A Supply Chain Production Inventory Model for Deteriorating Products with Stock Dependent Demand and Partial Backlogging

S K Shon, Vinesh Kumar
Department of P.G Studies and Research in Mathematics, Vardhaman College, Bijnor, India

Abstract

In the present study, we developed an Inventory Model in which the production rate is taken as a function of Demand rate and Demand rate is taken as a function of variable stock level to make the study close to reality .Shortages are allowed and occurring shortages are partially backlogged at a constant rate .The Model is developed under inflationary environment with Ramp type demand with the assumption of two Warehouse ,Deterioration and Shortages under learning and Inflationary Environment. Numerical examples and sensitivity are provided to illustrate the Model.

KEYWORDS - Supply Chain Production, Deterioration, Inflation ,Partial Backlogging.

Introduction

In the development of inventory models to assume the demand rate as a constant or time dependent is not enough for all type of business. There are so many products in our daily life for which the demand rate depends on displayed stock level. For these the demand rate may increase or decrease according to the available stock level.

Mandal and Phaujdar (1989) developed an economics production quantity model for deteriorating items with con-stant production rate in which consumption rate depends on stock linearly. It assumed that the more consumption takes place at more stock. Later on, Datta and Pal (1990) established an EOQ model in which the demand rate is a power function of the on-hand inventory displayed until down to a certain stock level, at which the demand rate becomes a constant. Pal et al. (1993) studied a deterministic inventory model for deteriorating items and the demand of item being stock dependent. Giri and Choudhury (1998) determined economic order quantity of perishable inventory with stock dependent demand rate and nonlinear holding cost. Kobbacy and Liang (1999) concerned with the development of an intelligent inventory management system which aims at bridging the substantial gap between the theory and the practice of inventory management. The models incorporated cover deterministic demand models including: constant, quasi-constant, trended and seasonal demand as well as stochastic demand models. Mishra and Singh (2000) presented the inventory models for damageable multi items and also focused on the consumption cost where demand depends on the available stock. Reddy and Sarma (2001) pre-sented a periodic review inventory problem assuming that demand is stock dependent. Datta and Paul (2001) discussed an inventory system in which demand is stock dependent and more sensitive in the sense of price of item. Sana and Chaudhuri (2003) developed an inventory model of a volume flexible manufacturing system for a deteriorating item taking a stock -dependent demand rate. Demand rate remains stock-dependent for an initial period after which a uniform demand rate follows as the stock comes down to a certain level. The unit production cost is taken to be a

function of the finite production rate. Ouyang et al. (2005) proposed an EOQ inventory mathematical model for deteriorating items with exponentially decreasing demand.
Berman and Perry (2006) presented a model in which demand rate is the function of inventory level and it is a piecewise constant function and a family of exponential functions. The demand functions and holding cost functions considered as quite general. Jain et al. (2007) presented an economic production quantity (EPQ) model for deteriorating items with stock-dependent demand and shortages. They assumed that a constant fraction of the on-hand inventory deteriorates and demand rate depends upon the amount of the stock level. Zhou and Shi (2008) developed a deterministic inventory model for deteriorating items with stock- dependent demand. Ghosh and Chakrabarty (2009) considered an order-level inventory model with two levels of storage for deteriorating items assuming that the demand is time- dependent. Panda et al. (2009) considered a single item economic order quantity model in which the demand is stock dependent. After a certain time the product starts to deteriorate and due to visualization effect and other aspects of deterioration the demand becomes constant. Singh et al. (2009) developed an inventory model in which demand follows power pattern. When fresh and new items arrive in stock they begin to decay after a fixed time interval. Skouri and Konstantaras (2009) considered an order level inventory model for seasonable and fashionable products subject to a period of increasing demand followed by a period of level demand and then by a period of decreasing demand rate (three branches ramp type demand rate). Tyagi et al. (2014) presented an optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and variable holding cost.

Deterioration is also an important and necessary feature for the development of inventory models. In our daily life all the products deteriorate with time. Some products have a high rate of deterioration and some products deteriorate at a lower rate. Hou (2006) introduced an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. Chang et al. (2010) developed an inventory model with stock and price dependent demand rate for deteriorating items based on limited shelf space. Tayal et al. (2014) presented a deteriorating production inventory problem with space restriction. Tayal et al. (2014) also introduced an inventory model for deteriorating items having seasonal demand and an option of an alternative market. But for many products deterioration do not occur instantly, the products start to deteriorate after some time. This type of products is known as non instantaneous deteriorating products. For such type of products an EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate was developed by Tayal et al. (2014). After that Tayal et al. (2015) developed an integrated production inventory model for perishable products with trade credit period and investment in preservation technology. In this model a preservation technology is applied to reduce the existing rate of deterioration. Further stock out is also a realistic problem in inventory models. There are so many models in existing literature which assumed that the demand during stock out is completely backlogged or completely lost, but generally, both of these conditions are not possible. Considering this phenomenon Singh et al. (2010) presented an EOQ model with Pareto distribution for deterioration, Trapezoidal type demand and backlogging under trade credit policy. In this model the occurring shortages are partially backlogged. Tayal et al. (2014) presented a multi item inventory model for deteriorating items with expiration date and allowable shortages.

