# Two-Warehouse Inventory Model with Time Dependent Demand and Partial Backlogging

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## Abstract

In this article, a two ware-house inventory model developed with time-dependent demand rate. Shortages are allowed and are partially backlogged. Let the considering that the stock is transferred from the RW to the OW in a permanent release pattern. Furthermore, holding cost in the RW is considered to be higher than that of the OW. In addition, it is seen that some but not all the customers will wait for backlogged items during a shortage period. The model is solved analytically by minimizing the total inventory cost. A numerical example is taken to discuss the sensitivity analysis has also been performed.

**KEYWORDS:** Two ware-house, Inventory model, Time-dependent demand, Partially backlogged, Shortages.

## 1. Introduction

It is generally seen that enterprises purchase more goods than can be held in their owned warehouses (OW) for many reasons, such as discounts on bulk purchases, etc. The excess units are stored in an additional storage space. This additional storage space may be a rented warehouse (RW). The holding cost in the RW is generally assumed to be higher than that in the OW. To reduce the inventory costs, it is imperative to consume the goods of the RW at the earliest time. As a result, the stocks of the OW will not be released until the stocks of the RW are exhausted. A two warehouse model was considered by Hartely (1976) under the assumption that the holding cost in the RW is greater than that in the OW, therefore, items in the RW are first transferred to the OW to meet the demand until the stock level in the RW drops to zero and then the items in the OW are released. On the other hand, Sarma (1983) developed a deterministic inventory model with an infinite replenishment rate. Sarma (1987) extended Hartely's model to cover the fixed transportation cost, independently of the quantity transferred from the RW to the OW. But he failed to consider shortages in his model. Goswami and Chaudhuri (1992) further extended the model with or without shortages with the assumption that the demand varies over time with a linearly increasing trend and that the transportation cost from the RW to the OW depends on the quantity being transported. In their model, the stock was transferred from the RW to the OW in an intermittent pattern. However, their work is for non-deteriorating items only. In addition to all of these, Pakkala and Achary (1992) further considered the two-warehouse model with finite replenishment rate and shortages, taking time as a discrete and continuous variable, respectively. Subsequently, many authors such as Lee (2006), Chung and Huang (2007), Rong et al. (2008) and many others have worked in the area of two-warehousing under different scenarios. Kumar et al. (2012) developed an inventory model with time – dependent demand and limited storage facility. Kumar et al. (2013) presented two-warehouse inventory model with K – release rule and learning effect.

In the past, many researchers worked on inventory problems for deteriorating items such as medicines, seasonal products and many others. Ghare and Schrader (1963) first proposed an economic order quantity (EOQ) model for items having a constant rate of time horizon. Their work was extended by Covert and Philip (1973) by introducing a variable rate of deterioration. A further generalization to the above models was proposed by Shah (1977) by considering a model allowing complete backlogging of the unsatisfied demand. Dave and Patel (1981) considered an inventory model for deteriorating items with time proportional demand and shortages. Kang and Kim (1983) provided a study on the price and production level of the inventory system. Sachan (1984) extended Dave and Patel's model to allow shortages. Datta and Pal (1988) developed an EOQ model by introducing a variable deterioration rate and power demand pattern. Aggarwal and Jaggi (1989) considered an ordering policy for decaying inventory. Then Hariga (1995) extended this work to allow shortages. So many researchers, namely, Chakrabarti and Chaudhuri (1997), Papachristos and Skouri (2003) continued their research in the area of inventory management for deteriorating items in various situations.

