b, c}, {a, b, d}, {a, c, d}, X}. Then A = {a, b} is πgα-closed as well as πgξ-closed but not closed.

**2.9 Example.** Let X = {a, b, c, d} and □ = {□, {a}, {d}, {a, d}, {c, d}, {a, c, d}, X}. Then A = {c} is πgα-closed as well as πgξ-closed but it is neither g-closed nor closed.

**2.10 Example.** Let X = {a, b, c, d} and □ = {□, {a}, {b}, {a, b}, {c, d}, {b, c, d}, {a, c, d}, X}. Then A = {a} and B = {b} are πg-closed as well as πgα-closed.

## 3. Quasi ξ -normal spaces

**3.1 Definition.** A space X is said to be **ξ-normal** [5] (resp. **α-normal** [2]) if for every pair of disjoint closed subsets A, B of X, there exist disjoint ξ-open (resp. α-open) sets U, V of X such that A □ U and B □ V.

**3.2 Definition.** A space X is said to be **□ξ–normal** (resp. **□-normal** [6], **□α-normal**) if for every pair of disjoint closed subsets A, B of X, one of which is □-closed, there exist disjoint ξ-open (resp. open, α-open) sets U, V of X such that A □ U and B □ V.

**3.3 Definition.** A space X is said to be **quasi ξ-normal** (resp. **quasi normal** [14], **quasi α-normal [1]**) if for every pair of disjoint □-closed subsets H, K of X, there exist disjoint ξ-open (resp. open, α-open) sets U, V of X such that H □ U and K □ V.

**3.4 Definition.** A space X is said to be **mildly ξ-normal** (resp. **mildly normal** [13], **mildly α-normal [1]**) if for every pair of disjoint regular closed subsets H, K of X, there exist disjoint ξ-open (resp. open, α-open) sets U, V of X such that H □ U and K □ V.

**3.5 Definition.** A space X is said to be **softly ξ-normal** (resp. **softly normal** [12], **softly α-normal**) if for any two disjoint closed subsets A, B of X, one of which is □-closed and other is regularly closed, there exist disjoint ξ-open (resp. open, α-open) sets U, V of X such that A □ U and B □ V.

**By the definitions stated above, we have the following diagram:**

normality □ □-normalily □□□□quasi normality □ softly normality □ mild-normality

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α-normality □□□ □α-normalily □□□ quasi α-normality □□□ softly α-normality □ mild α-normality

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ξ-normality □ □ξ-normalily □ quasi ξ-normality □□□ softly ξ-normality □ mild □-normality

**Where none of the implications is reversible as can be seen from the following examples:**

**3.6 Example.** Let X = {a, b, c, d} and □ = {□, {a, b}, {c, d}, X}. The pair of disjoint closed subsets of X are A = {a, b} and B = {c, d}. Also U = {a, b} and V = {c, d} are open sets such that A □ U and B □ V. Hence the space X is normal as well as α-normal.

**3.7 Example.** Let X = {a, b, c, d, e} and □ = {□, {e}, {a, b}, {c, d}, {a, b, e}, {c, d, e}, {a. b, c, d}, X}. The pair of disjoint π-closed subsets of X are A = {a, b} and B = {c, d}. Also U = {a, b, e} and V = {c, d} are open sets such that A □ U and B □ V. Hence the space X is quasi normal as well as quasi α-normal.

**3.8 Example.** Let X = {a, b, c, d, e} and □ = {□, {a}, {c, d}, {a, c, d}, {b, c, d, e}, X}. The pair of disjoint closed subsets of X are A = {a} and B = {b, e}. Also U = {a} and V = {b, c, d, e} are open sets such that A □ U and B □ V. Hence the space X is normal as well as α-normal. It is also ξ-normal space.

**3.9 Example.** Let X = {a, b, c} and □ = {□, {a, b}, {c}, X}. The pair of disjoint regularly closed subsets of X are A = {a, b} and B = {c}. Also U = {a, b} and V = {c} are open sets such that A □ U and B □ V. Hence the space X is mildly normal as well as mildly α-normal. It is also mildly ξ-normal space.

