A Study of Bulk Queues under Poisson Process

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Abstract

Suppose that customers arrive at a counter, according to a homogeneous Poisson process and are served in groups, according to the following policy: If there are less than L customers waiting at the time of a departure, the server must wait until there are L customers present, whereupon he serves them together. If there are L or more, but less than ν $(\nu \geqq L)$ customers waiting, all are served together. If there are ν or more customers waiting, a group of ν customers are served and the others must wait. The service times of successive groups are assumed to be conditionally independent given the bulk sizes, but may depend on their magnitude. We obtain a description of the output process, the queue length in discrete time, the distribution of the busy period the queue length in continuous time.

KEYWORDS: Poisson process, Bulk queue, Exponential, The queue length, Generating function.

1. Introduction

This paper deals with the study of bulk queuing model with the fixed batch size 'α' and customers arrive to the system with Poisson fashion with the rate λ and are severed exponentially with the rate μ. On formulating the mathematical model, we suppose that the expressions for mean waiting time in the queue, mean time spent in the system, mean number of customers work pieces in the queue and in the system by using generating function method.

The queuing process refers to the number of queues, and their respective lengths. The number of queues depends upon payout of a service system. Thus may be a single queue or multiple queues. For this queuing system we assume that customers arrive according to a poisson process an average rate of customers per unit of time and served on first come, first served basis at any of the servers. The most common stochastic models assume that the arrival rate and service rate follow a poisson distribution and p_ν be steady-state probability that there are ν customers the queuing system.

In these mathematical queuing models, customers arrive randomly according to a Poisson process and form a waiting line. In this situation, the work-pieces arrive at a typical manufacturing machine centre in batches and they leave in batches. A batch consists of identical work-pieces that are processed and then transported in batches for further processing. Such a situation can be modeled as queues with bulk arrivals. There is a discipline within the mathematical theory of probability, called a bulk queue (also called batch queue) where customers are served in groups of random size. There has been performed a number of researches on the bulk queuing system. Some of these previous works are briefly discussed here. Abolnikov et al. [1] threw the light on the study a class of bulk queuing systems with a compound Poisson input modulated by semi-Markov process, multilevel control service time and queue length dependent, service delay discipline. Bagyam et al. [2] analyzed bulk arrival general service retrial queuing system where server provides two phases of service-essential and optimal. After each service completion, the server searches for customers in the orbit. Customers may balk or renege at particular times and accidental and active breakdown of the server is considered. Baruah et al. [3] again studied the behavior of a batch arrival queuing system equipped with a single server providing general arbitrary service to customers with different service rates in two fluctuating modes of service. In addition, the server is subject to random breakdown. As soon as the server faces breakdown, the customer whose service is interrupted comes back to the head of the queue.

Baruah et al. [4] aimed at studying a queuing model with two stage heterogeneous service where customer arrival in batches and has a single server providing service in two stages, one after the other in succession. Chen [5] developed a nonlinear programming approach to derive the membership functions of the steady-state performance measures in bulk arrival queuing systems with varying batch sizes, in that the arrival rate and service rate are fuzzy numbers.

Haridass and Arunmuganathum [6] studied the operating characteristics of a $M^x/G/1$ queuing system with unreliable server and single vacation is analyzed. The server is subjected to fail, while it is on, and the arrival rate depends on the up and down states of the server. Jain and Bhargava [7] dealt with the analysis of unreliable server bulk arrival retrial queue with two class non-preemptive priority subscribers. The two types of subscribers arrive according to Poisson flow in which priority is assigned to class one, and class two subscribers are of non-priority types. Jeeva and Rathnakumari [8] dealt with a mathematical non-linear programming method to construct the membership function of the system characteristics of a *M/G/1*, bulk arrival queues with server vacations and feedback facility, in which arrival rate, service time, batch size departure probability, and vacation time are all fuzzy numbers. Khalaf et al. [9] studied a batch arrival queuing system in which the server may face occasional random breakdowns. The repair process does not start immediately after a breakdown and there is a delay time waiting for repairs to start. Khalaf et al. [10] dealt a queuing system with four different main server interruptions and a stand-by server replaces the main server during any potential stop. The main server has five statues in this system where it is either services the customers or it does not work.