In supermarkets, it has been observed that the demand rate is usually influenced by the amount of stock level, that is, the demand rate may go up or down with the on-hand stock level. Levin et al. (1972) explained the presence of inventory has a motivational effect on the people around it at times. It is commonly believed that large piles of goods displayed in a supermarket will lead the customers to buy more'. In the past numerous years, many researchers have given substantial attention to the situation where the demand rate is dependent on the level of the on-hand inventory. Gupta and Vrat (1986) were the first to build up models for stock-dependent consumption rate. Mandal and Phaujdar (1989) after that developed an economic production quantity model with constant production rate and linearly stock-dependent demand. Some of the works in this field may relate to Datta et al. (1998), Dye (2002) and so on. First time, Zhou and Yang (2005) studied stock-dependent demand without shortage with two-warehouse. Singh et al. (2010) presented an inventory model with stock-dependent demand under the effect of inflation and two shops of a single management. Singh et al. (2011) developed a deterministic two-warehouse inventory model with stock-dependent demand and shortages. Sarkar and Sarkar (2013) improved an inventory model with partial backlogging and stock-dependent demand. Singh and Pattnayak [2014] and Singh and Pattanayak [2015] made the valuable contribution in this direction. Kumar and Kumar (2016) introduced an inventory model with stock dependent demand rate. Kumar and Kumar (2016) developed an inventory model with stock dependent demand and partial backlogging. Recently Kumar and Kumar (2017) introduced an inventory model with partial backlogging using linear demand in fuzzy environment.

In this chapter, a two-warehouse inventory model developed with a linear trend in demand. Shortages are allowed and are partially backlogged. The scheduling period is taken to be variable and not constant. The results have been validated with the help of some numerical examples and comprehensive sensitivity analysis has also been performed.

## 2. Assumptions and notations

The mathematical models of the two-warehouse inventory problems are based on the following assumptions:

- The demand rate D(t) is given by a + bt, where a, & b are demand positive constants.
- Unsatisfied demand/shortages are allowed. Unsatisfied demand is partially backlogged, and the fraction of shortages backordered is a differentiable and decreasing function of time t, where t is the waiting time up to the next replenishment.
- The OW has a fixed capacity of W units and the RW has unlimited capacity.
- The goods of the RW are consumed only after consuming the goods kept in the OW.
- The inventory holding cost in the RW are higher than those in the OW.
- With the viewpoint of cost-minimization, the opportunity cost due to lost sale is the sum of the revenue loss and the cost of goodwill. Hence, the opportunity cost due to lost sale here is greater than the unit purchase cost.
- Replenishment rate is infinite.
- Lead time is zero.

**Notations:**

D(t) demand rate which is a linear function of time t (a+bt, a,& b > 0)
Q the replenishment quantity per replenishment
W the capacity of the owned warehouse (OW)
Z the initial inventory for the period
A replenishment cost per order
c purchasing cost per unit
s the shortage cost per unit time
L the unit opportunity cost due to lost sale, if the shortage is lost
H the holding cost per unit per unit time in the OW
F the holding cost per unit per unit time in the RW, F>H
TC the present value of the total relevant cost per unit time in a two-warehouse system
$I_o(t)$ the inventory level in the OW at time t
$I_r(t)$ the inventory level in the RW at time t
$e^{-\sigma t}$ the backlogging rate, where σ, the backlogging parameter, is a positive constant and t is the waiting time up to the next replenishment
B(t) the backlogged level at time t
L(t) the number of lost sales at time t
$t_1$ the time at which the inventory level reaches zero in the RW
$t_2$ the time at which the inventory level reaches zero in the OW
T the time at which the shortage level reaches the lowest point in the replenishment cycle

## 3. Formulation of the model

The inventory level at the time t = 0, a lot size of Q units enters the system from which a portion is used to meet the partial backlogged items towards previous shortages, and the initial inventory for the period is Z. Out of these Z units, W units are kept in the OW and

the rest (Z-W) units are stored in the RW provided Z-W, otherwise zero units are stored in the RW. The goods of the OW are consumed only after consuming the goods kept in the RW. By time $t_1$, inventory level in the RW reaches to zero due to the combined effect of demand. During ($t_1$, $t_2$), inventory level in the OW reaches to zero due to the combined effect of demand. By the time to, both warehouses are empty and there after shortages are allowed to occur. The partially backlogged quantity is supplied to the customers at the beginning of the next cycle. The inventory situation can be represented as shown in Figure 1. Hence, during the interval (0, $t_1$), the inventory levels at time t in the RW and the OW is governed by the following differential equations:
The inventory levels in RW and OW are given by the following differential equations:

$$I_1'(t) = -(a+bt), \qquad 0 \le t \le t_1 \qquad \ldots(1)$$

With the condition $I_1(t_1)=0$ and

$$I_2'(t) = 0, \qquad 0 \le t \le t_1 \qquad \ldots(2)$$

With the condition $I_2(t)=W$. The solution of equation (1) and (2) are given:

$$\mathrm{I}_1(\mathrm{t}) = \mathrm{b}(\mathrm{t}_1 - \mathrm{t}), \qquad 0 \le t \le t_1 \qquad \ldots(3)$$

$$\mathrm{I}_2(\mathrm{t}) = \mathrm{W}, \qquad 0 \le \mathrm{t} \le \mathrm{t}_1 \qquad \ldots(4)$$

