An Inventory Model for Decaying Items with Multivariate Demand in Inflationary Environment

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Abstract

In this paper a deterministic inventory model is developed for deteriorating items by assuming that the demand rate is taken as multivariate under inflationary conditions. In addition we allowed for shortages and partially backlogging with time dependent. An optimization framework is presented to derive optimal replenishment policy when the present value of profit is maximized .A theory is developed to obtain the optimal solution of the problem, it is then illustrated with the aid of numerical example .In addition ,the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out.

1 Introduction

As inventory represents a very important part of the company's financial assets, it is very much affected by the market's response to various situations, especially inflation. Inflation is a global phenomenon in present day times. Inflation can be defined as that state of disequilibrium in which an expansion of purchasing power tends to cause or is the effect of an increase in the price level. A period of prolonged, persistent and continuous inflation results in the economic, political, social and moral disruption of society.

Inflation is also viewed as a hidden risk pressure that provides an incentive for those with savings to invest them, rather than have the purchasing power of those savings erode through inflation. In investing inflation risks often cause investors to take on more systematic risk, in order to gain returns that will stay ahead of expected inflation. Inflation is also used as an index for cost of living adjustments and as a peg for some bonds. In effect, inflation is the rate at which previous economic transactions are discounted economically.

Almost everyone thinks inflation is evil, but it isn't necessarily so. Inflation affects different people in different ways. It also depends on whether inflation is anticipated or unanticipated. If the inflation rate corresponds to what the majority of people are expecting (anticipated inflation), then we can compensate, and the cost isn't high.

Buzacott (1975) developed the first *EOQ* model taking inflationary effects into account. In this model, a uniform inflation was assumed for all the associated costs and minimizing the average annual cost derived an expression for the *EOQ*. **Sarker and Pan (1994)** assumed a finite replenishment model and analyzed the effects of inflation and time-value of money on order quantity when shortages are allowed.

Bose *et al.* (1995) developed an economic order quantity inventory model for deteriorating items. Authors developed inventory model with linear trend in demand allowing inventory shortages and backlogging. The effects of inflation and time-value of money were incorporated into the model. It was assumed that the goods in the inventory deteriorate over time at a constant rate. **Ray and Chaudhuri (1997)** developed a finite time-horizon deterministic economic order quantity inventory model with shortages, where the demand rate at any instant depends on the on-hand inventory at that instant. The effects of inflation and time value of money were taken into account. **Liao *et al.* (2000)** developed an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible. In this inventory model, shortages are not allowed. The effect of the inflation rate, deterioration rate, initial-stock-dependent consumption rate and delay in payment are discussed. The discounted cash flow approach to investigate inventory replenishment problem for deteriorating items taking account of time value of money over a fixed planning horizon was proposed by **Chung and Lin (2001)**. **Wee and law (2001)** considered a deteriorating inventory model taking into account the time-value of money for a deterministic inventory system with price-dependent demand. In this study, they apply the discounted cash flows (DCF) approach for problem analysis. A heuristic approach is presented to derive the near optimal replenishment and pricing policy that tries to maximize the total net present-value profit. **Moon *et al* (2005)** developed models for ameliorating/ deteriorating items with time varying demand pattern over a finite planning horizon, taking into account the effects of inflation and time value of money. An inventory model for deteriorating items with stock-dependent consumption rate with shortages was produced by **Hou (2006)**. **Jaggi *et al.* (2007)** presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon. Shortages in inventory were allowed and completely backlogged and demand rate was assumed to be a function of inflation. **Dey *et al.* (2008)** discussed a finite time horizon inventory problem for a deteriorating item having two separate warehouses, one is a own warehouse (OW) of finite dimension and other a rented warehouse (RW), is developed with interval-valued lead-time under inflation and time value of money. **Yang *et al.* (2010)** considered a partial backlogging inventory lot-size model for deteriorating items with stock-dependent demand. They have shown that not only the optimal replenishment schedule exists uniquely, but also the total profit associated with the inventory system is a concave function of the number of replenishments. They also have simplified the search process by establishing an intuitively good starting value for the optimal number of replenishments. **Chang, H.J. and Lin, W.F. (2010)** developed a partial backlogging inventory model for non-instantaneous deteriorating items with stock-dependent consumption rate under inflation.