Maurya [11] studied some significant performance measures of a bulk arrival retrial queuing model with two phase service where first phase service is essential and the next second phase service is optional. Pang and Whitt [12] were motivated by large-scale service systems, and considered an infinite-server queue with batch arrivals, where the service times are dependent within each batch. We allow the arrival rate of batches to be time-varying as well as constant. Shinde and Patankar [13] investigated the state dependent bulk service queue with balking, reneging and multiple vacations where (a-1) arrivals waiting in the queue, server will wait for some time (called the change over time), in spite of going for a vacation.

Sikdar et al.[14] contributed to the analysis of a batch arrival single-server queue with renewal input and multiple exponential vacations. Singh et al. [15] investigated a single-server Poisson input queuing model, wherein arrivals of units are in bulk. The arrival rate of the units is state dependent, and service time is arbitrary distributed. It is also assumed that the system is subject to breakdown, and the failed server immediately joins the repair facility which takes constant duration to repair the server.

2. Main characteristics of queuing System :

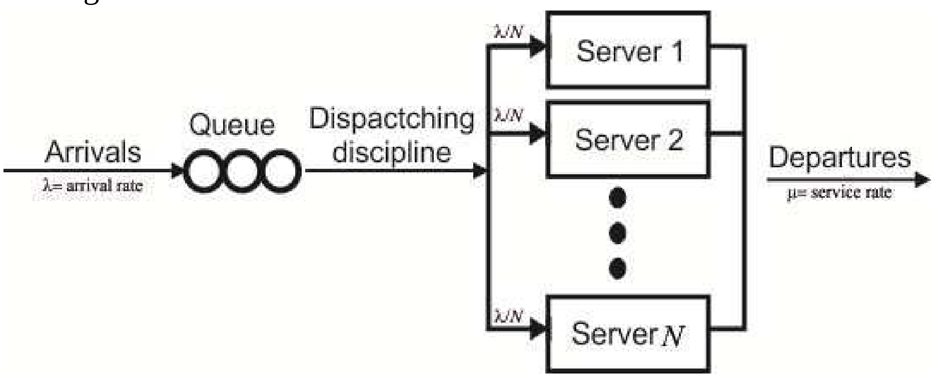
1. **Arrival pattern of customers:** In queuing the arrival process is usually stochastic. As a result it is necessary to determine the probability distribution of the interarrival times (times between successive customer arrivals) as well. Also customers can arrive in individually or simultaneously (batch or bulk arrivals).

2. **Service pattern of customers:** As in arrivals, a probability distribution is needed for describing the sequence of customer service time. Service may also be single or batch. The service process may depend on the number of customers waiting in queue for service. In this process, it is called state dependent service.

3. **Queue discipline:** Queue discipline refers to the manner in which customers are selected for service when a queue has formed. The default is FCFS i.e. is first come first served. Some others are LCFS (last come first served), RSS (random service selection) i.e. selection for service in random other order independent of the time of arrival and there are other priority systems where customers are given priorities upon entering the system, ones with higher priority are selected first.

4. **System capacity:** A queuing system can be finite or infinite. In certain queuing process there is a limitation on the length of the queue i.e. customers are not allowed to enter if the queue has reached a certain length. These are called finite queuing systems. If there is no restriction on the length of the queue then it is called an infinite queuing system.

5. **Number of service channels:** A queuing system can be single or a multiserver system. In a multiserver queuing system there are several parallel servers running to serve a single line.



A Multiserver Queuing System

6. **Number of service stages:** A queuing system may have only a single stage of service.

But as an example of a multistage queuing system consider the physical examination procedure, where each patient proceeds through various stages of medical examination, like throat check up, eye test, blood test etc.

3. Formulation of Mathematical Model:

Now we are having some notational convention in this paper is as follows:

□ = arrival rate □ = service rate

ρ = traffic intensity $\alpha =$ batch size

W_S = waiting time in the system W_q = waiting time in the queue

L_S = mean number of work piece in the system

L_q = mean number of work piece in the queue

The steady-state transition diagram for our model is shown in figure 1.