In the present model we have tried to combine all above mentioned realistic features in a single model. This is a production inventory model in which the production rate is taken as a function of demand rate and demand rate is taken as a function of available stock level to make the study close to reality. In this the shortages are allowed and occurring shortages are partially backlogged at a constant rate. The model is developed under inflationary environment. A numerical example and sensitivity analysis is also presented to illustrate the model.

Assumptions and Notations

1. The product considered in this model is deteriorating in nature.
2. The deterioration rate is a constant fraction of on hand inventory.
3. The model is developed for finite time horizon.
4. The demand for the products is stock dependent.
5. The production rate is also considered as a function of demand rate.
6. The shortages are allowed for retailer only and occurring shortages are partially backlogged.

α	positive constant
β	stock dependent parameter for demand, $0 \le \beta \le 1$
T	time horizon
r	rate of inflation
K	production coefficient, $K \ge 1$
θ	constant deterioration rate, $0<\theta<<1$
T_1	production period for vendor
c_m	production cost per unit for the vendor
h_1	holding cost per unit for the vendor
h_2	holding cost per unit for the retailer
A_1	set up cost per production run for the vendor
A_2	ordering cost per order for the retailer
s	shortage cost per unit for the retailer
l	lost sale cost per unit for the retailer
Q	initial inventory level for the retailer
Q_2	backordered quantity during shortages
p	purchasing cost per unit for the retailer
v	the time at which inventory level becomes zero for the retailer

Mathematical Modelling

For Vendor

In this section we have developed a mathematical inventory model for the manufacturer. The production starts at t=0. The production occurs during $[0,T_1]$ and after that during $[T_1, T]$ the inventory depletes due to combined effect of demand and deterioration. This procedure is shown in figure 1.

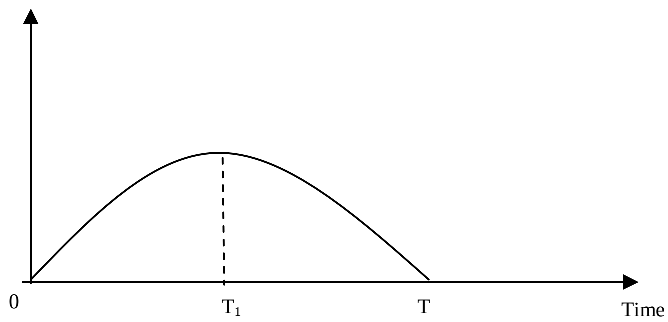

Fig. 1: Vendor's inventory time graph

The differential equations governing the transition of the system during production and non production period is given as follow:

$$\frac{dI_v(t)}{dt} = (K-1)(\alpha + \beta I_v(t)) - \theta I_v(t) \qquad 0 \le t \le T_1 \qquad (1)$$

$$\frac{dI_v(t)}{dt} = -(\alpha + \beta I_v(t)) - \theta I_v(t) \qquad T_1 \le t \le T \qquad (2)$$

Boundary conditions are:

$I_v(0)=0$ and $I_v(T)=0$ (3)

The solutions of these equations are given by:

$$I_v(t) = \frac{(K-1)\alpha}{\theta - \beta(K-1)}(1 - e^{-(\theta - K\beta + \beta)t}) \qquad 0 \le t \le T_1 \qquad (4)$$

$$I_v(t) = \frac{\alpha}{(\theta + \beta)}(e^{(\beta+\theta)(T-t)} - 1) \qquad T_1 \le t \le T \qquad (5)$$

For Retailer

The inventory time graph for the retailer is shown in figure 2. The inventory cycle starts at t=0. During the time period [0, v], the inventory level decreases due to demand and deterioration. At t=v the inventory level becomes zero and after it shortages occur. The occurring shortages are partially backlogged at a constant rate. The differential equations showing the behavior of the system are given as follows

