# An Inventory Model for Decaying Items with Ramp Type Demand and Backlogging Under Inflation

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## Abstract

In this paper, an inventory model is developed for decaying items which have backlogs under inflation. The demand rate of ramp type is used for fluctuating demand pattern which is time dependent also. The demand pattern of seasonal and fashionable products are considered which is increasing at start, steady after some time, decreasing when approached to season ending and become asymptotic at last. Shortages are allowed and effects of inflation are included in the model. Finally, numerical examples are presented implementing sensitivity analysis of optimal solution which show developed model and solution techniques.

**KEYWORDS:** Inventory Model, Ramp type Demand, Inflation, Backlogging

## 1. Introduction:

Demand is the most volatile of all the market forces, as it is the least controlled by management personnel. Even a slight change in the demand pattern for any particular item causes a lot of havoc with the market concerned. Overall, it means that every time the demand for any commodity goes a noticeable change, the inventory manager has to reformulate the complete logistics of management for that item. Here, one thing becomes very apparent, even if the firm is able to take the jolt of changed customer's preference, it will not be able to take the sweep of formulating a new inventory management theory every time. Previously two types of time dependent demands i.e. linear and exponential have been studied.

The main limitation in linear time-dependence of demand rate is that it implies a uniform change in the demand rate per unit time. This rarely happens in the case of any commodity in the market. On the other hand, an exponential rate of change in demand is extraordinarily high and the demand fluctuation of any commodity in the real market cannot be so high. It concludes that demand rate never has been constant or an increasing function or decreasing function of time. To observe these fluctuation in demand pattern, ramp type demand rate taken into consideration. At the beginning of the season it increases, in the mid of the season it becomes steady and towards the end of the season it decreases and becomes asymptotic. The demand rate increases with time up to a certain time and then ultimately stabilizes and becomes constant. The demand pattern assumed here is found to occur not for all types of seasonal products but also for fashionable goods, electronic items etc. It feels that a more realistic approach is to think of accelerated growth (or decline) in the demand rate can be best represented by a time dependent ramp type demand rate.

The ramp type demand is very commonly seen when some fresh come to the market. In case of ramp type demand rate, the demand increases linearly at the beginning and then the market grows into a stable stage such that the demand becomes a constant until the end of the inventory cycle. **Hill (1995)** first proposed a time dependent demand pattern by considering it as the combination of two different types of demand such as increasing demand followed by a constant demand in two successive time periods over the entire time horizon and termed it as ramp-type time dependent demand pattern. He derived the exact solution to compare with the Silver-Meal heuristic**. Mandal and Pal (1998)** extended the inventory model with ramp type demand for deteriorating items and allowing shortage. **Wee and Wang (1999)** studied a production-inventory model for deteriorating items with time varying demand, finite production rate and shortages, over a known planning horizon. **Wu and Ouyang (2000)** extended the inventory model to include two different replenishment policies: (a) models starting with no shortage and (b) models starting with shortage. In **(2001) Wee and Law** studied an EOQ model with Weibull deterioration, price dependent demand considering the time value of money. **Wu (2001)** further investigated the inventory model with ramp type demand rate such that the deterioration followed the Weibull distribution deterioration and partial backlogging. **Goyal and Giri (2003)** considered the production-inventory problem in which the demand, production and deterioration rates of a product were assumed to vary with time. Shortages of a cycle were allowed to be partially backlogged. **Giri *et al.* (2003)** extended the ramp type demand inventory model with a more generalized Weibull deterioration distribution. They noted a demand pattern for fashionable products which initially increases exponentially with time for a period of time after that it becomes steady rather than increasing exponentially. But for fashionable products as well as for seasonal products the steady demand after its exponential increment with time never be continued indefinitely. Rather it would be followed by exponential decrement with respect to time after a period of time and becomes asymptotic in nature. Thus the demand may be illustrated by three successive time periods classified time dependent ramp-type function, in which in the first phase the demand increases with time and after that it becomes steady and towards the end in the final phase it decreases and becomes asymptotic. An economic production quantity model for deteriorating items was discussed by **Teng and Chang (2005)**. **Manna and Chaudhuri (2006)** have developed a production inventory model with ramp-type two time periods classified demand pattern where the finite production rate depends on the demand. The demand increases linearly with time in the first period of time, and then it becomes steady for the remaining time of the production cycle. They noted that this type of demand pattern is generally followed by new brand of consumer goods coming to the market. Recently **Deng et al. (2007)** investigated an inventory model to amend the incompleteness of the models given by **Mandal and Pal (1998), Wu and Ouyang (2000). Dye et al. (2007)** analyzed an inventory model for deteriorating items in which demand and deterioration rate are continuous and differential function of price and time respectively, and shortages are fully backlogged. **Deng *et al.* (2007)** point out some questionable results of **Mandal and Pal (1998)** and **Wu and Ouyang (2000)**, and then resolved the similar problem by offering a rigorous ad efficient method to derive the optimal solution. **Panda et al. (2008)** developed a single item order level inventory model for a seasonal product with ramp-type demand rate. Shortages are not allowed over a finite time horizon. **Sugapriya (2008)** studied an EPQ model for non-instantaneous deteriorating item in which holding cost varies with time. It is a production problem of non-instantaneous deteriorating item in which production and demand rate are constant. **Manna et al. (2009)** developed an EOQ model for non-instantaneous deteriorating items with demand rate as time-dependent. In the model, shortages are allowed and partially backlogged. **Skouri et al. (2009)** determined an inventory model with general ramp type demand rate, time dependent (Weibull) deterioration rate and partial backlogging.