A deterministic inventory model is developed for deteriorating items by assuming that the demand rate is taken as multivariate under inflationary conditions. In addition we allow for shortages and partially backlogged. The backlogging rate is considered to be time dependent. An optimization framework is presented to derive optimal replenishment policy when the present value of profit is maximized. A theory is developed to obtain the optimal solution of the problem; it is then illustrated with the aid of numerical example. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried ou

2 Assumptions and Notations

To develop the mathematical model, the following notations and assumptions are given below.

Assumptions:

1. The demand rate D(t) at time t is assumed to be multivariate demand and is known as the function of time, selling price and stock.
   i.e. $D(t) = \alpha + \gamma t + \eta s$, where α, γ and δ are positive constants, $0< \alpha <1$, $0<\gamma<1$, $0<\eta<1$ and I(t) is the inventory level at time t.
2. A single item is considered with a constant rate of deterioration over a known and finite planning horizon of length *H*.
3. The replenishment rate is infinite and lead time is zero.
4. Product transactions are followed by instantaneous cash flow.
5. Shortages are allowed and partially backlogged. The backlogged rate is to be taken as $\frac{1}{1+\delta(T-t)}$ when inventory is negative, where backlogging parameter 'δ' is a positive constant.

Notations:

(1) H: planning horizon

(2) T: replenishment cycle

(3) N: number of replenishment during the planning horizon (a decision variable);
N = H/T.

(4) T_j: the j-th replenishment time; $T_j = T_{j-1}+T$ (j = 1,2,…,N)
where $T_N = H$ and $T_0 = 0$.

(5) t_1 :time with positive inventory (a decision variable)

(6) $T - t_1$: time when shortage occurs

(7) $I_1(t)$: the level of positive inventory at time t, where $0 \le t \le t_1$

(8) $I_2(t)$: the level of negative inventory at any time t, where $t_1 \le t \le T$

(9) Q: the 2nd, 3rd, …, Nth replenishment lot size (units)

(10) I_m: maximum inventory level per cycle

(11) q : deterioration rate fraction of the on-hand inventory

(12) r: discount rate, representing the time value of money

(13) i: inflation rate

(14) R: the net discount rate of inflation; R = r – i

(15) A: the replenishment cost per order, $/order

(16) c: the purchasing cost per unit, $/unit

(17) s: the selling price per unit, where s > c , $/unit

(18) c_1 : holding cost per unit per unit time.

