An Eoq Model with Ramp Type Demand Rate, Two Parameter Weibull Distribution Deterioration and Shortages with Partial Backlogging and Credit Period

Pankaj Aggrawal, Tarun Jeet Singh
Department of Applied Science and Humanities, Ajay Kumar Garg Engineering College, Ghaziabad

Abstract

In this paper, we develop an inventory model for retailers permissible delay in payment with shortage, which is partially backlogged. Items are perishable and deterioration is assumed to follow two parameter Weibull distribution. The demand rate is linear function of time in beginning of cycle, which becomes constant as passage of time.
Here, we consider two different cases. In first case, the trade-credit period (M) is less than or equal to the time interval $t_1$in which S units are depleted to zero due to demand. In second case, the trade-credit period (M) is greater than the time interval t_1. The principle objective of the introduced model is to minimize the average inventory cost by finding an optimal replenishment policy. Numerical examples are used to illustrate the model. Sensitivity analysis is also performed to study the effect of change in various parameters on the behavior of the model.
KEYWORDS: Inventory, Deterioration, Partial backlogging, Ramp type demand rate, Credit period

1. Introduction

Inventory is an important part of our manufacturing, distribution and retail infrastructure where demand plays an important role in choosing the best inventory policy. Researchers were engaged to develop the inventory models assuming the demand of the items to be constant, linearly increasing or decreasing, exponential increasing or decreasing with time. Inventory models with time-dependent demand were studied by Dave (1981) and Maiti et al. (2009).
Later, it has been realized that the above demand patterns do not precisely depict the demand of certain items such as newly launched fashion items, garments, cosmetics, automobiles etc, for which the demand increases with time as they are launched into the market and after some time, it becomes constant. In order to consider demand of such types, the concept of ramp-type demand is introduced. Ramp-type demand depicts a demand which increases up to a certain time after which it stabilizes and becomes constant. Mandal and Pal (1998) have developed inventory models with ramp type demand rate for deteriorating items. Panda et al. (2008) have developed optimal replenishment policy for perishable seasonal products taking ramp-type time dependent demand rate. Avinadav et al. (2013) have developed considered demand function sensitive to price and time. Models for seasonal products with ramp-type time-dependent demand are discussed by Wang and Huang (2014).

Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage and loss of utility or loss of marginal value of a commodity that reduces usefulness from original ones. Blood, fish, fruits and vegetables, alcohol, gasoline, radioactive chemicals, medicines, etc., lose their utility with respect to time. In this case, a discount price policy is implemented by the suppliers of these products to promote sales. Thus, decay or deterioration of physical goods in stock is a very realistic feature. Modelers felt the need to take this factor into consideration. Various types of order-level inventory models for items deteriorating at a constant rate were discussed by Shah and Jaiswal (1977) and Dave (1986). As time progressed, several researchers developed inventory models with variable deterioration rate. In this connection, researchers may consult the work by Covert and Philip (1973), Chakrabarti et al. (1998), Jalan et al. (1996)and Dye (2004), who have used Weibull distribution for representing deterioration rate. Manna and Chaudhuri (2006) have developed an EOQ model with ramp type demand rate and time dependent deterioration rate. An inventory system with Markovian demands, phase type distributions for perishability and replenishment is developed by Chakravarthy (2011). San-José et al. (2014) have studied inventory system with partial backlogging and mixture of dispatching policies.

When shortage for a product occurs, some customers will go away, while some would like to wait for backlogging after the next replenishment. But the willingness is diminishing with the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Thus practically, all shortages are not backlogged but only some part of shortages is backlogged. This phenomenon is called partial backlogging.

Chang and Dye (1999) developed an inventory model in which the demand rate is a time-continuous function and items deteriorate at a constant rate with partial backlogging rate which is the reciprocal of a linear function of the waiting time. Papachristos and Skouri (2000) developed an EOQ inventory model with time-dependent partial backlogging. Teng et al. (2003) then extended the backlogged demand to any decreasing function of the waiting time up to the next replenishment. The related analysis on inventory systems with partial backlogging have been performed by Teng and Yang (2004), Dye et al. (2006) etc. Singh and Singh (2007, 2009) studied inventory model with partial backlogging considering quadratic demand and power demand. San-Jose et al. (2015) have studied partial backlogging with non linear holding cost.

When the supplier delivered goods to their customers, they often do not require to be paid immediately. Instead, suppliers offer credit terms that allow the buyers to delay the payment. This is known as trade-credit. This is very beneficial to the customers because, they do not have to pay the supplier immediately, after receiving the product, but instead, can delay their payment until, the end of the allowed period. Chang et al**.** (2003) developed a model for deteriorating items under supplier credits linked to order quantity. Bhunia et al. (2014) applied the concepts of permissible delay in two-warehouse inventory model. Yong et al. (2015) discussed a single-period inventory and payment model with partial trade credit. Beatriz et al. (2016) applied permissible delay for a single-vendor two-buyer system. Salem (2017) discussed Coordination of a three-level supply chain (supplier–manufacturer–retailer) with permissible delay in payments and price discounts.

In this paper, an effort has been made to analyse an EOQ model for time-dependent deteriorating items with credit period. Supplier offers a credit period (M) to retailer. If retailer makes the payment before this period, he need not to pay any interest. The money obtained by selling the items in this period will earn some interest.
The demand rate is assumed to be a combination a linear and constant function of time. Such type of the demand pattern is generally seen in the case of any new brand of consumer goods coming to the market. The demand rate for such items increases with time up to certain time and then ultimately stabilizes and becomes constant. It is believed that such type of demand rate is quite realistic.

2. Assumptions and Notations:

To develop an inventory model with variable demand and partial backlogging, the following notations and assumptions are used:

i) Replenishment is instantaneous and lead time is zero
ii) $c_1$is the inventory holding cost per unit per unit of time.
iii) c_3 is the shortage cost per unit per unit of time.
iv) $c_4$is the unit cost of lost sales.
v) c_5 is the cost of each unit.
vi) Ordering cost is c'.
vii) Demand rate is a combination a linear and constant function of time defined by $f(t) = a\{t - (t - \mu)H(t - \mu)\}$ where a & μ are constants and $H(t - \mu)$ is Heviside's function defined as follows: $H(t - \mu) = \begin{cases} 0, t < \mu \\ 1, t \geq \mu \end{cases}$