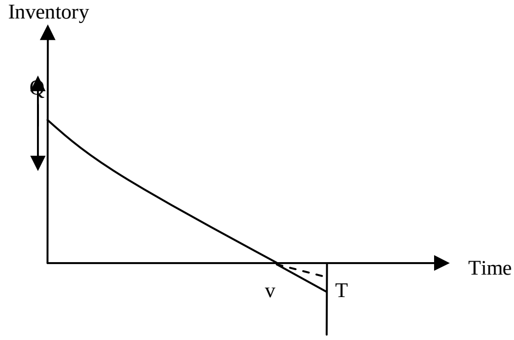

Fig. 2: Retailer's inventory time graph

$$\frac{dI_r(t)}{dt} = -(\alpha + \beta I_r(t)) - \theta I_r(t) \qquad 0 \le t \le v \qquad (6)$$

$$\frac{dI_r(t)}{dt} = -\alpha - \theta I_r(t) \qquad v \le t \le T \qquad (7)$$

with boundary condition $I_r(v) = 0$ (8)

The solutions of these equations are given by:

$$I_r(t) = \frac{\alpha}{(\theta + \beta)}(e^{(\beta+\theta)(v-t)} - 1) \qquad 0 \le t \le v \qquad (9)$$

$$I_r(t) = -\alpha(t - v) \qquad v \le t \le T \qquad (10)$$

Now from equation (10):

$$Q = I_r(0) = \frac{\alpha}{(\beta + \theta)}(e^{(\beta+\theta)v} - 1) \qquad (11)$$

Total cost for the vendor will be the sum of production cost, holding cost and set up cost.
$T.C_v$ = Production cost + Holding Cost + Set up cost (12)

For the retailer total cost will be the sum of purchasing cost, ordering cost, holding cost, shortage cost and lost sale cost.
$T.C_r$ = Purchasing cost + Holding cost + Ordering Cost + Shortage cost + Lost sale cost (13)

Cost Analysis for Vendor:

Production Cost:

Present value of production cost-

$$P.C. = c_m \int_0^{T_1} K(\alpha + \beta I_v(t))dt \qquad (14)$$

Holding Cost:

Present value of holding cost-

$$H.C. = h_1 \int_0^{T_1} I_v(t)dt + h_1 \int_{T_1}^{T} I_v(t)dt \qquad (15)$$

Setup Cost:

Present value of set up cost-

All set up is made at the beginning of cycle, so there will be no inflation at that time.

$$S.U.C. = A_1 \qquad (16)$$

Cost Analysis for Retailer:

Purchasing Cost:

Present value of purchasing cost-

Since the replenishment is made at the beginning of each cycle, so the present value of purchasing cost will be-

$$P.C. = (Q + Q_2)p \qquad (17)$$

Holding Cost:
Present value of holding cost-

$$H.C_r = h_2 \int_0^v I_r(t)dt \tag{18}$$

Shortage Cost:
Present value of shortage cost-

$$S.C_r = s \int_v^T \alpha dt \tag{19}$$

Lost Sale Cost:
Present value of lost sale cost-

$$L.S.C_r = l \int_v^T (1-\theta)\alpha dt \tag{20}$$

$$T.C_S = T.C_v + T.C_r$$

$$T.A.C_S = \frac{1}{T}[T.C_v + T.C_r] \tag{21}$$

Numerical Example:
The following data is used to illustrate the model numerically.

$\alpha = 500\,units,\ T = 10\,days,\ c_m = 5\,rs/unit,\ p = 12\,rs/unit,\ s = 6\,rs/unit,\ h_1 = 0.8\,rs/unit$

$h_2 = 0.7\,rs/unit,\ A_1 = 500\,rs,\ \theta = 0.01,\ \beta = 0.02,\ l = 7\,rs/unit$

For these input values the optimal value of v and T_1 come out to be 8.67279 days and 2.79674 days respectively. Corresponding to these, the optimal value of T.A.C for the supply chain is Rs. 25350.

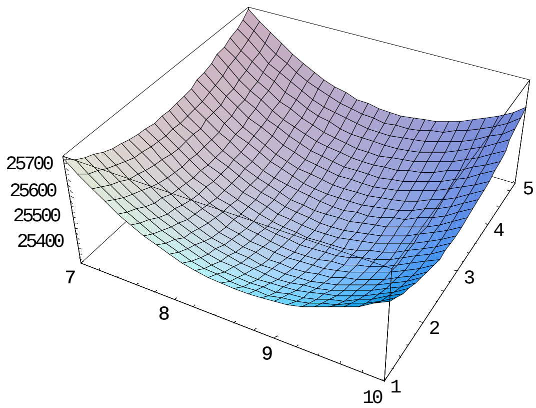

Fig. 3: Behaviour of the T.C. function

Sensitivity Analysis:
A sensitivity analysis with respect to different associated parameters is carried out to observe the change in T.A.C. with the change in these parameters. Table 1-