(19) c_2: the shortage cost per unit per unit time, \$/unit/unit time

(20) c_3: the cost of lost sales (i.e., goodwill cost) per unit, \$/unit

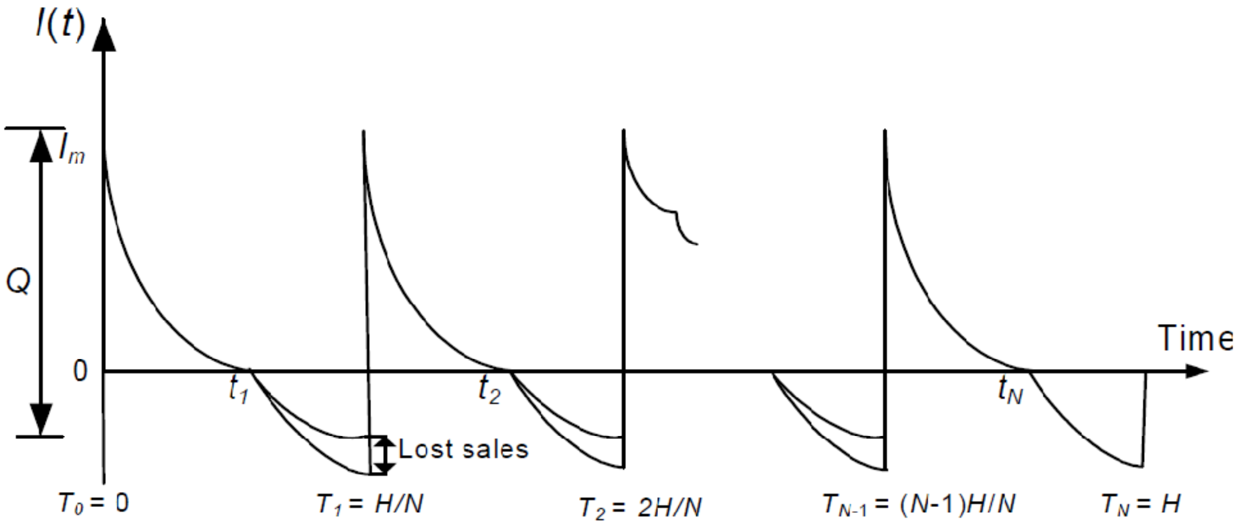

Figure 1. The Graphical Representation for the Inventory System

3 Mathematical Model

To establish the total present value function of profits over a finite planning horizon H, we suppose that the planning horizon H is divided into N equal parts of length T =H/N. Hence, the reorder times over the planning horizon H will be $T_j = jT$ (j = 0,1,2,…,N). The first replenishment lot size of Im is replenished at $T_0 = 0$. During the time interval $(T_j, t_{j+1}]$ (j = 0,1,2,…,N-1), the inventory is depleted due to demand and deterioration until it is zero at $t = t_j$ (j =1,2,…,N). During the time interval $[t_j, T_j)$ (j =1,2,…,N), the inventory level only depends on demand, and unsatisfied demand is backlogged at a rate $\frac{1}{1+\delta(T-t)}$, where $t \in [t_j, T_j]$ (j =1,2,…,N). A realization of the inventory level in the system is given in Figure 1. Hence, the inventory level during the first replenishment cycle can be represented by the following differential equations:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -[\alpha + \gamma t + \eta s] \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt} = -\frac{[\alpha + \gamma t + \eta s]}{1+\delta(T-t)} \qquad t_1 \le t \le T \qquad (2)$$

with the boundary conditions $I_1(t_1) = 0$ and $I_2(t_1) = 0$, the solution of (1) and (2) can be represented by:

$$I_1(t)=\frac{1}{\theta}\left[\left\{\alpha+\eta s-\frac{\gamma}{\beta+\theta}\right\}\left\{e^{(\theta)(t_1-t)}-1\right\}+\gamma\left\{t_1e^{(\theta)(t_1-t)}-t\right\}\right]\quad 0\le t\le t_1 \tag{3}$$

$$I_2(t)=\frac{\gamma}{\delta}(t-t_1)+\frac{1}{\delta}\left\{\alpha+\eta s+\frac{\gamma}{\delta}(1+\delta T)\right\}\{\log(1+\delta(T-t))-\log(1+\delta(T-t_1))\}\ t_1\le t\le T \tag{4}$$

Hence, the maximum inventory level during the first replenishment cycle is:

$$I_m=I_1(0)=\frac{1}{\theta}\left[\left\{\alpha+\eta s-\frac{\gamma}{\beta+\theta}\right\}\left\{e^{(\theta)t_1}-1\right\}+\gamma t_1e^{(\theta)t_1}\right] \tag{5}$$

During the time interval $(T_j, t_{j+1}]$ (j = 0,1,2,…,N-1), the replenished inventory is being consumed due to demand and deterioration. During the time interval $[t_j, T_j)$ (j =1,2,…,N) the inventory level only depends on demand, the unsatisfied demand during time interval $[t_j, T_j)$ will be partially backordered at the rate $\frac{1}{1+\delta(T-t)}$ with respect to the waiting time.

Under instantaneous cash transaction, the present value of sales revenue during the first cycle is:

$$S_a=s\int_0^{t_1}[\alpha+\gamma t+\eta s]e^{-Rt}dt+se^{-RT}\int_{t_1}^{T}\frac{[\alpha+\gamma t+\eta s]}{1+\delta(T-t)},$$

$$S_a=s\left[\left(\frac{\alpha+\eta s}{R}\right)\left(1-e^{-Rt_1}\right)-\frac{\gamma}{R}t_1e^{-Rt_1}-\frac{\gamma}{R^2}\left(e^{-Rt_1}-1\right)+\right.$$

$$\left.+\frac{se^{-RT}}{\delta}\left[\left\{\alpha+\eta s+\frac{\gamma}{\delta}(1+\delta T)\right\}\log(1+\delta(T-t_1))-\gamma(T-t_1)\right]\right. \tag{6}$$

Since replenishment in each cycle is done at the start of each cycle, the present value of ordering cost during the first cycle is:

$$C_r=A \tag{7}$$

Inventory occurs during time interval $[T_j, t_{j+1})$ (j = 0, 1, 2,…,N-1), therefore, the present value of holding cost during the first replenishment cycle is:

$$C_h=\int_0^{t_1}c_1I_1(t)\,e^{-Rt}dt$$

$$C_h=\frac{c_1}{\theta}\left[\left\{\alpha+\eta s-\frac{\gamma}{\theta}\right\}\left\{\frac{\left(e^{(\theta)t_1}-e^{-Rt_1}\right)}{(\theta+R)}+\frac{\left(e^{-Rt_1}-1\right)}{R}\right\}+\gamma\left\{\frac{t_1\left(e^{(\theta)t_1}-e^{-Rt_1}\right)}{(\theta+R)}+t_1\frac{e^{-Rt_1}}{R}+\frac{\left(e^{-Rt_1}-1\right)}{R^2}\right\}\right] \tag{8}$$