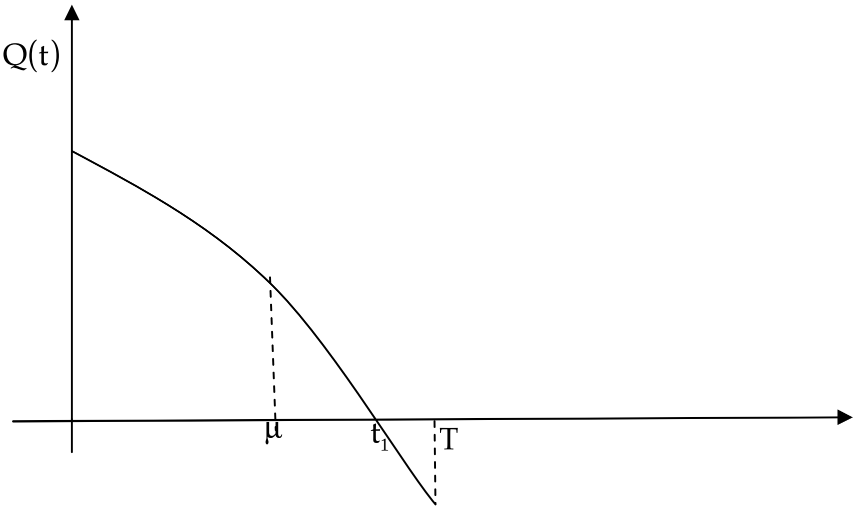
Thus demand can be written as $f(t) = \begin{cases} at, t < \mu \\ a\mu, t \geq \mu \end{cases}$

viii) Unsatisfied demand is backlogged at a rate $u^{-\lambda t}$, where t is the time up to next replenishment and λ is a positive constant.
ix) R is the total cost per production cycle and T is the time for each cycle.
x) $Q(t)$ be the inventory level at time t.
xi) The distribution of the time to deterioration of the items follows two parameter Weibull distribution.Thus a variable fraction $\theta(t) = \alpha\beta\, t^{\beta-1}$, $(0 < \alpha \ll 1, t \geq 0)$ is the deterioration rate.
xii) I_e is the interest earned per unit time and I_r is the interest charge which invested in inventory per unit time.
xiii) M is the permissible delay period for settling accounts in time units.
xiv) t_1 is the time at which shortage starts and T is the length of replenishment cycle.
xv) There is no repair or replenishment of deteriorated units.

3. Formulation and solution of the model:

At the start of the cycle, the inventory level reaches its maximum S units of item at time t=0. During the time interval $[0, t_1]$, the inventory depletes due mainly to demand and partly to deterioration. At time $t < t_1$, the inventory level depletes and at t_1, the inventory level is zero and all the demand hereafter (i.e. $T - t_1$) is partially backlogged. The demand varies with time up to a certain time and become constant thereafter. The deterioration rate is described by an increasing function of time $\theta(t) = \alpha\beta\, t^{\beta-1}$.

A Graphical representation of the considered inventory system is given below:



The differential equations governing the instantaneous states of $Q(t)$ in the interval $[0, T]$ are as follows:

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -f(t), 0 \le t \le \mu \qquad (1)$$

$$\frac{dQ}{dt} + \theta(t)Q(t) = -f(t), \mu \le t \le t_1 \qquad (2)$$

$$\frac{dQ}{dt} = -f(t)\, u^{-\lambda t} \;, t_1 \le t \le T \qquad (3)$$

Conditions are $Q(0) = S$, $Q(t_1) = 0$

When, we solve above equations by analytical method, sometimes integration become difficult due to presence of the term of the form e^{x^a}. This term is approximated as $e^{x^a} = 1 + x^a$ to make the integration possible. Also as α is small, its higher powers are neglected to avoid tedious calculation work. Thus, the solutions of equations (1) to (3) are given below:

$$Q(t) = \frac{4S - 2at^2 + 2S\beta - at^2\beta}{2(2+\beta)} + \frac{\alpha(-4St^\beta - 2St^\beta\beta + at^{2+\beta}\beta)}{2(2+\beta)}, \qquad 0 \le t \le \mu \qquad (4)$$

$$Q(t) = \frac{a\, t_1^{1+\beta}\alpha\mu}{1+\beta} + \frac{a(-t + \mathrm{t}_1 - t^\beta \mathrm{t}_1\alpha - t\beta + \mathrm{t}_1\beta + t^{1+\beta}\alpha\beta - t^\beta \mathrm{t}_1\alpha\beta)\mu}{1+\beta}, \qquad \mu \le t \le t_1 \qquad (5)$$

$$Q(t) = \frac{a(u^{-t\lambda} - u^{-\mathrm{t1}\lambda})\mu}{\lambda \mathrm{Log}[u]}, \; t_1 \le t \le T \qquad (6)$$

The inventory holding cost during the interval $(0, T)$ is given by

$$C_H = c_1 \int_0^{t_1} Q(t)\, dt$$

$$\Rightarrow C_H = c_1 \left[\int_0^{\mu} Q(t)dt + \int_{\mu}^{t_1} Q(t)\, dt\right]$$

$\Rightarrow C_H =$

$$c_1\left[\frac{S\mu}{1+\beta}+\frac{S\beta\mu}{1+\beta}-\frac{a\mu^3}{(2+\beta)(3+\beta)}-\frac{5a\beta\mu^3}{6(2+\beta)(3+\beta)}-\frac{a\beta^2\mu^3}{6(2+\beta)(3+\beta)}-\frac{at_1^{1+\beta}\alpha\mu(-t_1\beta+2\mu+\beta\mu)}{(1+\beta)(2+\beta)}+\frac{\alpha\mu^{1+\beta}(-12S-10S\beta-2S\beta^2+a\beta\mu^2+a\beta^2\mu^2)}{2(1+\beta)(2+\beta)(3+\beta)}+\frac{1}{2(1+\beta)(2+\beta)}a\mu\left\{2t_1^2+3t_1^2\beta+t_1^2\beta^2-4t_1\mu-6t_1\beta\mu-2t_1\beta^2\mu+2\mu^2+3\beta\mu^2+\beta^2\mu^2+4t_1\alpha\mu^{1+\beta}+2t_1\alpha\beta\mu^{1+\beta}-2\alpha\beta\mu^{2+\beta}\right\}\right]$$

(7)

The cost due to deterioration of units in the period $(0, T)$ is given by

$C_D = c_5$ (Initial inventory level-Total units sold)

$$= c_5\left[S-\int_0^{t_1} f(t)\,dt\right]$$

$$\Rightarrow C_D = c_5\left(S - at_1\mu + \frac{a\mu^2}{2}\right)$$

(8)

The cost due to shortages in the interval $(0, T)$ is given by

$$C_S = -c_3\left[\int_{t_1}^{T} Q(t)\,dt\right]$$

$$\Rightarrow C_S = -c_3\left(-\frac{au^{-T\lambda}\mu}{\lambda^2\mathrm{Log}[u]^2}+\frac{au^{-t_1\lambda}\mu}{\lambda^2\mathrm{Log}[u]^2}-\frac{aTu^{-t_1\lambda}\mu}{\lambda\mathrm{Log}[u]}+\frac{at_1u^{-t_1\lambda}\mu}{\lambda\mathrm{Log}[u]}\right)$$

(9)

The opportunity cost due to lost sales in the interval $(0, T)$ is given by

$$C_O = c_4\left[\int_{t_1}^{T}(1-u^{-\lambda t})f(t)dt\right]$$

$$\Rightarrow C_O = a\mu c_4\left(T+\frac{u^{-T\lambda}-u^{-\lambda t_1}}{\lambda\mathrm{Log}[u]}-t_1\right)$$

(10)

Now, the inventory available may last before the completion of permissible delay period M or it may last after the completion of permissible delay period M.

Thus, depending upon the permissible delay period M for settling the accounts, there arise two cases:

Case 1: When $M \le t_1$

Case 2: When $M > t_1$

Now, we shall discuss these cases separately one by one.