Again, in the interval ($t_1$, $t_2$), the inventory in OW is depicted due to the joint effect of the demand. The inventory level in OW is given by the following differential equation:

$$I_2'(t) = -(a+bt), \qquad t_1 \le t \le t_2 \qquad \ldots(5)$$

With the condition $I_2(t_2)=0$. The solution of equation (5) is similarly given by

$$\mathrm{I}_2(\mathrm{t}) = \mathrm{b}(\mathrm{t}_2 - \mathrm{t}), \qquad t_1 \le t \le t_2 \qquad \ldots(6)$$

From equations (4) and (6) and due to the continuity at $t = t_1$, We get

$$\mathrm{I}_2(\mathrm{t}_1) = \mathrm{W} = \mathrm{b}(\mathrm{t}_2 - \mathrm{t}_1)$$

which implies that

$$\mathrm{W} = \mathrm{b}(\mathrm{t}_2 - \mathrm{t}_1) \qquad \ldots(7)$$

From equation (7) reduces to

$$\mathrm{t}_2 = \mathrm{t}_1 + \frac{\mathrm{W}}{\mathrm{b}} \qquad \ldots(8)$$

Equation (8) shows that $t_2$ is a function of $t_1$, therefore $t_2$ is not a decision variable.

Furthermore, at time $t_2$ the inventory level becomes zero in OW and shortage occurs. In the interval $(t_2, T)$ the inventory level depends on the demand which is partially backlogged. The inventory level is given by the following differential equation:

$$I_3'(t) = e^{-\sigma(T-t)}(a+bt), \qquad t_2 \le t \le T. \qquad \ldots (9)$$

With the condition $I_3(t_2) = 0$. The solution of equation (9) is given by

$$I_3(t) = \frac{a}{\sigma}\left(e^{-\sigma(T-t)} - e^{-\sigma(T-t_2)}\right) + \frac{b}{\sigma}\left(te^{-\sigma(T-t)} - t_2 e^{-\sigma(T-t_2)}\right) - \frac{b}{\sigma^2}\left(e^{-\sigma(T-t)} - e^{-\sigma(T-t_2)}\right),$$
$$t_2 \le t \le T. \qquad \ldots (10)$$

Using the condition $I_1(t) = Z - W$ at $t = 0$ and $I_3(t) = Q - Z$ at $t = T$, we have

$$Z = W + bt_1$$

And

$$Q = Z + \frac{a}{\sigma}\left(1 - e^{-\sigma(T-t_2)}\right) + \frac{b}{\sigma}\left(T - t_2 e^{-\sigma(T-t_2)}\right) - \frac{b}{\sigma^2}\left(1 - e^{-\sigma(T-t_2)}\right)$$

Now the total cost consists of following components such as ordering cost, inventory holding cost in the RW, inventory holding cost in the OW, backlogging cost, lost sales cost.

Ordering cost = A … (13)

Holding cost per cycle in RW

$$HC_{RW} = F\int_0^{t_1} I_1(t)dt = F\int_0^{t_1} b(t_1 - t)dt = \frac{Fb}{2}t_1^2 \qquad \ldots (14)$$

Holding cost per cycle in OW

$$HC_{OW} = H\left[\int_0^{t_1} I_2(t)dt + \int_{t_1}^{t_2} I_2(t)dt\right] = H\left[\int_0^{t_1} Wdt + \int_{t_1}^{t_2} b(t_2 - t)dt\right]$$
$$= H\left[Wt_1 + \frac{b}{2}(t_2 - t_1)^2\right] \qquad \ldots (15)$$