Table 1: Sensitivity Analysis with respect to demand parameter (α):

% variation in α	**α**	**v**	**T_1**	**T.A.C.**
-20%	400	8.67279	2.79674	20310
-15%	425	8.67279	2.79674	21570
-10%	450	8.67279	2.79674	22830
-5%	475	8.67279	2.79674	24090
0%	500	8.67279	2.79674	25350
5%	525	8.67279	2.79674	26610
10%	550	8.67279	2.79674	27869.9
15%	575	8.67279	2.79674	29129.9
20%	600	8.67279	2.79674	30389.9

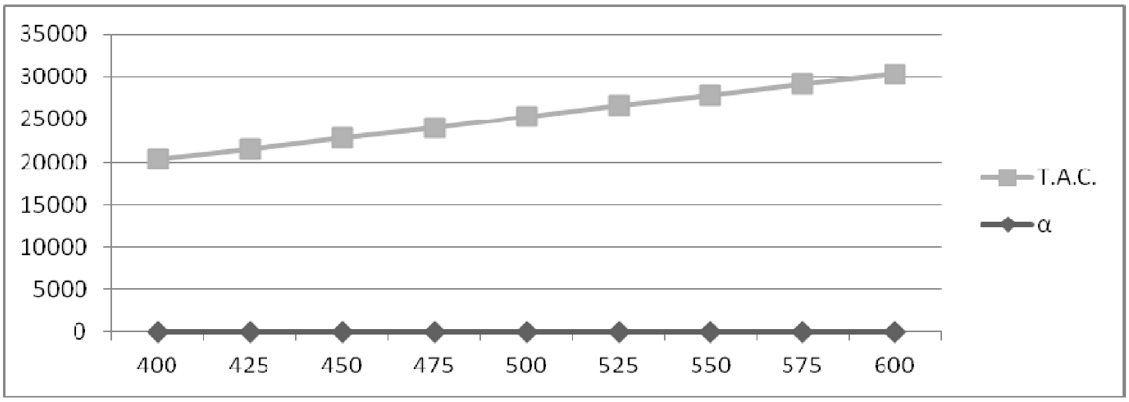

Fig. 4: T.A.C. v/s α

Table 2: Sensitivity Analysis with respect to deterioration parameter (θ):

% variation in θ	θ	v	T1	T.A.C.
-20%	0.008	9.09532	2.79017	24056.8
-15%	0.0085	8.9878	2.79184	24365.5
-10%	0.009	8.88155	2.7935	24683.7
-5%	0.0095	8.77654	2.79513	25011.6
0%	0.01	8.67279	2.79674	25350
5%	0.0105	8.57028	2.79833	25699.1
10%	0.011	8.46901	2.7999	26059.6
15%	0.0115	8.36898	2.80145	26432.1
20%	0.012	8.27018	2.80298	26817.2

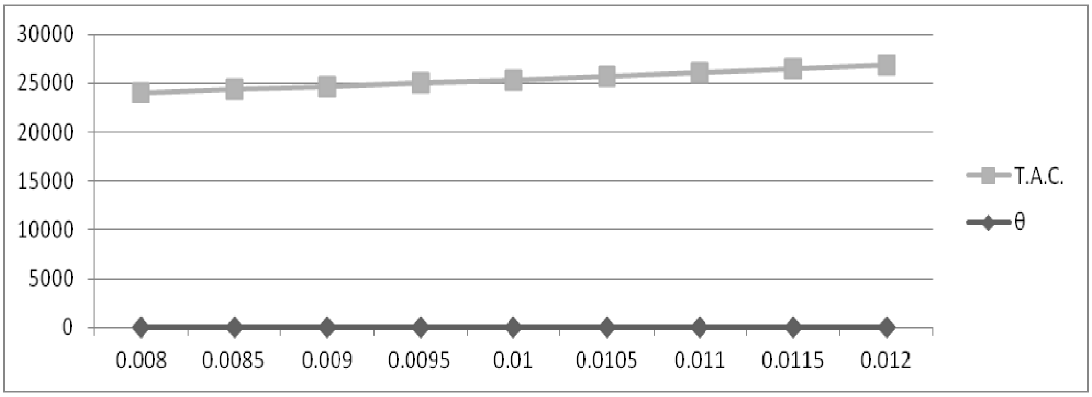

Fig. 5: T.A.C. v/s θ

Table 3: Sensitivity Analysis with respect to demand parameter (β):