4. Case I: $M \le t_1$

(i.e. when the permissible delay period is less than the inventory period)

In this case, since the credit period (M) is smaller than the length of period with positive inventory stock of the items, therefore the buyer can use the sale revenue to earn the interest with the rate I_e per unit time. In this case there again arise two sub cases.

5. Sub case 1: When $M < \mu$

In this situation, interest earned I_E is given by the following relation:

$$I_E = c_5\, I_e \int_0^M t\, f(t)dt$$

Substituting the value of $f(t)$ and integrating, we get,

$I_E = \frac{1}{3} a\, c_5 I_e M^3,$ (11)

Also, interest payable I_p is given by the following relation:

$$I_p = c_5\, I_r \int_M^{t_1} Q(t)dt$$

$$\Rightarrow\; I_p = c_5\, I_r \left[\int_M^{\mu} Q(t)dt + \int_{\mu}^{t_1} Q(t)dt\right]$$

Substituting the value of $Q(t)$, integrating and simplifying, we get,

$$I_p = c_5 I_r \Bigg[\frac{M}{6(1+\beta)(2+\beta)(3+\beta)}\{6aM^2 - 36S + 36M^{\beta}S\alpha + 11aM^2\beta - 66S\beta - 3aM^{2+\beta}\alpha\beta + 30M^{\beta}S\alpha\beta + 6aM^2\beta^2 - 36S\beta^2 - 3aM^{2+\beta}\alpha\beta^2 + 6M^{\beta}S\alpha\beta^2 + aM^2\beta^3 - 6S\beta^3\} + \frac{S\mu}{1+\beta} + \frac{S\beta\mu}{1+\beta} - \frac{a\mu^3}{(2+\beta)(3+\beta)} - \frac{5a\beta\mu^3}{6(2+\beta)(3+\beta)} - \frac{a\beta^2\mu^3}{6(2+\beta)(3+\beta)} - \frac{at_1^{1+\beta}\alpha\mu(-t_1\beta+2\mu+\beta\mu)}{(1+\beta)(2+\beta)} - \frac{\alpha\mu^{1+\beta}(12S+10S\beta+2S\beta^2-a\beta\mu^2-a\beta^2\mu^2)}{2(1+\beta)(2+\beta)(3+\beta)} + \frac{1}{2(1+\beta)(2+\beta)}a\mu\big(2t_1^2 + 3t_1^2\beta + t_1^2\beta^2 - 4t_1\mu - 6t_1\beta\mu - 2t_1\beta^2\mu + 2\mu^2 + 3\beta\mu^2 + \beta^2\mu^2 + 4t_1\alpha\mu^{1+\beta} + 2t_1\alpha\beta\mu^{1+\beta} - 2\alpha\beta\mu^{2+\beta}\big)\Bigg] \qquad (12)$$

The total cost R_1 in the system in this sub case in the interval $(0,T)$ is given by

$R_1 = c' + C_H + C_D + C_S + C_O + I_p - I_E$

In above relation, c' is constant, while $C_H, C_D, C_S \& C_O$ are given by the equations (7) to (10).

The average cost K_1 in the system in this sub case in the interval $(0,T)$ is given by

$K_1 = \frac{R_1}{T}$ (13)

The optimum values of $t_1$and T which minimize average cost K_1 are obtained by using the equations:

$$\frac{\partial K_1}{\partial t_1} = 0 \; and \; \frac{\partial K_1}{\partial T} = 0\,,$$

Now,

$$\frac{\partial K_1}{\partial t_1} = 0$$

$$\Rightarrow -\frac{a(c_1+c_5 I_r)t_1^{\beta}\alpha\mu(-t_1\beta+\mu+\beta\mu)}{1+\beta} - \frac{1}{1+\beta}au^{-t_1\lambda}\mu \big(-c_4 + c_3T - c_3t_1 + c_4u^{t_1\lambda} + c_5u^{t_1\lambda} - c_1t_1u^{t_1\lambda} - c_5I_rt_1u^{t_1\lambda} - c_4\beta + c_3T\beta - c_3t_1\beta + c_4u^{t_1\lambda}\beta + c_5u^{t_1\lambda}\beta - c_1t_1u^{t_1\lambda}\beta - c_5I_rt_1u^{t_1\lambda}\beta + c_1u^{t_1\lambda}\mu + c_5I_ru^{t_1\lambda}\mu + c_1u^{t_1\lambda}\beta\mu + c_5I_ru^{t_1\lambda}\beta\mu - c_1u^{t_1\lambda}\alpha\mu^{1+\beta} - c_5I_ru^{t_1\lambda}\alpha\mu^{1+\beta}\big) = 0 \qquad (14)$$

Also, $\frac{\partial K_1}{\partial T} = 0$ gives

$$-\frac{cc}{T}+\frac{ac_5I_eM^3}{3T} - \frac{c_5I_rM}{6T(1+\beta)(2+\beta)(3+\beta)}(6aM^2-36S+36M^\beta S\alpha+11aM^2\beta-66S\beta- 3aM^{2+\beta}\alpha\beta+30M^\beta S\alpha\beta+6aM^2\beta^2-36S\beta^2-3aM^{2+\beta}\alpha\beta^2+6M^\beta S\alpha\beta^2+ aM^2\beta^3-6S\beta^3) + ac_4u^{-T\lambda}(-1+u^{T\lambda})\mu - \frac{c_1S\mu}{T(1+\beta)} - \frac{c_5I_rS\mu}{T(1+\beta)} - \frac{c_1S\beta\mu}{T(1+\beta)} - \frac{c_5I_rS\beta\mu}{T(1+\beta)} + \frac{ac_1\mu^3}{T(2+\beta)(3+\beta)} + \frac{ac_5Ir\mu^3}{T(2+\beta)(3+\beta)} + \frac{5ac_1\beta\mu^3}{6T(2+\beta)(3+\beta)} + \frac{5ac_5I_r\beta\mu^3}{6T(2+\beta)(3+\beta)} + \frac{ac_1\beta^2\mu^3}{6T(2+\beta)(3+\beta)} + \frac{ac_5I_r\beta^2\mu^3}{6T(2+\beta)(3+\beta)} + \frac{a(c_1+c_5I_r)t_1^{1+\beta}\alpha\mu(-t_1\beta+2\mu+\beta\mu)}{T(1+\beta)(2+\beta)} - \frac{c_5(2S-2at_1\mu+a\mu^2)}{2T} - \frac{c_1\alpha\mu^{1+\beta}(-12S-10S\beta-2S\beta^2+a\beta\mu^2+a\beta^2\mu^2)}{2T(1+\beta)(2+\beta)(3+\beta)} - \frac{c_5I_r\alpha\mu^{1+\beta}(-12S-10S\beta-2S\beta^2+a\beta\mu^2+a\beta^2\mu^2)}{2T(1+\beta)(2+\beta)(3+\beta)} - \frac{1}{2T(1+\beta)(2+\beta)}ac_1\mu(2t_1^2+3t_1^2\beta+t_1^2\beta^2-4t_1\mu-6t_1\beta\mu-2t_1\beta^2\mu+2\mu^2+ 3\beta\mu^2+\beta^2\mu^2+4t_1\alpha\mu^{1+\beta}+2t_1\alpha\beta\mu^{1+\beta}-2\alpha\beta\mu^{2+\beta}) - \frac{1}{2T(1+\beta)(2+\beta)}ac_5I_r\mu(2t_1^2+ 3t_1^2\beta+t_1^2\beta^2-4t_1\mu-6t_1\beta\mu-2t_1\beta^2\mu+2\mu^2+3\beta\mu^2+\beta^2\mu^2+4t_1\alpha\mu^{1+\beta}+ 2t_1\alpha\beta\mu^{1+\beta}-2\alpha\beta\mu^{2+\beta}) + \frac{ac_3u^{-T\lambda-t_1\lambda}(u^{T\lambda}-u^{t_1\lambda})\mu}{\lambda \mathrm{Log}[u]} - \frac{ac_4u^{-T\lambda-t_1\lambda}\mu(-u^{T\lambda}+u^{t_1\lambda}+Tu^{T\lambda+t_1\lambda}\lambda\mathrm{Log}[u]-t_1u^{T\lambda+t_1\lambda}\lambda\mathrm{Log}[u])}{T\lambda\mathrm{Log}[u]} - \frac{c_3u^{-T\lambda-t_1\lambda}(-au^{T\lambda}\mu+au^{t_1\lambda}\mu+aTu^{T\lambda}\lambda\mu\mathrm{Log}[u]-at_1u^{T\lambda}\lambda\mu\mathrm{Log}[u])}{T\lambda^2\mathrm{Log}[u]^2}=0 \quad (15)$$