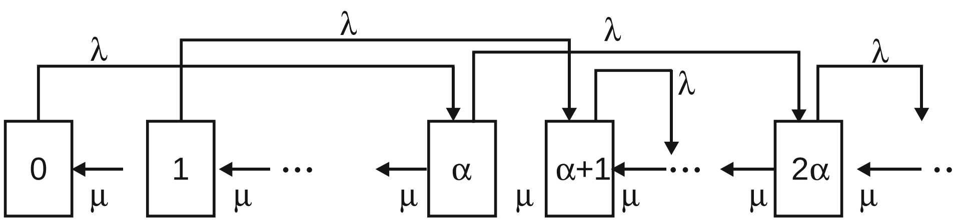


Figure 1: Transition diagram for all the states

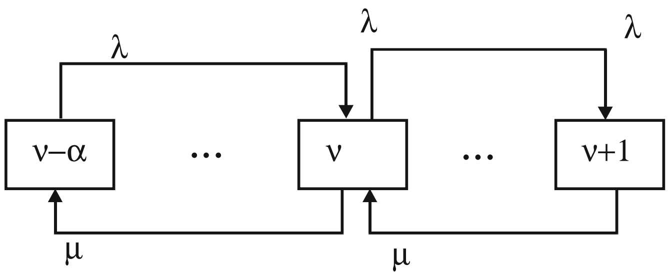


Figure 2: Transition diagram for general states

Let $p_{\nu(\nu=0,1,\ \ldots,n)}$ be steady-state probability that there are ν customers in the queuing system. The state balance equation can be written with the help of following diagram as follows:

$(\lambda+\mu)p_\nu$ = Flow rate out of ν state

$\lambda p_{\nu-\alpha}+\mu p_\nu,\ \nu\neq 1$= Flow rate into ν state

At equilibrium, we have

$(\lambda+\mu)p_\nu=\lambda p_{\nu-\alpha}+\mu p_{\nu+1};\nu\geq\alpha$ ....(1)

$(\lambda+\mu)p_\nu=\mu p_{\nu+1};\nu<\alpha,\nu=1,\ 2,\ \ldots\ldots.,\alpha-1$ ....(2)

From (2), $\lambda p_0=\mu p_1$

i.e.,

$\mu p_1 - \lambda p_0 = 0; \qquad \nu = 0$ ....(3)

Sum of multiplying (1) by x^ν under the summation from α to ∞ and multiplying (2) by x^ν under the summation from $\nu = 1$ to $\alpha - 1$ and adding these with (3) we get,

$$\sum_{\nu=\alpha}^{\infty} \lambda x^\nu p_{\nu-\alpha} + \sum_{\nu=\alpha}^{\infty} \mu x^\nu p_{\nu+1} - \sum_{\nu=\alpha}^{\infty} (\lambda+\mu) x^\nu p_\nu + \sum_{\nu=1}^{\alpha-1} \mu x^\nu p_{\nu+1}$$

$$-\sum_{\nu=1}^{\alpha-1} (\lambda+\mu) x^\nu p_\nu + (\mu p_1 - \lambda p_0) = 0$$

$$\Rightarrow x^\alpha \sum_{\nu=\alpha}^{\infty} \lambda x^{\nu-\alpha} p_{\nu-\alpha} + \sum_{\nu=1}^{\infty} \mu x^\nu p_{\nu+1} - \sum_{\nu=1}^{\infty} (\lambda+\mu) x^\nu p_\nu + (\mu p_1 - \lambda p_0) = 0 \quad ..(4)$$

We define generating function of p_ν as:

$$G(x) = \sum_{\nu=0}^{\infty} x^\nu p_\nu = x^0 p_0 + \sum_{\nu=1}^{\infty} x^\nu p_\nu$$

i.e., $G(x) - p_0 = \sum_{\nu=1}^{\infty} x^\nu p_\nu$ ....(5)

$$\Rightarrow \lambda x^\alpha \sum_{\nu=\alpha}^{\infty} x^{\nu-\alpha} p_{\nu-\alpha} + \mu x^{-1} \left[\sum_{\nu=-1}^{\infty} x^{\nu+1} p_{\nu+1} - x^0 p_0 + x^1 p_1 \right] -$$

$$\sum_{\nu=0}^{\infty} (\lambda+\mu) x^\nu p_\nu - (\lambda+\mu) p_0 + \mu p_1 - \lambda p_0 = 0$$

$$\Rightarrow \lambda x^\alpha G(x) + \mu x^{-1} \left[G(x) - p_0 - x p_1 \right] - (\lambda+\mu) G(x) + (\lambda+\mu) p_0 + \mu p_1 - \lambda p_0 = 0$$

$$\Rightarrow G(x) \left[\lambda x^\alpha + \mu x^{-1} - (\lambda+\mu) \right] + \mu p_0 - \mu x^{-1} p_0 = 0$$

$$\Rightarrow G(x) \frac{\lambda x^{\alpha+1} + \mu - (\lambda+\mu) x}{x} + \mu \left(1 - \frac{1}{x} \right) p_0 = 0$$