Note that the unsatisfied demand during time interval [*tj*,*Tj*) will be partially backordered and t ε[t_j, T_j) (j = 1,2,…,N). So, the present value of shortage cost during the first cycle is:

$$C_s = c_2 \int_{t_1}^{T} (-\mathrm{I}_2(\mathrm{t}))\, e^{-Rt} dt$$

$$C_s = c_2 \left[\frac{1}{\delta}\left\{\alpha + \eta \mathrm{s} + \frac{\gamma}{\delta}(1+\delta T)\right\}\left\{\left(\log(1+\delta(T-t_1)) - \delta T + \frac{\delta}{R}\right)\frac{\left(e^{-Rt_1} - e^{-RT}\right)}{R} + \delta \frac{\left(t_1 e^{-Rt_1} - Te^{-RT}\right)}{R}\right\} - \frac{\gamma}{\delta}\left\{\frac{\left(t_1 e^{-Rt_1} - Te^{-RT}\right)}{R} + \left(t_1 - \frac{1}{R}\right)\frac{\left(e^{-RT} - e^{-Rt_1}\right)}{R}\right\}\right] \quad (9)$$

And the opportunity cost due to lost sales during the first cycle is:

$$C_o = c_3 \int_{t_1}^{T} [\alpha + \gamma \mathrm{t} + \eta \mathrm{s}]\left[1 - \frac{1}{1+\delta(T-t)}\right] e^{-Rt} dt$$

$$C_o = c_3 \left[(\alpha + \eta \mathrm{s})\left\{\frac{\left(e^{-Rt_1} - e^{-RT}\right)}{R} - \frac{1}{\delta}\log(1+\delta(T-t_1))\right\} + \gamma\left\{\frac{\left(t_1 e^{-Rt_1} - Te^{-RT}\right)}{R} - \frac{\left(e^{-RT} - e^{-Rt_1}\right)}{R^2} + \frac{R}{\delta}\left(\frac{T^2 - t_1^2}{2}\right)\right\} + \frac{1}{\delta}\{(\alpha + \eta \mathrm{s})R + \gamma\{1 + R(1+\delta T)\}\}\left\{T - t_1 - \frac{(1+\delta T)}{\delta}\log(1+\delta(T-t_1))\right\}\right] \quad (10)$$

Replenishment is done at $t = T_j$ (j = 0,1,2,…,N), the replenishment items are consumed by demand as well as deterioration during time interval [t_j, T_j) (j = 1,2,…,N). So, the present value of purchase cost during the first cycle is:

$$C_p = c\,\mathrm{I_m} + ce^{-RT}\int_{t_1}^{T}\left[\frac{\alpha + \gamma \mathrm{t} + \eta \mathrm{s}}{1+\delta(T-t)}\right] dt$$

$$C_p = \frac{c}{\theta}\left[\left\{\alpha + \eta \mathrm{s} - \frac{\gamma}{\theta}\right\}\left\{e^{\theta t_1} - 1\right\} + \gamma t_1 e^{\theta t_1}\right] + \frac{ce^{-RT}}{\delta}\left[\left\{\alpha + \eta \mathrm{s} + \frac{\gamma}{\delta}(1+\delta T)\right\}\log(1+\delta(T-t_1)) - \gamma(T-t_1)\right] \quad (11)$$

Consequently, the total present value of profits during the first replenishment cycle can be formulated as:

$$\pi = S_a - C_r - C_h - C_s - C_o - C_p \quad (12)$$

There are N cycles during the planning horizon. Since inventory is assumed to start and end at zero, an extra replenishment at t = H is required to meet the unsatisfied demand of the last cycle in the planning horizon. Therefore, there are N+1 replenishments in the entire time horizon H, the first replenishment lot size is I_m, and the 2nd, 3rd,…, Nth replenishment lot size is:

$$Q = I_m + \int_{t_1}^{T} \left[\frac{\alpha + \gamma t + \eta s}{1+\delta(T-t)} \right] dt \tag{13}$$

And the last or (N+1)th replenishment cost size is:

$$I_b = \frac{1}{\delta}\left[\left\{\alpha+\eta s+\frac{\gamma}{\delta}(1+\delta T)\right\}\log(1+\delta(T-t_1))-\gamma(T-t_1)\right] \tag{14}$$