Solving (14) and (15) simultaneously, we can obtain optimal solution for t_1 & T.

5. **Numerical Example:** To illustrate the model numerically, we use the following parameter values:
$c'=100, c_1=2.4, c_3=5, c_4=10, c_5=8,,\ \alpha=0.002, \beta=3,\ a=900, \mu=1.5, u=e,\ \lambda=0.1,\ I_r=0.2, I_e=0.15, M=1$ and $S=8000$

Applying the subroutine FindRoot in Mathematica 8, we obtain the optimal solution for t_1 and T as follows:
$t_1=5.3846, T=7.6309$
Also, the optimal average cost for these parameters is 15132.1

6. Sensitivity Analysis:

Sensitivity analysis is performed by changing (increasing and decreasing) the most of the parameters by 10%, 30% & 50% and taking one parameter at a time, keeping the remaining parameters at their original values. Thus following table is formed:

Table 1

Changing Parameter	**% Change**	t_1	T	**Average Cost**	**% change in average cost**
S	+50	6.0399	10.0560	20765.6	37.23
	+30	5.7980	9.0993	18694.1	23.54
	+10	5.5307	8.1276	16390.5	8.32
	−10	5.2281	7.1236	13788.1	-8.88
	−30	4.8742	6.0613	10777.2	-28.78
	−50	4.4388	4.8941	7152.0	-52.74

c'	+50	5.3853	7.6334	15138.7	0.04
	+30	5.3850	7.6324	15136.1	0.03
	+10	5.3847	7.6314	15133.4	0.01
	−10	5.3844	7.6304	15130.8	-0.01
	−30	5.3841	7.6294	15128.2	-0.03
	−50	5.3838	7.62838	15125.6	-0.04
c_1	+50	5.0254	8.1297	18410.3	21.66
	+30	5.1539	7.9293	17161.7	13.41
	+10	5.3018	7.7300	15832.0	4.63
	−10	5.4743	7.5323	14405.9	-4.80
	−30	5.6791	7.3370	12862.7	-15.00
	−50	5.9275	7.1458	11173.1	-26.16
c_3	+50	5.4585	7.0722	15586.4	3.00
	+30	5.4345	7.2526	15434.8	2.00
	+10	5.4036	7.4861	15245.6	0.75
	−10	5.3623	7.8010	15002.4	-0.86
	−30	5.3038	8.2498	14677.7	-3.00
	−50	5.2141	8.9447	14219.9	-6.03
c_4	+50	5.4098	6.9971	15959.4	5.47
	+30	5.3950	7.2364	15657.4	3.47
	+10	5.3864	7.4941	15316.9	1.22
	−10	5.3846	7.7736	14937.6	-1.29
	−30	5.3904	8.0787	14518.7	-4.05
	−50	5.4046	8.4145	14059.3	-7.09
c_5	+50	5.6383	8.5420	17433.9	15.21
	+30	5.5265	8.1023	16315.7	7.82
	+10	5.4601	7.8713	15733.0	3.97
	−10	5.2981	7.3789	14510.6	-4.11
	−30	5.1978	7.1124	13865.2	-8.37
	−50	4.9400	6.5195	12483.9	-17.50
α	+50	5.2033	7.5027	15365.6	1.54
	+30	5.2698	7.5493	15277.3	0.96
	+10	5.3440	7.6019	15182.4	0.33
	−10	5.4279	7.6621	15079.7	-0.35
	−30	5.5242	7.7324	14967.3	-1.09
	−50	5.6374	7.8165	14843.1	-1.91
β	+50	4.1682	6.8068	16983.8	12.24
	+30	4.6338	7.1011	16149.2	6.72
	+10	5.1424	7.4535	15421.8	1.91
	−10	5.5960	7.7990	14903.9	-1.51
	−30	5.8810	8.0084	14622.9	-3.37
	−50	6.0032	8.1015	14504.3	-4.15
a	+50	4.8077	5.8740	15326.9	1.29
	+30	5.0035	6.4366	15433.5	1.99

	+10	5.2426	7.1697	15304.2	1.14
	−10	5.5465	8.1828	14874.2	-1.70
	−30	5.9565	9.7173	14037.6	-7.23
	−50	6.5722	12.4645	12597.5	-16.75
μ	+50	5.4882	6.6129	16018.8	5.86
	+30	5.4194	6.9378	15970.0	5.54
	+10	5.3846	7.3628	15516.6	2.54
	−10	5.3999	7.9518	14637.6	-3.27
	−20	5.4353	8.3447	14027.5	-7.30
	−30	5.4971	8.8397	13293.9	-12.15
u	+50	5.4068	7.7782	15342.3	1.39
	+30	5.3985	7.7197	15275.7	0.95
	+10	5.3894	7.6606	15187.7	0.37
	−10	5.3796	7.6013	15065.9	-0.44
	−30	5.3693	7.5434	14885.6	-1.63
	−50	5.3605	7.4921	14587.8	-3.60
λ	+50	5.4126	7.8212	15382.0	1.65
	+30	5.4007	7.7344	15294.0	1.07
	+10	5.3897	7.6621	15190.3	0.38
	−10	5.3798	7.6027	15069.4	-0.41
	−30	5.3713	7.5549	14929.2	-1.34
	−50	5.3647	7.5179	14767.3	-2.41
I_r	+50	5.0434	7.6498	16570.9	9.51
	+30	5.1707	7.6407	16020.9	5.87
	+10	5.3099	7.6336	15437.7	2.02
	−10	5.4630	7.6288	14816.3	-2.09
	−30	5.6326	7.6265	14150.7	-6.49
	−50	5.8216	7.6271	13433.1	-11.23
I_e	+50	5.3819	7.6218	15108.5	-0.16
	+30	5.3829	7.6255	15118.0	-0.09
	+10	5.3840	7.6291	15127.4	-0.03
	−10	5.3851	7.6327	15136.8	0.03
	−30	5.3862	7.6364	15146.3	0.09
	−50	5.3873	7.6400	15155.7	0.16
M	+40	5.3021	7.3606	14423.4	-4.68
	+30	5.3232	7.4291	14604.6	-3.49
	+10	5.3644	7.5642	14958.8	-1.15
	−10	5.4045	7.6971	15303.1	1.13
	−30	5.4434	7.8281	15638.5	3.35
	−50	5.4814	7.9576	15966.1	5.51

From Table 1, the following points are noted:
(i) It is seen that the percentage change in the optimal cost is almost equal for both positive and negative changes of all the parameters except S, β, a and μ

(ii) It is observed that the model is more sensitive for a negative change than an equal positive change in the parameter S, α and μ. The trend is reversed in case of parameter β.