**3.10 Theorem.** For a space X, the following are equivalent:
(a) X is quasi ξ-normal.
(b) For every pair of □-open subsets U and V of X whose union is X, there exist ξ-closed subsets G and H of X such that G□□ U, H □□V and G □ H = X.
(c) For any □-closed set A and every □-open set B in X such that A □ B, there exists a ξ-open subset U of X such that A □□U □ ξcl(U) □ B.
(d) For every pair of disjoint □-closed subsets A and B of X, there exist ξ-open subsets U and V of X such that A □□U, B □□V and ξcl(U)□□□ξcl(V) = □.

**Proof.** (a) □□(b), (b) □□(c), (c) □□(d) and (d) □□(a).

(a) □□(b). Let U and V be any □-open subsets of a quasi ξ-normal space X such that U □ V = X. Then, X – U and X – V are disjoint □-closed subsets of X. By quasi ξ-normality of X, there exist disjoint ξ-open subsets $U_1$ and $V_1$ of X such that X - U □ $U_1$ and X - V □ $V_1$. Let G = X - $U_1$ and H = X - $V_1$. Then, G and H are ξ-closed subsets of X such that G □□U, H □□V and G □□H = X.

(b) □□(c). Let A be a □-closed and B is a □-open subsets of X such that A □□B. Then, X - A and B are □-open subsets of X such that (X - A) □ B = X. Then, by part (b), there exist ξ-closed sets G and H of X such that G □ (X - A), H □□ B and G □ H = X. Then, A □ (X - G), (X - B) □□□X - H) and (X - G) □ (X - H) = □. Let U = X – G and V = (X - H). Then U and V are disjoint ξ-open sets such that A □ U □□X - V □ B. Since X - V is ξ-closed, then we have ξcl(U) □ (X - V). Thus, A □ U □□ξcl(U) □ B.

(c) □□(d). Let A and B be any disjoint □-closed subset of X. Then A □□X - B, where X – B is □-open. By the part (c), there exists a ξ-open subset U of X such that A □ U □□ξcl(U) □ X - B. Let V = X - ξcl(U). Then, V is a ξ-open subset of X. Thus, we obtain A □□U, B □□□V and ξcl(U) □ ξcl(V) = □.

(d) □□(a). It is obvious.

**3.11 Proposition.** Let f : X □□Y be a function, then:
(a) The image of ξ-open subset under an open continuous function is ξ-open.
(b) The inverse image of ξ-open (resp. ξ-closed) subset under an open continuous function is □ξ-open (resp. ξ-closed) subset.
(c) The image of ξ-closed subset under an open and a closed continuous surjective function is ξ-open.

**3.12 Theorem.** The image of a quasi ξ-normal space under an open continuous injective function is a quasi ξ-normal.

**Proof.** Let X be a quasi ξ-normal space and let f : X □□Y be an open continuous injective function. We need to show that f(X) is a quasi ξ-normal. Let A and B be any two disjoint □-closed sets in f(X). Since the inverse image of a □-closed set under an open continuous function is a □-closed. Then, $f^{-1}$(A) and $f^{-1}$(B) are disjoint □-closed sets in X. By quasi ξ-normality of X, there exist ξ-open subsets U and V of X such that $f^{-1}$(A) □□U, $f^{-1}$(B) □□V and U □□V = □. Since f is an open continuous injective function, we have A □ f(U), B □□f(V) and f(U) □□f(V) = □. By **Proposition 3.11**, we obtain f(U) and f(V) are disjoint ξ-open sets in f(X) such that A □□f(U) and B □□f(V). Hence f(X) is quasi ξ-normal.

From the above theorem, we have the following corollary.

**3.13 Corollary.** Quasi□ξ-normality is a topological property.

The following lemma helps us to show that quasi ξ-normality is a hereditary with respect to closed domain subspaces.

**3.14 Lemma.** Let M be a closed domain subspace of a space X. If A is a ξ-open set in X, then A □□M is ξ-open set in M.

**3.15 Theorem.** Quasi ξ-normality is a hereditary with respect to closed domain subspaces.

**Proof.** Let M be a closed domain subspace of a quasi ξ-normal space X. Let A and B be any disjoint □-closed sets in M. Since M is a closed domain subspace of X, then we have A and B be any disjoint □-closed sets of X. By quasi ξ-normal of X, there exist disjoint ξ-open subsets U and V of X such that A □ U and B □ V. By the **Lemma 3.14,** we obtain U □□M and V □□M are disjoint ξ-open sets in M such that A □□U □□M and B □ V □□M. Hence, M is quasi ξ-normal subspace.