Backlogging cost per cycle

$$SC=-C_s\int_{t_2}^{T} I_3(t)\,dt$$

$$=-C_s\int_{t_2}^{T}\left(\frac{a}{\sigma}\left(e^{-\sigma(T-t)}-e^{-\sigma(T-t_2)}\right)+\frac{b}{\sigma}\left(te^{-\sigma(T-t)}-t_2e^{-\sigma(T-t_2)}\right)-\frac{b}{\sigma^2}\left(e^{-\sigma(T-t)}-e^{-\sigma(T-t_2)}\right)\right)dt$$

$$=\left(-C_s\frac{a}{\sigma}\left(\frac{1}{\sigma}\left(1-e^{-\sigma(T-t_2)}\right)-(T-t_2)e^{-\sigma(T-t_2)}\right)+\frac{b}{\sigma}\left(\frac{T}{\sigma}-\frac{t_2}{\sigma}e^{-\sigma(T-t_2)}-\frac{1}{\sigma^2}+\frac{e^{-\sigma(T-t_2)}}{\sigma^2}-t_2(T-t_2)e^{-\sigma(T-t_2)}\right)\right.$$

$$\left.-\frac{b}{\sigma^2}\left(\frac{1}{\sigma}\left(1-e^{-\sigma(T-t_2)}\right)-(T-t_2)e^{-\sigma(T-t_2)}\right)\right) \quad \ldots(17)$$

Opportunity cost due to lost sales per cycle

$$OC=L\int_{t_2}^{T}\left(1-e^{-\sigma(T-t)}\right)(a+bt)\,dt$$

$$=L\left(a(T-t_2)+\frac{b}{2}\left(T^2-t_2^2\right)-\frac{a}{\sigma}\left(1-e^{-\sigma(T-t_2)}\right)-b\left(\frac{T-e^{-\sigma(T-t_2)}}{\sigma}-\frac{1-e^{-\sigma(T-t_2)}}{\sigma^2}\right)\right)$$

… (18)

Therefore total average cost per unit time is

$$TC(t_1,T)=(1/T)\left[OC+HO_{RW}+HO_{OW}+SC+OP\right]$$

$$TC(t_1,T)=\frac{1}{T}\left\{A+\frac{Fb}{2}t_1^2+H\left[Wt_1+\frac{b}{2}(t_2-t_1)^2\right]-C_s\left(\frac{a}{\sigma}\left(\frac{1}{\sigma}\left(1-e^{-\sigma(T-t_2)}\right)-(T-t_2)e^{-\sigma(T-t_2)}\right)\right.\right.$$

$$\left.+\frac{b}{\sigma}\left(\frac{T}{\sigma}-\frac{t_2}{\sigma}e^{-\sigma(T-t_2)}-\frac{1}{\sigma^2}+\frac{e^{-\sigma(T-t_2)}}{\sigma^2}-t_2(T-t_2)e^{-\sigma(T-t_2)}\right)-\frac{b}{\sigma^2}\left(\frac{1}{\sigma}\left(1-e^{-\sigma(T-t_2)}\right)-(T-t_2)e^{-\sigma(T-t_2)}\right)\right)$$

$$\left.+L\left(a(T-t_2)+\frac{b}{2}\left(T^2-t_2^2\right)-\frac{a}{\sigma}\left(1-e^{-\sigma(T-t_2)}\right)-b\left(\frac{T-e^{-\sigma(T-t_2)}}{\sigma}-\frac{1-e^{-\sigma(T-t_2)}}{\sigma^2}\right)\right)\right\}$$

…(19)

Now, for minimizing the total average cost per unit time, the optimal values of $t_1$ and T (say $t_1^*$ and $T^*$) can be obtained by solving the following equations simultaneously:

$$\frac{\partial TC(t_1,T)}{\partial t_1}=0 \text{ and } \frac{\partial TC(t_1,T)}{\partial T}=0 \qquad \text{---(20)}$$

which also satisfies the conditions

$$\left.\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right|_{(t_1^*,T^*)}>0, \left.\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right|_{(t_1^*,T^*)}>0$$

and $$\left.\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1\partial T}\right)^2\right|_{(t_1^*T^*)}>0$$

## 4. Numerical examples

In this section, to illustrate the preceding theory, let us consider an inventory system with the following data in appropriate units based on the previous research work: a = 200, b = 15, C = 10, H = 1, F = 2.5, A = 85, $A_1$ = 110, s = 5, $C_1$ = 12, σ = 0.90, W = 100. The optimal values are: $t_1$ = 0.20, $t_2$ = 0.59, Z = 129.2, Q = 146.3 and TC = 258.4.