(iii) The optimal cost increases (decreases) and decreases (increases) with the increase (decrease) and decrease (increase) in the value of all parameters except I_e & M . The trend is reversed for these two parameters.

(iv) Model is highly sensitive to changes in c_1, c_5 & S and low sensitive to changes in

c', α, u, λ and I_e . It is moderately sensitivity to changes in $c_3, c_4, \beta, a, I_r, \mu$ and M.

(v) From the above points, it is clear that much care is to be taken to estimate c_1, c_5 & S.

7. Sub case 2: When $M > \mu$

In this situation, interest earned I_E is given by the following relation:

$$I_E = c_5\, I_e \int_0^M t\, f(t)dt$$

$$= c_5\, I_e \left[\int_0^{\mu} t\, f(t)dt + \int_{\mu}^{M} t\, f(t)dt\right]$$

Substituting the value of $f(t)$, integrating and simplifying, we get,

$$I_E = \mathrm{c_5 I_e}\left(\frac{1}{2}aM^2\mu - \frac{a\mu^3}{6}\right)$$

(16)

Also, interest payable I_p is given by the following relation:

$$I_p = c_5\, I_r \int_M^{t_1} Q(t)dt$$

Substituting the value of $Q(t)$, integrating and simplifying, we get,

$$I_p = -\frac{a\mathrm{c_5 I_r}Mt_1^{1+\beta}\alpha(2M+M\beta-\mathrm{t_1}\beta)}{(1+\beta)(2+\beta)} + \frac{a\mathrm{c_5 I_r}M}{2(1+\beta)(2+\beta)}\left(2M^2 - 4M\mathrm{t_1} + 2t_1^2 + 4M^{1+\beta}\mathrm{t_1}\alpha + 3M^2\beta - 6M\mathrm{t_1}\beta + 3t_1^2\beta - 2M^{2+\beta}\alpha\beta + 2M^{1+\beta}\mathrm{t_1}\alpha\beta + M^2\beta^2 - 2M\mathrm{t_1}\beta^2 + t_1^2\beta^2\right)$$

(17)

The total cost R_2 in the system in this sub case in the interval $(0, T)$ is given by

$$R_2 = c' + C_H + C_D + C_S + C_O + I_p - I_E$$

In above relation, c' is constant, while C_H, C_D, C_S&C_O are given by the equations (7) to (10).

The average cost K_2 in the system in this sub case in the interval $(0, T)$ is given by

$$K_2 = \frac{R_2}{T}$$

(18)

The optimum values of $t_1$and T which minimize average cost K_2 are obtained by using the equations:

$$\frac{\partial K_2}{\partial t_1} = 0 \; and \; \frac{\partial K_2}{\partial T} = 0\,,$$

Now,

$$\frac{\partial K_2}{\partial t_1} = 0$$

$$\Rightarrow a(c_4 - c_3T + c_3t_1)u^{-t_1\lambda}\mu \;\; -\frac{a}{1+\beta}\Big(c_5I_rM^2 - c_5I_rMt_1 - c_5I_rM^{2+\beta}\alpha + c_5I_rM^2t_1^{\beta}\alpha + c_5I_rM^2\beta - c_5I_rMt_1\beta + c_5I_rM^2t_1^{\beta}\alpha\beta - c_5I_rMt_1^{1+\beta}\alpha\beta + c_4\mu + c_5\mu - c_1t_1\mu + c_4\beta\mu + c_5\beta\mu - c_1t_1\beta\mu - c_1t_1^{1+\beta}\alpha\beta\mu + c_1\mu^2 + c_1t_1^{\beta}\alpha\mu^2 + c_1\beta\mu^2 + c_1t_1^{\beta}\alpha\beta\mu^2 - c_1\alpha\mu^{2+\beta}\Big) = 0$$

(19)

Also, $\frac{\partial K_2}{\partial T} = 0$ gives

$$\Rightarrow -\frac{c'}{T} \;\; -\frac{ac_5I_rM}{2T(1+\beta)(2+\beta)}\big(2M^2 - 4Mt_1 + 2t_1^2 + 4M^{1+\beta}t_1\alpha + 3M^2\beta - 6Mt_1\beta + 3t_1^2\beta - 2M^{2+\beta}\alpha\beta + 2M^{1+\beta}t_1\alpha\beta + M^2\beta^2 - 2Mt_1\beta^2 + t_1^2\beta^2\big) \;\; +ac_4u^{-T\lambda}(-1 + u^{T\lambda})\mu \; -\frac{c_1S\mu}{T(1+\beta)} - \frac{c_1S\beta\mu}{T(1+\beta)} + \frac{ac_1\mu^3}{T(2+\beta)(3+\beta)} + \frac{5ac_1\beta\mu^3}{6T(2+\beta)(3+\beta)} + \frac{ac_1\beta^2\mu^3}{6T(2+\beta)(3+\beta)} - \frac{c_5(2S-2at_1\mu+a\mu^2)}{2T} + \frac{at_1^{1+\beta}\alpha(2c_5I_rM^2+c_5I_rM^2\beta-c_5I_rMt_1\beta-c_1t_1\beta\mu+2c_1\mu^2+c_1\beta\mu^2)}{T(1+\beta)(2+\beta)} - \frac{c_1\alpha\mu^{1+\beta}(-12S-10S\beta-2S\beta^2+a\beta\mu^2+a\beta^2\mu^2)}{2T(1+\beta)(2+\beta)(3+\beta)} - \frac{c_5I_e(-3aM^2\mu+a\mu^3)}{6T} - \frac{1}{2T(1+\beta)(2+\beta)}ac_1\mu\;\big(2t_1^2 + 3t_1^2\beta + t_1^2\beta^2 - 4t_1\mu - 6t_1\beta\mu - 2t_1\beta^2\mu + 2\mu^2 + 3\beta\mu^2 + \beta^2\mu^2 + 4t_1\alpha\mu^{1+\beta} + 2t_1\alpha\beta\mu^{1+\beta} - 2\alpha\beta\mu^{2+\beta}\big) + \frac{ac_3u^{-T\lambda-t_1\lambda}(u^{T\lambda}-u^{t_1\lambda})\mu}{\lambda\mathrm{Log}[u]} - \frac{ac_4u^{-T\lambda-t_1\lambda}\mu(-u^{T\lambda}+u^{t_1\lambda}+Tu^{T\lambda+t_1\lambda}\lambda\mathrm{Log}[u]-t_1u^{T\lambda+t_1\lambda}\lambda\mathrm{Log}[u])}{T\lambda\mathrm{Log}[u]} - \frac{c_3u^{-T\lambda-t_1\lambda}(-au^{T\lambda}\mu+au^{t_1\lambda}\mu+aTu^{T\lambda}\lambda\mu\mathrm{Log}[u]-at_1u^{T\lambda}\lambda\mu\mathrm{Log}[u])}{T\lambda^2\mathrm{Log}[u]^2} = 0$$

(20)

Solving (19) and (20) simultaneously, we can obtain optimal solution for t_1 & T.