% variation in β	**β**	**v**	**T_1**	**T.A.C.**
-20%	0.016	9.5419	2.79366	28497.2
-15%	0.017	9.31707	2.79455	27627.6
-10%	0.018	9.09728	2.79536	26817.1
-5%	0.019	8.88252	2.79609	26059.6
0%	0.02	8.67279	2.79674	25350
5%	0.021	8.46805	2.79732	24683.6
10%	0.022	8.26826	2.79782	24056.6
15%	0.023	8.07338	2.79825	23465.5
20%	0.024	7.88335	2.79862	22907.2

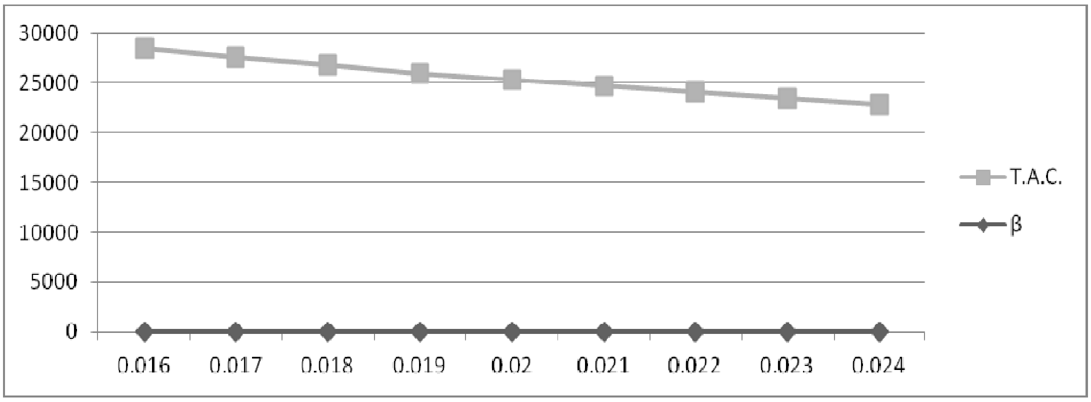

Fig. 6: T.A.C. v/s θ

Table 4: Sensitivity Analysis with respect to holding cost (h_1):

% variation in h_1	**h_1**	**v**	**T_1**	**T.A.C.**
-20%	0.64	8.67279	1.87883	25145.2
-15%	0.68	8.67279	2.1374	25201.7
-10%	0.72	8.67279	2.37493	25254.3

-5%	0.76	8.67279	2.594	25303.5
0%	0.8	8.67279	2.79674	25350
5%	0.84	8.67279	2.98499	25393.9
10%	0.88	8.67279	3.16028	25435.7
15%	0.92	8.67279	3.32394	25475.6
20%	0.96	8.67279	3.47712	25573.8

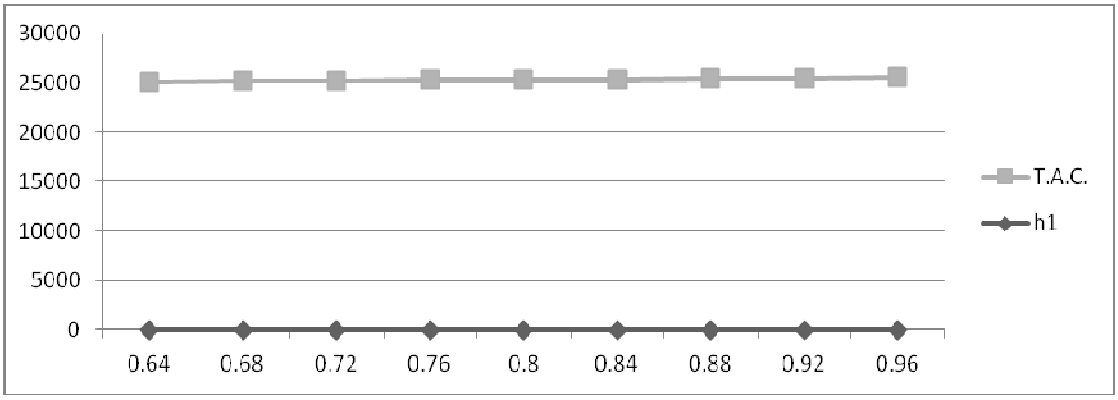

Fig. 7: T.A.C. v/s h_1

Table 5: Sensitivity Analysis with respect to holding cost (h_2):

% variation in h_2	**h_2**	**v**	**T_1**	**T.A.C.**
-20%	0.56	8.11772	2.79674	21873.1
-15%	0.595	8.27259	2.79674	22747.5
-10%	0.63	8.41593	2.79674	23618.1
-5%	0.665	8.54897	2.79674	24485.15
0%	0.7	8.67279	2.79674	25350
5%	0.735	8.78831	2.79674	26211.7
10%	0.77	8.89635	2.79674	27071.1
15%	0.805	8.9976	2.79674	27928.4
20%	0.84	9.09268	2.79674	28889