So, the total present value of profits over a finite planning time horizon H is:

$$P(t_1)=\sum_{j=0}^{N-1}\pi e^{-RjT}-Ae^{-RH} \qquad =\pi\left(\frac{1-e^{-RNT}}{1-e^{-RT}}\right)-Ae^{-RH}$$

$$=\left\{s\left[\left(\frac{\alpha+\eta s}{R}\right)\left(1-e^{-Rt_1}\right)-\frac{\gamma}{R}t_1e^{-Rt_1}-\frac{\gamma}{R^2}\left(e^{-Rt_1}-1\right)\right]+\frac{se^{-RT}}{\delta}\left[\left\{\alpha+\eta s+\frac{\gamma}{\delta}(1+\delta T)\right\}\log(1+\delta(T-t_1))-\gamma(T-t_1)\right.\right.$$

$$-A-\frac{c_1}{\theta}\left[\left\{\alpha+\eta s-\frac{\gamma}{\theta}\right\}\left\{\frac{\left(e^{(\theta)t_1}-e^{-Rt_1}\right)}{(\theta+R)}+\frac{\left(e^{-Rt_1}-1\right)}{R}\right\}+\gamma\left\{\frac{t_1\left(e^{(\theta)t_1}-e^{-Rt_1}\right)}{(\theta+R)}+t_1\frac{e^{-Rt_1}}{R}+\frac{\left(e^{-Rt_1}-1\right)}{R^2}\right\}\right]$$

$$-c_2\left[\frac{1}{\delta}\left\{\alpha+\eta s+\frac{\gamma}{\delta}(1+\delta T)\right\}\left\{\left(\log(1+\delta(T-t_1))-\delta T+\frac{\delta}{R}\right)\right.\right.$$

$$\frac{\left(e^{-Rt_1}-e^{-RT}\right)}{R}+\delta\frac{\left(t_1e^{-Rt_1}-Te^{-RT}\right)}{R}\Bigg\}-\frac{\gamma}{\delta}\left\{\frac{\left(t_1e^{-Rt_1}-Te^{-RT}\right)}{R}+\left(t_1-\frac{1}{R}\right)\frac{\left(e^{-RT}-e^{-Rt_1}\right)}{R}\right\}\Bigg]-c_3\left[(\alpha+\eta s)\left\{\frac{\left(e^{-Rt_1}-e^{-RT}\right)}{R}\right.\right.$$

$$-\frac{1}{\delta}\log(1+\delta(T-t_1))\Bigg\}+\gamma\left\{\frac{\left(t_1e^{-Rt_1}-Te^{-RT}\right)}{R}-\frac{\left(e^{-RT}-e^{-Rt_1}\right)}{R^2}+\frac{R}{\delta}\left(\frac{T^2-t_1^2}{2}\right)\right\}+\frac{1}{\delta}\{(\alpha+\eta s)R+\gamma\{1+R(1+\delta T)\}\}$$

$$\left\{T-t_1-\frac{(1+\delta T)}{\delta}\log(1+\delta(T-t_1))\right\}\Bigg]-\frac{c}{\theta}\left[\left\{\alpha+\eta s-\frac{\gamma}{\theta}\right\}\left\{e^{(\theta)t_1}-1\right\}+\gamma t_1e^{(\theta)t_1}\right]-\frac{ce^{-RT}}{\delta}$$

$$\left[\left\{\alpha+\eta s+\frac{\gamma}{\delta}(1+\delta T)\right\}\log(1+\delta(T-t_1))-\gamma(T-t_1)\right]\Bigg\}\left(\frac{1-e^{-RH}}{1-e^{-R\frac{H}{N}}}\right)-Ae^{-RH} \tag{15}$$

This is our objective function which needs to be maximized.

4 Solution Procedure

To maximize total average profit per unit time P (t_1), the optimal values of t_1 can be obtained by solving the following equations simultaneously

$$\frac{dP(t_1)}{dt_1}=0 \tag{16}$$

provided, they satisfy the following conditions

$$\frac{d^2P(t_1)}{dt_1^{\,2}}>0, \tag{17}$$

Equations (16) are highly non-linear and hence are solved with the help of mathematical software ***MATHEMATICA 5.2***. With the use of these optimal values equation (15) provides maximum total average profit per unit time of the system in consideration.