8. Numerical Example: To illustrate the model numerically, we use the following parameter values:

$c' = 100, c_1 = 2.4, c_3 = 5, c_4 = 10, c_5 = 8,\;\; \alpha = 0.002, \beta = 3,\; a = 900, \mu = 1,$
$u = e,\; \lambda = 0.1,\; I_r = 0.2, I_e = 0.15, M = 4$ and $S = 8000$

Applying the subroutine FindRoot in Mathematica 8, we obtain the optimal solution for t_1 and T as follows:

$t_1 = 5.1418, T = 7.7043$

Also, the optimal average cost for these parameters is 7367.9

9. Sensitivity Analysis:

Sensitivity analysis is performed by changing (increasing and decreasing) the most of the parameters by 10%, 30% & 50% and taking one parameter at a time, keeping the remaining parameters at their original values. Thus following table is formed:

Table 2

Changing Parameter	% Change	t_1	T	Average Cost	% change in average cost
S	+50	5.6498	10.4257	11096.1	50.60
	+30	5.4577	9.3410	9721.2	31.94
	+10	5.2517	8.2541	8198.5	11.27
	−10	5.0261	7.1464	6480.52	-12.04
	−30	4.7722	5.9907	4486.6	-39.11
	−50	4.4730	4.7385	2057.6	-72.07
c'	+50	5.1425	7.7076	7375.9	0.11
	+30	5.1422	7.7063	7372.7	0.07
	+10	5.1419	7.7049	7369.5	0.02
	−10	5.1416	7.7036	7366.3	-0.02
	−30	5.1414	7.7023	7363.1	-0.07
	−50	5.1411	7.7010	7359.9	-0.11
c_1	+50	4.9157	8.4199	9997.6	35.69
	+30	5.0006	8.1392	8998.6	22.13
	+10	5.0927	7.8513	7931.0	7.64
	−10	5.1931	7.5549	6783.0	-7.94
	−30	5.3032	7.2482	5539.4	-24.82
	−50	5.4244	6.9290	4180.0	-43.27
c_3	+50	5.1941	7.0405	7433.1	0.88
	+30	5.1770	7.2548	7413.5	0.62
	+10	5.1551	7.5323	7386.0	0.25
	−10	5.1263	7.9063	7345.9	-0.30
	−30	5.0862	8.4393	7284.8	-1.13
	−50	5.0260	9.2657	7185.7	-2.47
c_4	+50	5.1506	7.0741	7684.0	4.29
	+30	5.1444	7.3133	7575.4	2.82
	+10	5.1417	7.5693	7443.0	1.02
	−10	5.1429	7.8445	7286.8	-1.10
	−30	5.1486	8.1420	7106.7	-3.55
	−50	5.1593	8.4657	6902.3	-6.32
c_5	+50	5.2744	8.5213	7703.0	4.55
	+30	5.2184	8.1245	7530.9	2.21
	+10	5.1834	7.9180	7448.3	1.09
	−10	5.0917	7.4811	7289.4	-1.07
	−30	5.0303	7.2455	7212.2	-2.11
	−50	4.8535	6.7158	7058.4	-4.20
α	+50	5.0560	7.6548	7572.4	2.78
	+30	5.0885	7.6732	7492.3	1.69
	+10	5.1233	7.6934	7410.0	0.57
	−10	5.1610	7.7158	7325.1	-0.58
	−30	5.2021	7.7409	7237.4	-1.77

	−50	5.2473	7.7693	7146.5	-3.00
β	+50	4.38731	7.3008	9079.9	23.24
	+30	4.7263	7.4585	8168.5	10.87
	+10	5.0257	7.6308	7569.5	2.74
	−10	5.2294	7.7614	7220.0	-2.01
	−30	5.3296	7.8268	7043.5	-4.40
	−50	5.3678	7.8506	6962.8	-5.50
a	+50	4.7255	5.7881	6166.8	-16.30
	+30	4.8637	6.3969	6774.3	-8.06
	+10	5.0368	7.1967	7218.5	-2.03
	−10	5.2637	8.3155	7460.4	1.26
	−30	5.5830	10.0404	7437.6	0.95
	−50	6.0931	13.2300	7039.3	-4.46
μ	+50	5.1478	6.3407	8431.1	14.43
	+30	5.1376	6.7731	8107.0	10.03
	+10	5.1376	7.3436	7647.2	3.79
	−10	5.1489	8.1365	7056.5	-4.23
	−30	5.1734	9.3354	6340.7	-13.94
	−50	5.2168	11.4434	5504.1	-25.30
u	+50	5.1573	7.9258	7594.3	3.07
	+30	5.1514	7.8383	7516.6	2.02
	+10	5.1451	7.7494	7423.0	0.75
	−10	5.1384	7.6588	7305.3	-0.85
	−30	5.1317	7.5673	7147.1	-3.00
	−50	5.1261	7.4775	6910.5	-6.21
λ	+50	5.1615	7.9901	7644.8	3.76
	+30	5.1529	7.8603	7537.2	2.30
	+10	5.1452	7.7517	7425.7	0.78
	−10	5.1386	7.6611	7308.5	-0.81
	−30	5.1330	7.5860	7184.0	-2.50
	−50	5.1288	7.5247	7050.2	-4.31
I_r	+50	4.8932	7.5155	6603.7	-10.37
	+30	4.9786	7.5807	6926.7	-5.99
	+10	5.0817	7.6589	7227.5	-1.91
	−10	5.2088	7.7547	7500.4	1.80
	−30	5.3688	7.8743	7737.9	5.02
	−50	5.5766	8.0278	7929.8	7.63
I_e	+50	5.0838	7.4219	5690.3	-22.77
	+30	5.1072	7.5351	6374.3	-13.49
	+10	5.1303	7.6480	7040.9	-4.44
	−10	5.1532	7.7605	7690.8	4.38
	−30	5.1757	7.8727	8325.0	12.99
	−50	5.1983	7.9846	8943.9	21.39
M	+50	6.0923	7.7213	2296.0	-68.84

	+30	5.6541	7.6248	4382.8	-40.51
	+10	5.2874	7.6462	6394.5	-13.21
	−10	5.0281	7.7952	8308.6	12.77
	−30	4.9202	8.0762	10043.9	36.32
	−50	5.0163	8.4745	11516.0	56.30

From Table 2, the following points are noted:
(i) It is seen that the percentage change in the optimal cost is almost equal for both positive and negative changes of all the parameters except S, β, a, M and μ.
(ii) It is observed that the model is more sensitive for a negative change than an equal positive change in the parameter S, a and μ. The trend is reversed in case of parameters β & M.
(iii) The optimal cost increases (decreases) and decreases (increases) with the increase (decrease) and decrease (increase) in the value of all parameters except a, I_r, I_e & M . The trend is reversed for parameters I_r, I_e & M. The average cost decreases when we change a in any direction.
(iv) Model is highly sensitive to changes in $c_{1,}$ β, μ, I_e, M& S and low sensitive to changes in
c', c_3, α, u & λ . It is moderately sensitivity to changes in c_4, c_5, a & I_r.
(v) From the above points, it is clear that much care is to be taken to estimate $c_{1,}$ β, μ, I_e, M& S.