## 5. Sensitivity analysis

The change in the values of parameters may happen due to uncertainties in any decision-making situations. To analyses these changes, sensitivity analysis might be of great help. Using the numerical example presented above, the sensitivity analysis of parameters with respect to the various parameters has been done.

**Table 1:** Sensitivity of the optimal solution with respect to the various parameters of the model

| **Changing parameter** | **% change in parameter value** | **% change in optimal values** | | |
|---|---|---|---|---|
| | | $t_1$ | $t_2$ | **TC** |
| **a** | +50 | 0.31 | -0.03 | 43.56 |
| | +20 | 0.15 | -0.01 | 23.89 |
| | -20 | -0.21 | 0.03 | -21.26 |
| | -50 | -1.01 | 0.07 | -39.46 |
| **b** | +50 | 0.68 | -0.06 | 186.42 |
| | +20 | 0.33 | -0.04 | 53.79 |
| | -20 | -0.52 | 0.06 | -34.56 |
| | -50 | -1.46 | 0.20 | -53.45 |
| **c** | +50 | 0.68 | -0.06 | 189.84 |
| | +20 | 0.35 | -0.04 | 54.36 |
| | -20 | -0.53 | 0.06 | -32.43 |
| | -50 | -1.89 | 0.20 | -56.53 |
| **F** | +50 | -8.81 | -8.42 | 32.49 |
| | +20 | -4.57 | -3.58 | 11.47 |
| | -20 | 4.96 | 4.99 | -14.89 |
| | -50 | 13.47 | 13.68 | -43.46 |

| | | | | |
|---|---|---|---|---|
| **H** | +50 | -0.35 | -0.9 | 0.43 |
| | +20 | -0.05 | -0.04 | 0.19 |
| | -20 | 0.05 | 0.03 | -0.24 |
| | -50 | 0.10 | 0.06 | 0.46 |
| **W** | +50 | -0.47 | 0.08 | -0.16 |
| | +20 | -0.18 | 0.04 | -0.11 |
| | -20 | 0.16 | -0.1 | 0.12 |
| | -50 | 0.47 | -0.05 | 0.18 |
| **s** | +50 | 6.37 | 7.18 | 5.43 |
| | +20 | 3.15 | 3.12 | 2.69 |
| | -20 | -5.46 | -4.30 | -3.63 |
| | -50 | 18.68 | 16.21 | -13.46 |
| **L** | +50 | 2.58 | 1.67 | 2.49 |
| | +20 | 0.79 | 0.69 | 1.23 |
| | -20 | -0.79 | -0.68 | -1.27 |
| | -50 | -2.58 | -2.07 | -2.46 |

From Table 2, the following observations can be drawn:
1. The change in the consumption rate (a), time-dependent consumption rate (b) and the stock-dependent consumption rate (c) leads a positive change in the present value of the total cost (TC). The parameters b and c are highly sensitive in comparison of the parameter (a).
2. The change in the holding cost (H) for the own warehouse leads a positive change in the present value of the total cost (TC).
3. The change in the holding cost (F) for the rented warehouse leads a negative change in the present value of the total cost (TC).
4. When the capacity of the own warehouse (W) increases, then the present value of the total cost (TC) increases.
6. The change in the shortage cost (s) leads a positive change in the present value of the total cost (TC).
6. The change in the opportunity cost (L) leads a positive change in the present value of the total cost (TC).