Since every closed and open (clopen) subset is a closed domain, then we have the following corollary.

## 4. Preservation Theorems

The following result is useful for giving some other characterizations of quasi ξ-normal spaces.

**4.1 Lemma.** A subset A of a space X is □gξ-open if and only if F □ ξint(A) whenever F□□□A and F is □-closed.

**4.2 Theorem.** For a space X, the following are equivalent:
(a) X is quasi ξ-normal.
(b) For any disjoint □-closed sets H and K, there exist disjoint gξ-open sets U and V such that
H□□ U and K □□V.
(c) For any disjoint □-closed sets H and K, there exist disjoint □gξ-open sets U and V such that H □ U and K □V.
(d) For any □-closed set H and any □-open set V containing H, there exists a gξ-open set U of X such that H □□U □ ξcl(U) □ V.□
(e) For any □-closed set H and any □-open set V containing H, there exists a □gξ-open set U of X such that H □□ U □ ξcl(U) □ V.

**Proof.** (a) □ (b), (b) □ (c), (c) □ (d), (d) □ (e) and (e) □ (a).

(a) □ (b). Let X be quasi ξ-normal space. Let H, K be disjoint □-closed sets of X. By assumption, there exist disjoint ξ-open sets U, V such that H □ U and K □ V. Since every ξ-open set is gξ-open, U and V are gξ-open sets such that H □ U and K □ V.

(b) □ (c): Let H, K be two disjoint □-closed sets. By assumption, there exist disjoint gξ-open sets U and V such that H □ U and K □ V. Since every gξ-open set is □gξ-open, U and V are □gξ-open sets such that H □ U and K □ V.

(c) □ (d): Let H be any □-closed set and V be any □-open set containing H. By assumption, there exist disjoint □gξ-open sets U and W such that H □ U and X-V □

W. By **Lemma 4.1**, we get X-V □ ξint(W) and ξcl(U) □ ξint(W) = □. Hence H □ U □ ξcl(U) □□ X-ξint(W) □ V.

(d) □ (e): Let H be any □-closed set and V be any □-open set containing H. By assumption, there exist gξ-open set U of X such that H □ U □ ξcl(U) □ V. Since, every gξ-open set is □gξ-open, there exists □gξ-open sets U of X such that H □ U □ ξcl(U) □ V.

(e) □ (a): Let H, K be any two disjoint □-closed sets of X. Then H □ X - K and X - K is □-open. By assumption, there exists □gξ-open set G of X such that H □ G □ ξcl(G) □ X - K. Put U = ξint(G), V = X - ξcl(G). Then U and V are disjoint ξ-open sets of X such that H □ U and K □ V.

**4.3 Definition.** A function f : X □ Y is said to be
(a) **ξ-closed** [3] (resp. □**gξ-closed**) if f(F) is ξ-closed (resp. □gξ-closed) in Y for every closed set F of X.
(b) **rc-preserving** [11] (resp. **almost closed** [13], **almost ξ-closed**, **almost □gξ-closed**) if f(F) is regular closed (resp. closed, ξ-closed, □gξ-closed) in Y for every F □ RC(X).
(c) □**-continuous** [4] (resp. **almost □-continuous** [4]) if $f^{-1}$(F) is □-closed in X for every closed (resp. regular closed) set F of Y.
(d) **almost continuous** [13] if $f^{-1}$(V) is open in X for every regular open set V of Y.
(e) □**gξ-continuous** (resp. **almost □gξ-continuous**) if $f^{-1}$(F) is □gξ-closed in X for every closed (resp. regular closed) set F of Y.

**4.4 Theorem.** If f : X □□Y is an almost □-continuous and □gξ-closed function, then f(A) is □gξ-closed in Y for every □gξ-closed set A of X.

**Proof.** Let A be any □gξ-closed set of X and V be any □-open set of Y containing f(A). Since f is almost □-continuous, $f^{-1}$(V) is □-open in X and A □□$f^{-1}$(V). Therefore, we have ξcl(A) □ $f^{-1}$(V) and hence f(ξcl(A)) □ V. Since f is □gξ-closed, f(ξCl(A)) is □gξ-closed in Y and hence we obtain ξcl(f(A)) □□ξcl(f(ξcl(A)))□□□V. Hence f(A) is □gξ-closed in Y.