10.Case II: $\boldsymbol{M > t_1}$

(i.e. when the permissible delay period is more than the inventory period)

In this case, since the credit period (M) is greater than the length of period with positive inventory stock of the items, therefore the buyer can use the sale revenue to earn the interest on the sold items with the rate I_e per unit time.

In this situation, interest earned I_E is given by the following relation:

$$I_E = c_5\, I_e \left[\int_0^{t_1} t\, f(t)dt + (M - t_1)\int_0^{t_1} f(t)dt\right]$$

Substituting the value of $f(t)$ and integrating, we get,

$$I_E = \mathrm{c_5 I_e}\left(aM\mathrm{t_1}\mu - \frac{1}{2}at_1^2\mu - \frac{1}{2}aM\mu^2 + \frac{1}{2}a\mathrm{t_1}\mu^2 - \frac{a\mu^3}{6}\right) \quad (21)$$

Also, since permissible delay period is more than inventory period, so interest payable will be zero in this case. Thus,

$$I_p = 0$$

The total cost R_3 in the system in this case in the interval $(0, T)$ is given by

$$R_3 = c' + C_H + C_D + C_S + C_O - I_E$$

In above relation, c' is constant, while C_H, C_D, C_S&C_O are given by the equations (7) to (10).

The average cost K_3 in the system in this case in the interval $(0, T)$ is given by

$$K_3 = \frac{R_3}{T} \tag{22}$$

The optimum values of t_1 and T which minimize average cost K_3 are obtained by using the equations:

$$\frac{\partial K_3}{\partial t_1} = 0 \; and \; \frac{\partial K_3}{\partial T} = 0 \,,$$

Now,

$\frac{\partial K_3}{\partial t_1} = 0$ gives,

$$-ac_4\mu - ac_5\mu + ac_4u^{-t_1\lambda}\mu - ac_3Tu^{-t_1\lambda}\mu + ac_3t_1u^{-t_1\lambda}\mu + \frac{6ac_1t_1\mu}{(1+\beta)(2+\beta)(3+\beta)} + \frac{11ac_1t_1\beta\mu}{(1+\beta)(2+\beta)(3+\beta)} + \frac{6ac_1t_1\beta^2\mu}{(1+\beta)(2+\beta)(3+\beta)} + \frac{ac_1t_1\beta^3\mu}{(1+\beta)(2+\beta)(3+\beta)} - \frac{6ac_1\mu^2}{(1+\beta)(2+\beta)(3+\beta)} - \frac{11ac_1\beta\mu^2}{(1+\beta)(2+\beta)(3+\beta)} - \frac{6ac_1\beta^2\mu^2}{(1+\beta)(2+\beta)(3+\beta)} - \frac{ac_1\beta^3\mu^2}{(1+\beta)(2+\beta)(3+\beta)} + \frac{6ac_1\alpha\mu^{2+\beta}}{(1+\beta)(2+\beta)(3+\beta)} + \frac{5ac_1\alpha\beta\mu^{2+\beta}}{(1+\beta)(2+\beta)(3+\beta)} + \frac{ac_1\alpha\beta^2\mu^{2+\beta}}{(1+\beta)(2+\beta)(3+\beta)} - \frac{ac_1t_1^{\beta}\alpha\mu(-t_1\beta+\mu+\beta\mu)}{1+\beta} - \frac{1}{2}c_5I_e(2aM\mu - 2at_1\mu + a\mu^2) = 0 \tag{23}$$

Also, $\frac{\partial K_3}{\partial T} = 0$ gives,

$$-\frac{cc}{T} + \frac{ac_4t_1\mu}{T} - \frac{6c_1S\mu}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{3ac_1t_1^2\mu}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{11c_1S\beta\mu}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{11ac_1t_1^2\beta\mu}{2T(1+\beta)(2+\beta)(3+\beta)} - \frac{6c_1S\beta^2\mu}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{3ac_1t_1^2\beta^2\mu}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{c_1S\beta^3\mu}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{ac_1t_1^2\beta^3\mu}{2T(1+\beta)(2+\beta)(3+\beta)} + \frac{6ac_1t_1\mu^2}{T(1+\beta)(2+\beta)(3+\beta)} + \frac{11ac_1t_1\beta\mu^2}{T(1+\beta)(2+\beta)(3+\beta)} + \frac{6ac_1t_1\beta^2\mu^2}{T(1+\beta)(2+\beta)(3+\beta)} + \frac{ac_1t_1\beta^3\mu^2}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{2ac_1\mu^3}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{11ac_1\beta\mu^3}{3T(1+\beta)(2+\beta)(3+\beta)} - \frac{2ac_1\beta^2\mu^3}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{ac_1\beta^3\mu^3}{3T(1+\beta)(2+\beta)(3+\beta)} + \frac{6c_1S\alpha\mu^{1+\beta}}{T(1+\beta)(2+\beta)(3+\beta)} + \frac{5c_1S\alpha\beta\mu^{1+\beta}}{T(1+\beta)(2+\beta)(3+\beta)} + \frac{c_1S\alpha\beta^2\mu^{1+\beta}}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{6ac_1t_1\alpha\mu^{2+\beta}}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{5ac_1t_1\alpha\beta\mu^{2+\beta}}{T(1+\beta)(2+\beta)(3+\beta)} - \frac{ac_1t_1\alpha\beta^2\mu^{2+\beta}}{T(1+\beta)(2+\beta)(3+\beta)} + \frac{5ac_1\alpha\beta\mu^{3+\beta}}{2T(1+\beta)(2+\beta)(3+\beta)} + \frac{ac_1\alpha\beta^2\mu^{3+\beta}}{2T(1+\beta)(2+\beta)(3+\beta)} + \frac{ac_1t_1^{1+\beta}\alpha\mu(-t_1\beta+2\mu+\beta\mu)}{T(1+\beta)(2+\beta)} - \frac{c_5(2S-2at_1\mu+a\mu^2)}{2T} - \frac{c_5I_e(-6aMt_1\mu+3a\,t_1^2\mu+3aM\mu^2-3at_1\mu^2+a\mu^3)}{6T} + \frac{ac_3u^{-t_1\lambda}\mu}{T\lambda^2\text{Log}[u]^2} + \frac{ac_4u^{-t_1\lambda}\mu}{T\lambda\text{Log}[u]} + \frac{ac_3t_1u^{-t_1\lambda}\mu}{T\lambda\text{Log}[u]} - \frac{au^{-T\lambda}\mu(c_3+c_4\lambda\text{Log}[u])}{T\lambda^2\text{Log}[u]^2} - \frac{au^{-T\lambda}\mu(c_3+c_4\lambda\text{Log}[u])}{\lambda\text{Log}[u]} = 0 \tag{24}$$

Solving (23) and (24) simultaneously, we can obtain optimal solution for t_1 & T.