$$\Rightarrow G(x) = \frac{\mu (1-x) p_0}{\mu - (\lambda+\mu) x + \lambda x^{\alpha+1}} = \frac{(1-x) \mu p_0}{\mu - \mu x - \lambda x + \lambda x x^\alpha}$$

$$\Rightarrow G(x) = \frac{\mu p_0 (1-x)}{(1-x)\mu - \lambda x (1-x) \left[1 + x + x^2 + ... + x^{\alpha-1} \right]} = \frac{\mu p_o}{\mu - \lambda x \sum_{j=0}^{\alpha-1} x^j}$$

$$\Rightarrow G(x) = \frac{\mu p_0}{\mu - \lambda \sum_{j=1}^{\alpha} x^j} \quad .....(6)$$

$$\Rightarrow G(x)(at\ x=1)=\frac{\mu p_0}{\mu-\lambda\sum_{j=1}^{\alpha}(1)^j}=\frac{\mu p_0}{\mu-\lambda\alpha}$$

$$1=\frac{\mu p_0}{\mu-\lambda\alpha}\left[Since\sum_0^{\infty}x^v p_v=G(x)\Rightarrow G(x)(at\,x=1)=\sum_{v=0}^{\infty}1p_v=1\right]$$

Since $\rho=\frac{\lambda\alpha}{\mu}$ is the utilization of the server, we have

$$1=\frac{p_0}{1-\frac{\lambda\alpha}{\mu}}=\frac{p_0}{1-\rho}$$

$$\therefore p_0=1-\rho$$

Putting the value of $p_0=1-\rho$ in equation (6) then we get

$$G(x)=\frac{\mu(1-\rho)}{\mu-\lambda\sum_{j=1}^{\alpha}x^j}$$

4. Measurement of the queuing System

In this section, we calculate the mathematical expressions for mean number of work pieces and mean waiting time in the system. Also, average number work piece and average waiting time of work piece in the queue has been calculated. These four different mathematical expressions of this model are termed as the performance measure of the system and are described as follows:

(i) Mean number of work pieces in the system

$$L_s=\sum_{v=0}^{\infty}v\,p_v=\frac{d}{dx}G(x)\ at\,x=1$$

$$=\frac{d}{dx}\left[\frac{\mu(1-\rho)}{\mu-\lambda\sum_{j=1}^{\alpha}x^j}\right]at\ x=1$$

$$=\mu(1-\rho)\frac{d}{dx}\left[\mu-\lambda\sum_{j=1}^{\alpha}x^j\right]^{-1}at\,x=1$$

$$=\mu(1-\rho)(-1)\left[\mu-\lambda\sum_{j=1}^{\alpha}x^j\right]^{-2}\cdot(-\lambda)\sum_{j=1}^{\alpha}jx^{j-1}$$

$$= \frac{\mu(1-\rho)\lambda \sum_{j=1}^{\alpha} j x^{j-1}}{\left[\mu - \lambda \sum_{j=1}^{\alpha} x^{j}\right]^2}, \; at\; x = 1$$

$$= \frac{\lambda\mu(1-\rho)\left(\frac{1+\alpha}{2}\right)}{(\mu-\lambda\alpha)^2} = \frac{\rho(1+\alpha)}{2(1-\rho)} \qquad .....(4.1)$$

(ii) Mean waiting time of work piece in the system

$$L_s = \lambda\alpha W_s$$

$$W_s = \frac{L_s}{\lambda\alpha} = \frac{1+\alpha}{2\mu(1-\rho)} \qquad ....(4.2)$$

(iii) Average number work piece in the queue is

$$L_q = L_S - \rho = \frac{\rho(\alpha-1+2\rho)}{2(1-\rho)} \qquad ....(4.3)$$

(iv) Average waiting time of work piece in the queue is

$$W_q = W_s - \frac{1}{\mu} = \frac{1+\alpha}{2\mu(1-\rho)} - \frac{1}{\mu} = \frac{\alpha+2\rho-1}{2\mu(1-\rho)} \qquad ......(4.4)$$

5. Conclusion

In this paper, the explicit formula obtained have their real life applications such as vehicle dispatching strategies for bulk arrival which consist of some combination of vehicle holding and cancellation strategies. We have obtained some performance measures explicitly by using the probability generating function method which may draw the attention of queuing theorist. These numerical results show that our model has its practical applicability in several real world situations. The queuing model under our study has widespread applications in the manufacturing system, transportation system, assembly line system and in overall supply chain management systems.

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