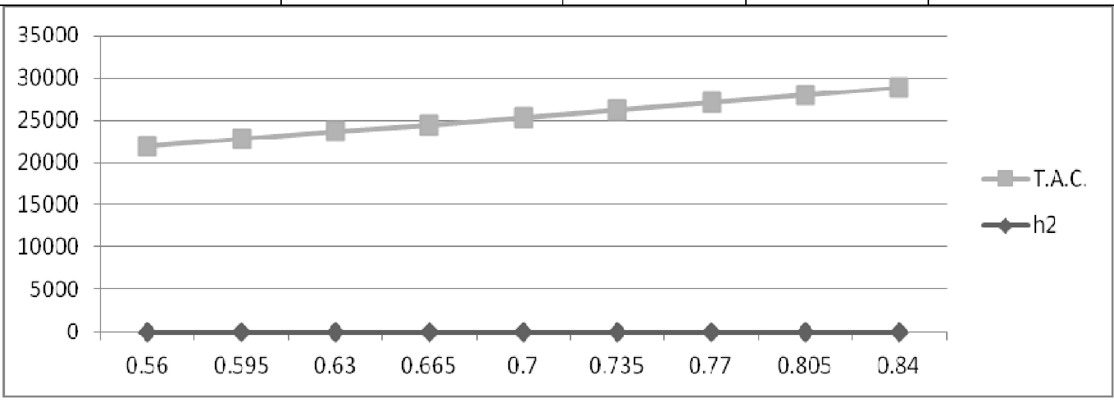

Fig. 8: T.A.C. v/s h_2

Table 6: Sensitivity Analysis with respect to m:

% variation in m	**m**	**v**	**T_1**	**T.A.C.**
-20%	4	8.867279	3.62081	25175.2
-15%	4.25	8.867279	3.40628	25222.7
-10%	4.5	8.867279	3.19761	25267.6
-5%	4.75	8.867279	2.99452	25310
0%	5	8.867279	2.79674	25350
5%	5.25	8.867279	2.60401	25387.4
10%	5.5	8.867279	2.41607	25422.4
15%	5.75	8.867279	2.2327	25455
20%	6	8.867279	2.05369	25485.2

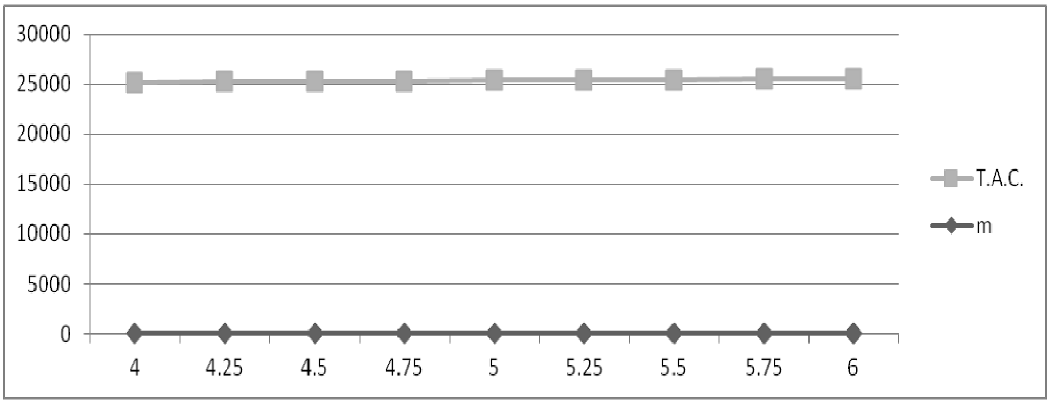

Fig. 9: T.A.C. v/s m

Table 7: Sensitivity Analysis with respect to p:

% variation in p	**p**	**v**	**T1**	**T.A.C.**
-20%	9.6	9.50203	2.79674	24110.5
-15%	10.2	9.29165	2.79674	24432.2
-10%	10.8	9.08334	2.79674	24746
-5%	11.4	8.87706	2.79674	25051.8
0%	12	8.67279	2.79674	25350
5%	12.6	8.47049	2.79674	25640.4
10%	13.2	8.27015	2.79674	25923.3
15%	13.8	8.07172	2.79674	26198.8
20%	14.4	7.87519	2.79674	26467