11. Numerical Example: To illustrate the model numerically, we use the following parameter

values:

$c' = 100, c_1 = 2.4, c_3 = 5, c_4 = 10, c_5 = 8, , \ \alpha = 0.002, \beta = 4, \ a = 900, \mu = 1,$
$u = e, \ \lambda = 0.1, \ I_r = 0.2, I_e = 0.15, M = 6$ and $S = 8000$

Applying the subroutine FindRoot in Mathematica 8, we obtain the optimal solution for t_1 and T as follows:
$t_1 = 5.1199, T = 7.2681$
Also, the optimal average cost for these parameters is 9862.4

12. Sensitivity Analysis:

Sensitivity analysis is performed by changing (increasing and decreasing) the most of the parameters by 10%, 30% & 50% and taking one parameter at a time, keeping the remaining parameters at their original values. Thus following table is formed:

Table 3

Changing Parameter	**% Change**	t_1	T	**Average Cost**	**% change in average cost**
S	+50	5.5103	9.8423	14754.8	49.61
	+30	5.3686	8.8282	12970.8	31.52
	+10	5.2090	7.7966	10967.1	11.20
	−10	5.0228	6.72667	8673.1	-12.06
	−15	4.9705	6.4497	8041.6	-18.46
	−20	4.9154	6.1675	7382.22	-25.15
c'	+50	5.1205	7.2713	9869.2	0.07
	+30	5.1203	7.2701	9866.5	0.04
	+10	5.1200	7.2688	9863.7	0.01
	−10	5.1198	7.2675	9861.0	-0.01
	−30	5.1196	7.2662	9858.2	-0.04
	−50	5.1194	7.2649	9855.5	-0.07
c_1	+50	4.7522	7.9425	12575.0	27.50
	+30	4.8804	7.6776	11563.4	17.25
	+10	5.0323	7.4064	10458.2	6.04
	−10	5.2176	7.1280	9231.7	-6.39
	−30	5.4539	6.8417	7839.2	-20.51
	−50	5.7774	6.5483	6197.7	-37.16
c_3	+50	5.1548	6.6993	10163.4	3.05
	+30	5.1435	6.8830	10062.6	2.03
	+10	5.1289	7.1208	9937.3	0.76
	−10	5.1094	7.4412	9777.0	-0.87
	−30	5.0821	7.8974	9564.2	-3.02
	−50	5.0403	8.6032	9266.7	-6.04
c_4	+50	5.1317	6.6656	10398.8	5.44
	+30	5.1249	6.8954	10202.4	3.45
	+10	5.1209	7.1398	9981.8	1.21
	−10	5.1198	7.4010	9736.7	-1.27

	−30	5.1220	7.6817	9466.6	-4.01
	−50	5.1280	7.9848	9171.1	-7.01
c_5	+50	5.4651	7.9118	10574.8	7.22
	+30	5.3068	7.6139	10250.1	3.93
	+10	5.2176	7.4479	10065.2	2.06
	−10	5.0120	7.0722	9638.6	-2.27
	−30	4.8912	6.8569	9390.3	-4.79
	−50	4.5955	6.3477	8797.3	-10.80
α	+50	4.8616	7.1623	10290.3	4.34
	+30	4.9525	7.1984	10135.1	2.77
	+10	5.0590	7.2423	9959.6	0.99
	−10	5.1873	7.2973	9757.4	-1.06
	−30	5.3479	7.3697	9518.1	-3.49
	−50	5.5616	7.4716	9223.0	-6.48
β	+50	3.5438	6.6635	12839.1	30.18
	+30	4.0379	6.8150	11733.8	18.98
	+10	4.7068	7.0771	10496.3	6.43
	−5	5.3455	7.3809	9553.5	-3.13
	−10	5.5814	7.5047	9256.5	-6.14
	−15	5.8237	7.6378	8977.0	-8.98
a	+20	4.9523	6.3551	9387.0	-4.82
	+15	4.9912	6.5578	9533.4	-3.34
	+10	5.0319	6.7760	9662.1	-2.03
	−10	5.2185	7.8554	9978.0	1.17
	−30	5.4618	9.4837	9900.9	0.39
	−50	5.8161	12.3969	9261.6	-6.09
μ	+40	5.0442	6.0352	9298.6	-5.72
	+30	5.0523	6.2790	9530.3	-3.37
	+10	5.0888	6.8843	9817.3	-0.46
	−10	5.1621	7.7291	9832.5	-0.30
	−30	5.2928	9.0132	9506.3	-3.61
	−50	5.5290	11.2847	8708.6	-11.70
u	+50	5.1297	7.3695	10010.2	1.50
	+30	5.1260	7.3290	9962.9	1.02
	+10	5.1221	7.2883	9901.0	0.39
	−10	5.1177	7.2482	9816.5	-0.47
	−30	5.1132	7.2101	9693.0	-1.72
	−50	5.1094	7.1785	9491.7	-3.76
λ	+50	5.1322	7.3994	10038.9	1.79
	+30	5.1270	7.3391	9975.8	1.15
	+10	5.1222	7.2894	9902.9	0.41
	−10	5.1178	7.2491	9818.9	-0.44
	−30	5.1141	7.2175	9722.7	-1.42
	−50	5.1112	7.1940	9612.7	-2.53

I_r	+50	5.1199	7.2681	9862.4	0.00
	+30	5.1199	7.2681	9862.4	0.00
	+10	5.1199	7.2681	9862.4	0.00
	−10	5.1199	7.2681	9862.4	0.00
	−30	5.1199	7.2681	9862.4	0.00
	−50	5.1199	7.2681	9862.4	0.00
I_e	+50	5.0761	6.7355	8552.7	-13.28
	+30	5.0936	6.9494	9088.0	-7.85
	+10	5.1111	7.1621	9607.9	-2.58
	−10	5.1287	7.3739	10113.3	2.54
	−30	5.1464	7.5849	10605.2	7.53
	−50	5.1642	7.7954	11084.2	12.39
M	+50	5.2017	6.5411	7674.0	-22.19
	+30	5.1703	6.8389	8581.9	-12.98
	+10	5.1372	7.1273	9445.7	-4.23
	−5	5.1111	7.3377	10067.1	2.08
	−10	5.1023	7.4068	10269.6	4.13
	−15	5.0933	7.4753	10469.8	6.16

From Table 3, the following points are noted:

(i) It is seen that the percentage change in the optimal cost is almost equal for both positive and negative changes of all the parameters except S,c_1 and μ.

(ii) It is observed that the model is more sensitive for a negative change than an equal positive change in the parameter c_1 and μ. The trend is reversed in case of parameter S.

(iii) The optimal cost increases (decreases) and decreases (increases) with the increase (decrease) and decrease (increase) in the value of all parameters except a, μ, I_e & M . The trend is reversed for parameters I_e & M. The average cost decreases when we change a & μ in any direction. The average optimal cost is remain unaffected by change in parameter I_r.

(iv) Model is highly sensitive to changes in S, c_1, β, I_e& M and low sensitive to changes in c', u & λ . It is moderately sensitivity to changes in c_3, c_4, c_5, α, a & μ. Model has no sensitivity for the parameter I_r.

(v) From the above points, it is clear that much care is to be taken to estimate S, c_1, β, I_e& M.

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