**6. Conclusion**

A major inventory problem is to how, when and where to stock the goods. Usually it is assumed that the warehouse has boundless capacity. But due to various factors such as bulk purchase discounts, re-ordering costs, seasonality of products, inflation-induced demand, etc., the organization orders more than the capacity of the OW. To store these excess units, an extra storage space is required which is termed a rented warehouse. Here, a two-warehouse inventory model has been presented by incorporating some of the realistic phenomena, namely time-dependent demand and partially backlogged shortages, assuming that the stock is transferred from the RW to the OW in a continuous release pattern. Moreover, holding cost in the RW is considered to be higher than that of the OW. The present model differs from the existing models, as here , time-dependent demand has been considered; since the time-dependent demand is very much applicable to a lot of consumer goods whose demand changes steadily along with a steady increase in population density and display area. Moreover, it is seen that some but not all the customers will wait for backlogged items during a shortage period, such as fashionable commodities or high tech products; thus shortages at the owned warehouse are also allowed subject to partial backlogging.

A solution procedure has also been presented and findings have been validated with the help of some numerical example. Sensitivity analysis of the optimal solution with respect to various parameters has also been presented.

A future study would be to extend the proposed model for finite replenishment rate, inflation-induced demand, price dependent demand, fuzzy demand and many more.

**Reference**


1. Aggarwal, S.P., and Jaggi, C.K. (1989), Ordering Policy for Decaying Inventory, International Journal of Systems Science, 20, 151–155.
2. Chakrabarti, T., and Chaudhuri, K.S. (1997), An EOQ with Linear Trend in Demand and Shortages in all Cycles, International Journal of Production Economics, 49, 205–213.
3. Chung, K.J., and Huang, T.S. (2007), The Optimal Retailer's Ordering Policies with Limited Storage Capacity under Trade Credit Financing, International Journal of Production Economics, 106, 127–145.
4. Covert, R.P., and Philip, G.C. (1973), An EOQ Model for Items with Weibull Distribution, American Institute of Industrial Engineering Transactions, 5, 323–326.
5. Datta T.K., Paul K., Pal A.K. (1998), Demand promotion by up-gradation under Stock-Dependent Demand Situation – a Model. *International Journal of Production Economics*, 55(1), 31–38.
6. Datta, T.K., and Pal, A.K. (1988), Order Level Inventory System with Power Demand Pattern for Items with Variable Rate, Indian Journal of Pure and Applied Mathematics, 19, 1043–1053.
7. Dave, U., and Patel, L.K. (1981), Policy Inventory Model with Time Proportional Demand, Journal of the Operational Research Society, 32, 137–142.
8. Dye C. Y. (2002), An inventory model with stock-dependent demand and partial backlogging under conditions of permissible delay in payment. *Opsearch*, 39, 189–201.
9. Ghare, P.M., and Shrader, G.F. (1963), A Model for Exponentially Decaying Inventories, Journal of Industrial Engineering, 14, 238–243.
10. Goswami, A., and Chaudhuri, K.S. (1992), An Economic Order Quantity Model for Items with Two Levels of Storage for a Linear Trend in Demand, Journal of Operational Research Society, 43, 157–167.
11. Gupta R., Vrat P. (1986), Inventory Model with Multi-Items under Constraint Systems for Stock Dependent Consumption Rate. *Operations Research*, 24, 41–42.
12. Hariga, M.A. (1995), An EOQ Model for Deteriorating Items with Shortages and Time-varying Demand, Journal of the Operational Research Society, 46, 398–404.
13. Hartely, V.R. (1976), Operations Research: A Managerial Emphasis, Santa Monica, CA: Good Year, pp. 315–317 (Chapter 12).