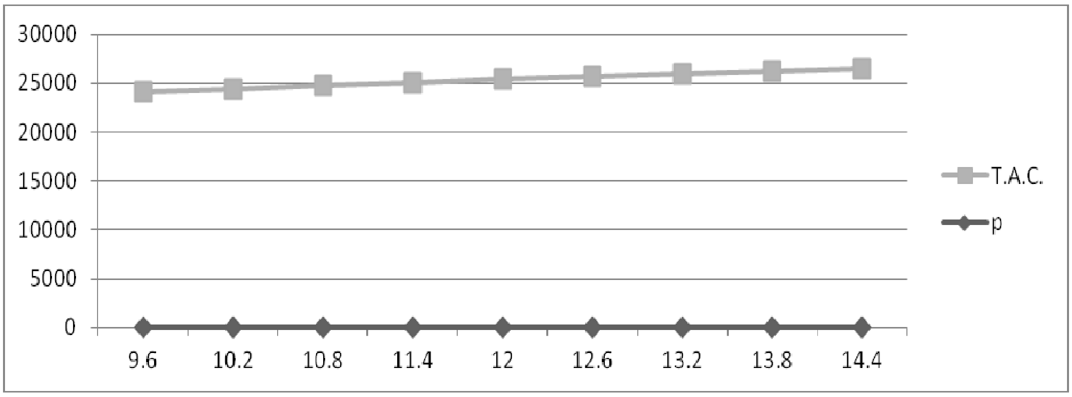

Fig. 10: T.A.C. v/s p

Observations:

1. Table 1 shows the behavior of T.A.C. with the variation in demand parameter α. From this table it is observed that as the value of demand parameter α increases, the T.A.C. of the system also increases.
2. From table 2 we observe the behavior of T.A.C. with the variation in deterioration rate θ, and it is observed that with the increment in deterioration rate θ, the T.A.C. of the supply chain increases.
3. Table 3 lists the variation in demand parameter (β) and it is observed that an increment in β results an increment in T.A.C.
4. Table 4 and table 5 show the variation in holding cost h_1 and h_2 and it is observed that the increment in both h_1 and h_2, increase the T.A.C. of the supply chain.
5. Table 6 shows the production cost m at different points and other variable unchanged. From this table it is observed that with the increment in m, T.A.C. of the system increases.
6. Table 7 lists the variation in purchasing cost (p) and it is observed that T.A.C. of the supply chain increases with the increment in purchasing cost (p).

Conclusion:

This is a two echelon supply chain model for deteriorating items for vendor and buyer. The demand rate considered in this model is stock dependent. The production rate is taken as a function of demand rate. During stock out the occurring shortages are partially backlogged. These all factors together make the study close to reality. This model and demand pattern is applicable for seasonal products, fashion products and bakery products. The numerical example and sensitivity analysis is presented to illustrate this model and its significant features. This model has a further scope of extension with permissible delay period.

References:

1. B.N. Mandal and S. Phaujdar (1989), "An Inventory Model for Deteriorating Items and Stock Dependent Consumption Rate", *Journal of Operational Research Society, 40, 483–488.*
2. T.K. Datta and A.K. Pal (1990), "A Note on an Inventory Model with Inventory Level Dependent Demand Rate", *Journal of the Operational Research Society, 41, 971-975.*

3. S. Pal, A. Goswami, and K.S. Chaudhuri (1993), "A Deterministic Inventory Model for Deteriorating Items with Stock-Dependent Rate", *International Journal of Production Economics 32: 291-299*

4. B.C. Giri, and K.S. Choudhury (1998), "Deterministic Models of Perishable Inventory with Stock Dpendent Demand Rate and Nonlinear Holding Cost", *European Journal of Opera-tional Research 105: 467- 474*

5. K.A.H. Kobbacy and Y. Liang (1999), "Towards the development of an intelligent inventory management system", *Integrated Manufacturing Systems 10(6): 354- 366*

6. S.S. Mishra, and R.S. Singh, (2000), "Inventory Models for Damageable Multi Items with Stock-Dependent Demand And Consumption Cost", *OPSEARCH 37: 82-87*

7. G.S.N., Reddy, and K.V.S., Sarma, (2001), "A Periodic Review Inventory Problem with Variable Stock Dependent Demand", *OPSEARCH, 38, 332-341*

8. S. Sana, and K.S. Chaudhuri, (2003), "On a Volume Flexible Stock-Dependent Inventory Model", *Advanced Modeling and Optimization 5(3): 197-210*

9. L.Y. Ouyang, K.S. Wu, and M.C. Cheng (2005), "An Inventory Model for Deteriorating Items with Exponential Declining Demand and Partial Backlogging", *Yugoslav Journal of Operations Research 15(2): 277-288*

10. O. Berman, and D. Perry (2006), "An EOQ Model with State-Dependent Demand Rate", *European Journal of Operation Research, 255-272*