We can find the maximum inventory level to be:

$$\mathrm{I_m} = \frac{1}{\theta}\left[\left\{\alpha+\eta s - \frac{\gamma}{\theta}\right\}\left\{e^{(\theta)t_1^*}-1\right\}+\gamma t_1 e^{(\theta)t_1^*}\right],$$

Subsititute t_1* into (13) to derive the 2nd, 3rd,……Nth replenishment lot size, we have:

$$Q^* = \mathrm{I_m} + \frac{1}{\delta}\left[\left\{\alpha+\eta s+\frac{\gamma}{\delta}\left(1+\delta\frac{H}{N}\right)\right\}\log\left(1+\delta\left(\frac{H}{N}-t_1^*\right)\right)-\gamma\left(\frac{H}{N}-t_1^*\right)\right]$$

5 Numerical Example and Sensitivity Analysis

A numerical example is used to illustrate the result. The necessary parameters and the optimal solutions are presented respectively.

S=16, □□650, □□0.04, R=0.3, □=0.4, T=0.54, H=10, N=19, c=4, c_1=1.5, c_2=2, c_3=4, □=0.5, □=0.4, A=200, h=0.3,□□=0.4

Optimum time of t_1 and Total Profit:

t_1 = 0.1059, Total optimal profit (P) = 2715.03

6 Sensitivity Analysis:

We now study the effects of changes in the values of the system parameters □**,** □**,** □**,** □□□**R** on the different costs, Ordering quantity and optimal total profit. The sensitivity analysis is performed by changing each of the parameters by −15%, −10%, -5%, 5%, 10% and 15%, taking one parameter at a time and keeping the remaining parameters unchanged.

Table: 1 Effects of Parameters

Parameter ‘□’	**Percentage Variation in t_1 and Total Profit**					
	-15%	**-10%**	**-5%**	**5%**	**10%**	**15%**
t_1	0.1051	0.1055	0.1056	0.1060	0.1062	0.1069
Profit	2308.52	2412.46	2564.64	2913.32	3015.24	3123.23
Parameter ‘□’	**Percentage Variation in t_1 and Total Profit**					
	-15%	**-10%**	**-5%**	**5%**	**10%**	**15%**
t_1	0.1015	0.1027	0.1043	0.1062	0.1068	0.1083
Profit	2641.07	2728.2	2872.61	2960.07	3003.99	3092.16

Parameter '□'	Percentage Variation in t_1 and Total Profit					
	-15%	**-10%**	**-5%**	**5%**	**10%**	**15%**
t_1	0.1180	0.1116	0.1085	0.1035	0.1027	0.0974
Profit	3248.93	3066.24	2987.69	2850.3	2790.19	2682.99

Parameter '□'	Percentage Variation in t_1 and Total Profit					
	-15%	**-10%**	**-5%**	**5%**	**10%**	**15%**
t_1	0.0838	0.0917	0.0995	0.1037	0.1075	0.1147
Profit	2436.68	2578.94	2615.12	2789.53	2863.31	3017.22

Parameter R	Percentage Variation in t_1 and Total Profit					
	-15%	**-10%**	**-5%**	**5%**	**10%**	**15%**
t_1	0.1002	0.1034	0.1055	0.1063	0.1067	0.1077
Profit	2356.55	2569.29	2603.61	2827.87	2877.64	2954.53

Observations:

- It has been observed that if we increase the value of demand parameter '□' then total average profit of the system increases.
- It has been observed that if we decrease the value of backlogging parameter 'δ' then the value of total profit increases.
- The total average profit of the system increase with the increment in deterioration rate 'θ'.
- When the discount rate of inflation R is increasing the total present value of profit P will increase.
- When the time dependent rate □□is increasing the total average profit will increase i.e. the change in □ will lead to the positive change in P.

7 Conclusion

This chapter deals an inventory model for deteriorating items with multi-variate demand. In our study, we have taken a more realistic demand rate that depends on two factors, one is the stock level available, and the second is the selling price of the item. As the selling price increases, the demand suffers a setback, and the sales are reduced. This whole setup is very close to the reality that is observed in the market. The environment of the whole study has been taken as inflationary, as any study done otherwise cannot justify itself under any circumstances. Inflation in itself has become a very vital aspect of every economy worldwide. Partially backlogged shortages are permitted in this model. Backlogging rate is taken as waiting time for the next replenishment. This whole setup is very practical and can be applied to many commodities in today's market. The model conforms to present day economic condition worldwide and is beneficial for obtaining optimal industrial output.

Further cases for stochastic demand and in more realistic conditions can be developed.

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