14. Hsieh, T.P., Dye, C.Y., and Ouyang, L.Y. (2008), Determining Optimal Lot Size for a Two-warehouse System with Shortages Using Net Present Value, European Journal of Operational Research, 191, 180–190.
15. Kang, S., and Kim, I. (1983), A Study on the Price and Production Level of the Inventory System, International Journal of Production Research, 21, 449–460.
16. Kumar, N., Singh, S.R., Kumari, R. (2012), An Inventory Model with Time – Dependent Demand and Limited Storage Facility under Inflation, Advances in Operations Research, 12, 17.
17. Kumar, N., Kumar, S., (2016), Effect of learning and salvage worth on an inventory model with inventory-dependent demand rate and partial backlogging with capability constraints. *Uncertain Supply Chain Management*, 4(2): 123-136.
18. Kumar, N., Kumar, S., (2016), Inventory model with stock dependent demand, partial backlogging, and inflation over a finite time horizon. *International journal of Supply and operations Management*, 3(1): 1168-1191.
19. Kumar, N., Kumar, S., and Smarandache, F., (2016), Two Warehouse Fuzzy Inventory Model With K-Release Rule. *Uses of Sampling Techniques & Inventory Control with Capacity Constraints. Pons publishing Brussels, Belgium.* 53 – 68.
20. Kumar, N., Kumar, S., (2017), An inventory model with partial backlogging using linear demand in fuzzy environment. *Cogent Business and Management*, 4(1): 1307687.
21. Kumar, N., Singh, S.R., Kumari, R. (2013), Two-warehouse inventory model with K – release rule and learning effect, International Journal of Procurement Management, 6(1), 76-91.
22. Lee, C.C. (2006), Two-warehouse Inventory Model under FIFO Dispatching Policy, European Journal of Operational Research, 174, 861–873.
23. Levin R.I., McLaughlin C.P., Lamone R.P., Kottas J.F. (1972). *Productions operations management: contemporary policy for managing operating systems*. McGraw-Hill, New York, 373.
24. Mandal B.N., Phaujdar S. (1989), An inventory model with stock-dependent consumption rate. *Journal Operational Research Society*, 40, 483–488.
25. Pakkala, T.P.M., and Achary, K.K. (1992), An inventory Model with two warehouses and finite replenishment rate, European Journal of Operational Research, 57, 71–76.
26. Papachristos, S., and Skouri, K. (2003), An Inventory Model with Quantity Discount and Time-dependent Partial Backlogging, International Journal of Production Economics, 83, 247–256.
27. Rong, M., Mahapatra, N.K., and Maiti, M. (2008), A Two Warehouse Inventory Model with Partially/fully Backlogged Shortage and Fuzzy Lead Time, European Journal of Operational Research, 189, 59–75.
28. Sachan, R.S. (1984), On (T, Si) Policy Inventory Model with Time Proportional Demand, Journal of the Operational Research Society, 35, 1013–1019.
29. Sarkar, B., Sarkar, S. (2013), An improved inventory model with partial backlogging, and stock-dependent demand, Economic Modelling, 30, 924–932.
30. Sarma, K.V.S. (1983), An inventory model with Two Levels of Storage and an Optimum Release Rate, Opsearch, 20, 175–180, Abstract-INSPEC.

31. Sarma, K.V.S. (1987), A Deterministic Order-level Inventory Model with Two Storage Facilities, European Journal of Operational Research, 29, 70–72.
32. Shah, Y.K. (1977), An Order-level Lot-size Inventory for weakening Items, American Institute of Industrial Engineering Transactions, 9, 108–112.
33. Singh, S.R., Kumar, N., Kumari, R., (2010), An inventory model with shortages and stock-dependent demand under inflation for two-shops under one management, Opsearch, 47(4), 311-329.
34. Singh, T.,Pattnayak, H.,( 2014), A two-warehouse inventory model with linear demand under conditionally permissible delay in payment. *International Journal o f Management Science and Engineering Management*, 9(2): 104-113.
35. Singh, T., Pattanayak, H., (2015), An ordering policy with linear demand and permissible delay in payment. *Smart Innovation, Systems and Technologies*, 33(3): 649-658.
36. Singh, S.R., Kumari, R., Kumar, N. (2010), Replenishment Policy with Stock-dependent Demand and Partial Back Logging with Two-storage Facilities under Inflation, International Journal of Operations Research and Optimization, 1(1), 161-179.
37. Singh, S.R., Kumari, R., Kumar, N., (2011), A deterministic two-warehouse inventory model with sock-dependent demand and shortages under the conditions of permissible delay, *International Journal of mathematical Modelling and Optimisation*, 11(3), 357-375.
38. Vrat, P., and Padmanabhan, G. (1990), An EOQ Model for Items with Stock Dependent Consumption Rate and Exponential Decay, Engineering Costs and Production Economics, 19, 379–383.
39. Zhou Y.W., Yang S.L. (2005). A two-warehouse inventory model for items with stock-level-dependent demand rate. *Int. J. Production Economics*, 95, 215–228.