11. M. Jain, G.C. Sharma, and S. Rathore (2007), "Economic Production Quantity Models with Shortage, Price and Stock-Dependent Demand for Deteriorating Items", *IJE Transactions A, Basics 20(2):159-168*

12. F. Zhou and Y. Shi (2008), "An EOQ model for deteriorating items with stock-dependent Demand and exponential partial backlogging", *Applied Mathematical Modelling, (Elsevier) 6898:1-13*

13. S. Ghosh and T. Chakrabarty (2009), "An Order-Level Inventory Model under Two Level Storage System with Time Dependent Demand", *OPSEARCH, 46(3), 335– 344*

14. S. Panda, S. Saha and M. Basu (2009), "An EOQ model for perishable products with discounted selling price and stock dependent demand" *CEJOR, 17, 31–53*

15. T.J. Singh, S. R. Singh and R. Dutt (2009), "An EOQ Model for Perishable Items with Power Demand and Partial Backlogging", *IJOQM,15(1), 65-72*
16. K. Skouri and I. Konstantaras (2009), "Order Level Inventory Models for Deteriorating Seasonable/Fashionable Products with Time Dependent Demand and Shortages", *Mathematical Problems in Engineering 679736,1- 24*
17. A. P. Tyagi, R. K. Pandey and S. R. Singh (2014), "An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and variable holding cost", *International Journal of Operational Research*, 21 (4), 466-488.
18. C. Singh and S. R. Singh (2012), "An EPQ model with power form stock dependent demand under inflationary environment using genetic algorithm," *International Journal of Computer Applications*, 48 (15), 25-30.
19. C. T. Chang, Y. J. Chen, T. R. Tsai and S. J. Wu (2010), "Inventory models with stock and price dependent demand rate for deteriorating items based on limited shelf space," *Yugoslav Journal of Operation Research*, 21(1), 55-69.
20. G. Padmanabhan and P. Vrat (1988), "Inventory models for perishable items under stock dependent consumption rate," *Paper presented at the XXIst Annual ORSI Convention, Trivandrum, India.*
21. H. M. Wee and S. T. Law (2001), "Replenishment and pricing policy for deteriorating items taking into account the time value of money," *International Journal of Production Economics*, 71, 213–220.
22. J. A. Buzacott (1975), "Economic order quantities with inflation," *Operational Research Quarterly*, 26, 553–558.
23. J. Ray and K. S. Chaudhuri (1997), "An EOQ model with stock-dependent demand, shortage, inflation and time discounting," *International Journal of Production Economics*, 53, 171–180.
24. K. L. Hou (2006), "An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting," *European Journal of Operational Research*, 168, 463–474.
25. N. Kumar and S. R. Singh (2009), "Two warehouse inventory model with stock-dependent demand for deteriorating items with shortages," *The IUP Journal of Computational Mathematics*, 2 (3), 7-23.
26. R. Gupta and P. Vrat (1986), "Inventory model for stock-dependent consumption rate," *Opsearch*, 23, 19–24.
27. S. R. Singh, V. Gupta and P. Bansal (2013), "EOQ model with volume agility, variable demand rate, Weibull deterioration rate and inflation," *International Journal of Computer Applications*, 72 (23), 1-6.
28. S. Tayal, S. R. Singh and R. Sharma (2014), "A multi item inventory model for deteriorating items with expiration date and allowable shortages," *Indian Journal of Science and Technology*, 7(4), 463-471.
29. S. Tayal, S. R. Singh and R. Sharma (2014), "An inventory model for deteriorating items with seasonal products and an option of an alternative market," *Uncertain Supply Chain Management*, 3, 69–86.
30. S. Tayal, S. R. Singh and R. Sharma (2015), "An integrated production inventory model for perishable products with trade credit period and investment in

preservation technology," *Int. J. Mathematics in Operational Research*, Article ID 76334.

31. S. Tayal, S. R. Singh, A. Chauhan and R. Sharma (2014), "A deteriorating production inventory problem with space restriction," *Journal of Information & Optimization Sciences*, 35 (3), 203–229.
32. S. Tayal, S. R. Singh, R. Sharma and A. Chauhan (2014), "Two echelon supply chain model for deteriorating items with effective investment in preservation technology," *International Journal Mathematics in Operational Research*, 6 (1), 78-99.
33. S. Tayal, S. R. Singh, R. Sharma and A. P. Singh (2015), "An EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate," *Int. J. Operational Research*, Vol. 23, No. 2, pp. 145-161.
34. T. K. Datta and A. K. Pal (1991), "Effects on inflation and time value of money on an inventory model with linear time dependent demand rate and shortages," *European Journal of Operational Research*, 52, 326–333.