Studies on Minimization Procedures Using Derivatives

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Abstract

The unconstrained minimization problem is that of finding a point form a set of point that gives the least value of the objective f(x) where it is not expected that the least value will be taken on by a point on the boundary of the region of constraints. In this paper algorithms for unconstrained non linear and non smooth models are given. In this paper we study pre established models and results and gathered information. Where we shall develop algorithms for unconstrained non linear and non smooth optimization models.

Introduction :- In this paper we describe second derivative methods. We consider many solution, techniques and these techniques have been selected from the view point of their effectiveness by themselves and in connection with the algorithm discussed in subsequent paper describing constrained non linear and non smooth programming. Because several of the important constrained non linear programming algorithms require the use of effective unconstrained minimization procedure the general non linear model without constraints reduces to just.

Minimize : f(x) $\quad x\in E^n$ ------- (1)

Where f(x) is the objective function E^n is the Euclidean space. Here we derive methods and algorithms that leads to a stationary point of f(x). that is $\nabla f(x^*) = 0$ by using first and second partial derivatives of f(x)

We first describe the search directions prescribed by steepest descent, then Newton method, conjugate directions and finally some of the methods of approximating the direction given by Newton method by using only first derivatives. The second derivative methods among which the best known is Newton's method. Originates from the quadratic approximation of f(x) by Taylor's series. These methods use second order information of f(x) with respect to the independent variables.

Result and Discussion - The transition point from a point $x^{(k)}$ to another point $x^{(k+1)}$ can be given by expression

$x^{(K+1)} = x^{(k)} + \Delta x^{(k)}$

The steepest descent method is due to cauchy in which the gradient of the objective function f(x) at any point x is a vector in the direction of the greatest local increase in f(x) The direction of steepest descent defined at $x^{(k)}$ by.

$$\hat{s}^{(k)} = -\frac{\nabla f(x^{(k)})}{\|\nabla f(x^{(k)})\|} \qquad \text{------- (2)}$$

Putting this value in $x^{(k+1)} = x^{(k)} + \lambda^{(k)}\ \hat{s}^{(k)}$

$$x^{(k+1)} = x^{(k)} - \frac{\lambda^{(k)}\nabla f(x^{(k)})}{\|\nabla f(x^{(k)})\|} \qquad \text{-------- (3)}$$

Or

$$x^{(k+1)} = x^{(k)} - \lambda^{*(k)}.\, \nabla f\left(x^{(k)}\right) \qquad \text{------ (4)}$$

Where $\lambda^{*(k)} = \frac{\lambda^{(k)}}{\|\nabla f(x^{(k)})\|}$

The objective function is minimized with respect to λ and in the other a fixed or variable value in selected for λ convergence of this method can be demonstrated by *A. A. Goldstein*.

The search direction S for Newton's method is chosen as follows. If $(x - x^{(k)})$ is Taylor's series is replaced by

$\Delta x^{(k)} = x^{(k+1)} - x^{(k)}$

Then the quadratic approximation of f(x) in terms of $\Delta x^{(k)}$ is

$$f\left(x^{(k+1)}\right) = f\left(x^{(k)}\right) + \nabla^T f^{(k)} \Delta x^{(k)} + \frac{1}{2}\left(\Delta x^{(k)}\right) . \nabla^2 f\left(x^{(k)}\right) \quad \Delta x^k$$

Minimizing f(x) in the direction of $\Delta x^{(k)}$

$$\Delta x^{(k)} = -[\nabla^2 f\left(x^{(k)}\right)]^{-1} \ \nabla f\left(x^{(k)}\right) \qquad \text{------- (5)}$$

Where $[\nabla^2 f\left(x^{(k)}\right)]^{-1}$ is the inverse of Hession matrix $H\left(x^{(k)}\right)$. Putting in $x^{(k+1)} = x^{(k)} + \Delta x^{(k)}$ the value of $\Delta x^{(k)}$

$$x^{(k+1)} = x^{(k)} - [\nabla^2 f\left(x^{(k)}\right)]^{-1} \ \nabla f\left(x^{(k)}\right) \qquad \text{-----(6)}$$