**4.5 Theorem.** A surjection f : X □□Y is almost □gξ-closed if and only if for each subset S of Y and each U □□RO(X) containing $f^{-1}$(S), there exists a □gξ-open set V of Y such that S □ V and $f^{-1}$(V) □□U.

**Proof. Necessity**. Suppose that f is almost □gξ-closed. Let S be a subset of Y and U □ RO(X) containing $f^{-1}$(S). If V = Y-f (X-U), then V is a □gξ-open set of Y such that S □□□V and $f^{-1}$ (V) □□U.

**Sufficiency**. Let F be any regularly closed set of X. Then $f^{-1}$(Y-f (F)) □ (X- F) and (X-F)□□ RO(X). There exists a □gξ-open set V of Y such that Y - f (F) □□V and $f^{-1}$ (V) □ (X - F). Therefore, we have f(F) □ (Y-V) and F □ X – $f^{-1}$(V) □ $f^{-1}$ (Y -V) . Hence we obtain f(F) = Y - V and f(F) is □gξ-closed in Y, which shows that f is almost □gξ-closed.

**4.6 Theorem.** If f : X □ Y is an almost □gξ-continuous, rc-preserving injection and Y is quasi ξ-normal then X is quasi ξ-normal.

**Proof.** Let A and B be any disjoint □-closed sets of X. Since f is an rc-preserving injection, f(A) and f(B) are disjoint □-closed sets of Y. Since Y is quasi ξ-normal, there exist disjoint ξ-open sets U and V of Y such that f(A) □ U and f(B) □ V.

Now if G = int(cl(U)) and H = int(cl(V)). Then G and H are regularly open sets such that f(A) □ G and f(B) □ H. Since f is almost □gξ-continuous, $f^{-1}$(G) and $f^{-1}$(H) are disjoint □gξ-open sets containing A and B which shows that X is quasi ξ-normal.

**4.7 Theorem.** If f : X □□Y is □-continuous, almost ξ-closed surjection and X is quasi ξ-normal space then Y is ξ-normal .

**Proof.** Let A and B be any two disjoint closed sets of Y. Then $f^{-1}$(A) and $f^{-1}$(B) are disjoint □-closed sets of X. Since X is quasi ξ-normal, there exist disjoint ξ-open sets U and V such that $f^{-1}$(A) □ U and $f^{-1}$(B)□□ V.

Let G = int(cl(U)) and H = int(cl(V)). Then G and H are disjoint regularly open sets of X such that $f^{-1}$(A) □□□G and $f^{-1}$(B) □ H. Now, we set K =Y - f(X-G) and L = Y- f (X-H). Then K and L are ξ-open sets of Y such that A □ K, B □ L, $f^{-1}$(K) □ G and $f^{-1}$(L) □ H. Since G and H are disjoint, K and L are disjoint. Since K and L are ξ-open and we obtain A □ ξint(K), B □ ξint(L) and ξint(K) □ ξint(L) = □. Therefore, Y is ξ-normal.

**4.8 Theorem.** Let f : X □ Y be an almost □-continuous and almost □gξ-closed surjection. If X is quasi ξ-normal space then Y is quasi ξ-normal.

**Proof.** Let A and B be any disjoint □-closed sets of Y. Since f is almost □-continuous, $f^{-1}$(A) and $f^{-1}$(B) are disjoint □-closed sets of X. Since X is quasi ξ-normal, there exist disjoint ξ-open sets U and V of X such that $f^{-1}$(A) □ U and $f^{-1}$(B) □ V.

Put G = int(cl(U)) and H = int(cl(V)). Then G and H are disjoint regularly open sets of X such that $f^{-1}$(A)□□□ G and $f^{-1}$(B) □ H. By **Theorem 4.5,** there exist gξ-open sets K and L of Y such that A □ K, B □ L, $f^{-1}$(K) □ G and $f^{-1}$(L) □ H. Since G and H are disjoint, so are K and L by **Lemma 4.1,** A □ ξint(K), B □ ξint(L) and ξint(K) □ ξInt(L) = □. Therefore, Y is quasi ξ-normal.

**4.9 Corollary**. If f : X □ Y is an almost continuous and almost closed surjection and X is a normal space, then Y is quasi ξ-normal.

**Proof.** Since every almost closed function is almost □gξ-closed by **Theorem 4.8,** Y is quasi ξ-normal.

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