By introducing the parameter for the step length λ into equation (6) we have

$$x^{(k+1)} = x^{(k)} - \frac{\lambda^{(k)} \ [\nabla^2 f(x^{(k)})]^{-1} \ \ [\nabla f(x^{(k)})]^{-1} [\nabla f(x^{(k)})]}{\| [\nabla^2 f(x^{(k)})]^{-1} [\nabla f(x^{(k)})] \|}$$

The ratio $\frac{\lambda^{(k)}}{\| [\nabla^2 f(x^{(k)})]^{-1} [\nabla f(x^{(k)})] \|} = \lambda^{*(k)}$ ------- (7)

Say Some Scalar, than

$$x^{(k+1)} = x^{(k)} - \lambda^{*(k)} H^{-1}\left(x^{(k)}\right) \nabla f\left(x^{(k)}\right) \qquad \text{-------- (8)}$$

The search direction S in now given by

$$s^{(k)} = -H^{-1}\left(x^{(k)}\right) \nabla f\left(x^{(k)}\right)$$

The equation (8) is applied iteratively.

This method provides a sequences of step lengths

Corresponding to the distance to minimize the respective quadratic approximation of f(x)at successive values of $x^{(k)}$

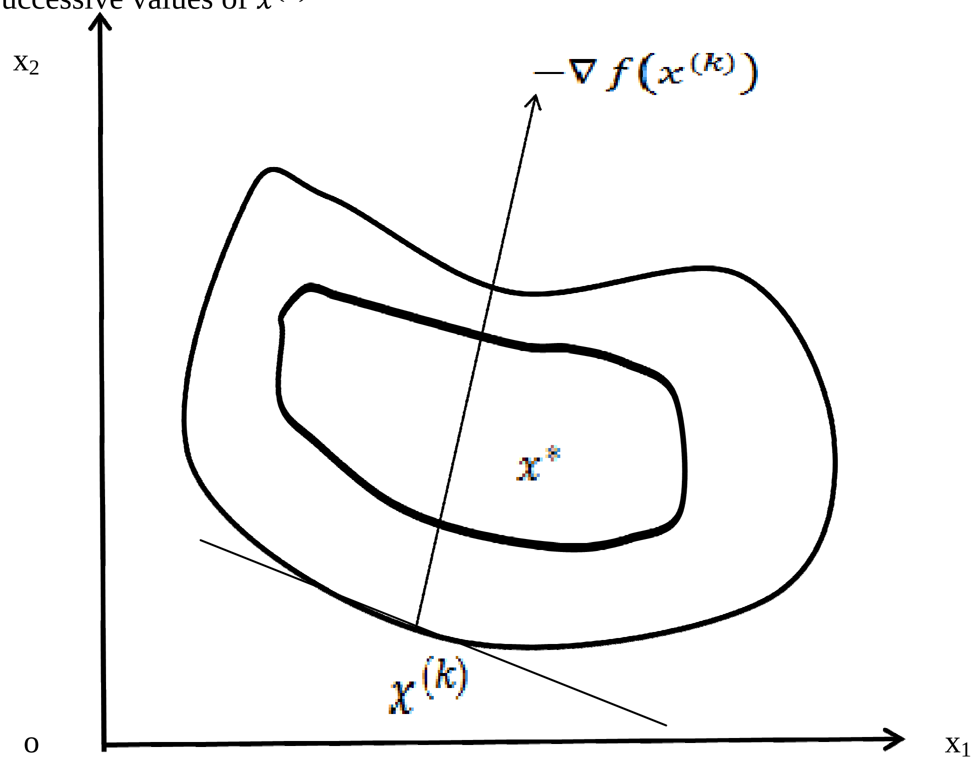

Fig.1 : Steepest Descent : First order approximation of f(x)

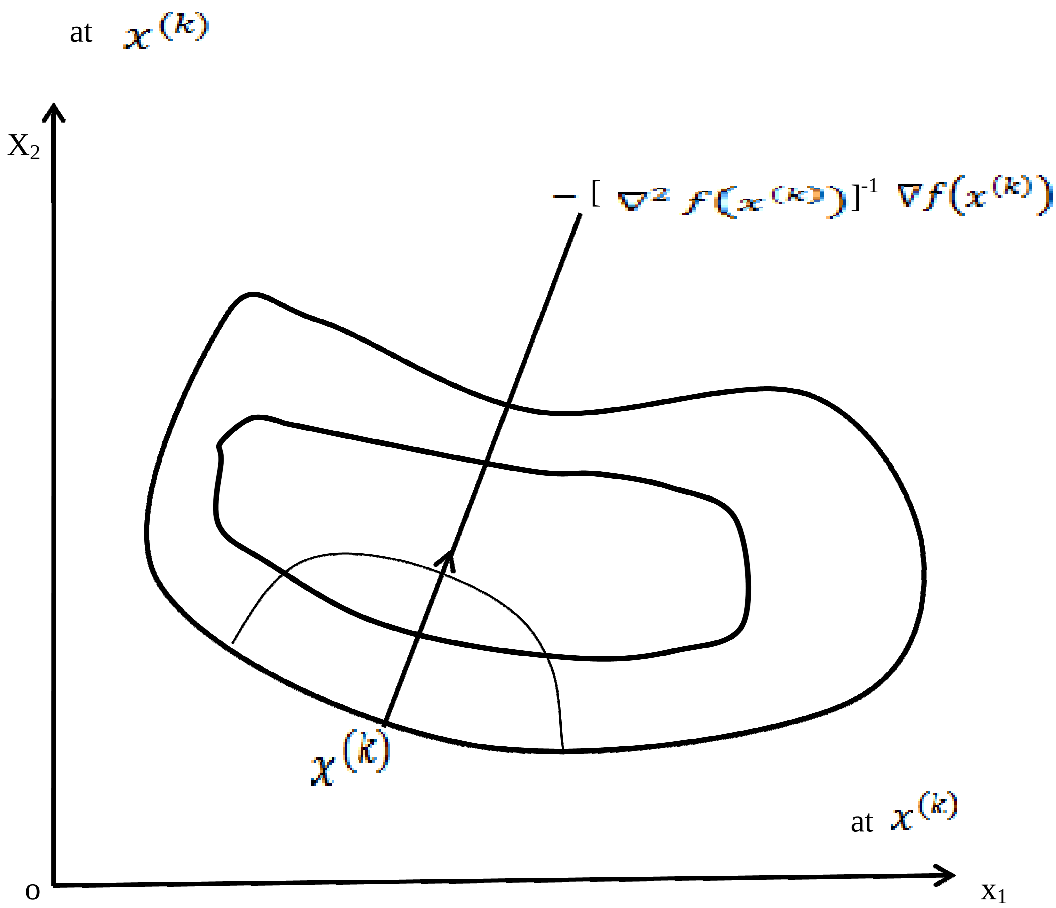

Fig.2 : Newton's Method : Second order approximation of f(x) .

A major drawback to Newton’s method is that the value of the objective function is guaranteed to the improved on each cycle only if the Hessian Matrix of the objective function is positive definite. It is necessary to guarantee converge the real symmetric matrix is positive. Consequently in minimization we observe, that when the eigen value of $H(\square^{(\square)})$ are positive, the quadratic approximation correspond a minimum.

Conclusion:-

Having been discussed many algorithms and methods in this paper. I got some conclusion as follows, to compare successfully the performance of different algorithms. I took into account three criteria viz. success in obtaining the optimal solution for a wide range of problems, number among all the Procedures. *Davidon-Fletcher Powell* procedure was superior as a general unconstrained non linear programming tool for obtaining correct objective function Newton’s method always was the best and the other methods, which failed in some tests, were roughly of the same effectiveness.

The first derived bundle Newton method in based on the following model, which generalizes a long – Kuhn cutting plane method due to *Kelley* and *Cheney* and *Goldstein*

Other ides which made the algorithm rapidly convergent based on QP sub problem can be found in *Miffin*. The trust region algorithm was tested with a variety of problems including *Yuan’s*.

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