In this paper an order level inventory system for non-instantaneous decaying items with ramp type demand rate has been developed. The three-parameter Weibull distribution rate of deterioration was applied to on-hand inventory. Shortages of a cycle were allowed to be partially backlogged. The backlogging rate is any non-increasing function of the waiting time up to the next replenishment. The model is fairly general in practice, as the demand of some items such as fashionable items increases up to the time point of its stabilization. The purpose of this study is to determine an optimal ordering policy for minimizing the expected total relevant inventory cost. Numerical examples are also discussed in the concluding phase of the model. Sensitivity analysis was applied on the parameter effects of the optimal time and the total cost.

## 2. Assumptions and Notations:

We develop the inventory model under the following assumptions and notations.

**Assumptions:**

1. The demand rate $D(t)$ is a ramp type function of time given by

$$D(t)=\begin{cases} f(t) & t<\mu \\ f(\mu) & t\geq\mu \end{cases}$$

   $f(t)$ is a positive, continuous function of $t\in[0, T]$.

2. The life time of the item in inventory is random and follows a Weibull distribution having probability density function

$$f(t)=\alpha\beta(t-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta+1}} \qquad \alpha,\beta>0,\ t>\gamma$$

   Hence the instantaneous rate of deterioration of the on hand inventory is

$$\theta(t)=\frac{f(t)}{1-F(t)}=\alpha\beta(t-\gamma)^{\beta-1} \qquad \alpha,\beta>0,\ t>\gamma$$

   $F(t)$ being the distribution function of Weibull distribution.

3. There is no replacement of deteriorated items during the period $T$.
4. Shortages are allowed and are backlogged at a rate $g(x)$. Which is non-increasing function of $x$ i.e. $g'(x)\leq 0$ with the assumption $0\leq g(x)\leq 1$, and $x$ is the waiting time up to the next replenishment, i.e. $x = T–t_1$, moreover we assume that $g(x)$

satisfies the relation $Rg(x) + g'(x) \geq 0$, where $g'(x)$ is the derivative of $g(x)$. The cases $g(x) = 0$ (or 1) correspond to complete backlogging (or complete lost sales) models.

**Notations:**

$T$ Scheduling period (cycle) which is constant

$t_1$ The time when inventory level reaches zero.

$A$ Ordering cost.

$c_h$ Holding cost per unit per unit of time.

$c_s$ Shortage cost per unit per unit of time.

$c_d$ Deterioration cost per unit per unit of time.

$c_o$ Opportunity cost per unit per unit of time due to lost sale.

$\mu$ Parameter of the ramp type demand function (time point).

$I_m$ Maximum inventory level

**3. The mathematical formulation of the model:**

**Case: I ($t_1 \leq \mu$)**

The replenishment at the beginning of the cycle brings the inventory level up to $I_m$ There is no deterioration during interval $[0, \gamma]$, the depletion of inventory during this interval is only due to demand. During the interval $[\gamma, t_1]$ the inventory level decreases with the combined effect of demand and deterioration, and falls to zero at $t = t_1$, thereafter shortages occur during the period $[t_1, T]$, which are partially backlogged. The backlogged demand is satisfied at the next replenishment.

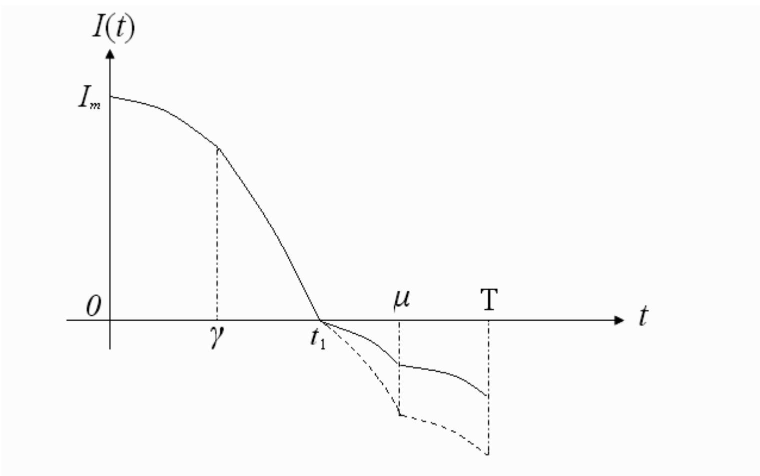


**Fig.1. Inventory Level When $T_1 \leq \mu$**

Let $I(t)$ be the inventory level at any time $t$, $0 \le t \le T$. The differential equations governing the instantaneous state of inventory level $I(t)$ in the interval $[0, T]$ are given by:

$$I'(t) = -f(t) \qquad 0 \le t \le \gamma \qquad (1)$$

$$I'(t) + \alpha\beta(t-\gamma)^{\beta-1}I(t) = -f(t) \text{ with } I(t_1) = 0 \qquad \gamma \le t \le t_1 \qquad (2)$$

$$I'(t) = -f(t)\, g(T-t) \qquad t_1 \le t \le \mu \qquad (3)$$

$$I'(t) = -f(\mu)\, g(T-t) \qquad \mu \le t \le T \qquad (4)$$

**3.1 The solutions of equations (1) – (4) are given by**

$$I(t) = \int_t^{\gamma} f(x)dx + \int_{\gamma}^{t_1} f(x)e^{\alpha(x-\gamma)^{\beta}}dx \qquad (5)$$

$$I(t) = e^{-\alpha(t-\gamma)^{\beta}} \int_t^{t_1} f(x)e^{-\alpha(x-\gamma)^{\beta}}dx \qquad (6)$$

$$I(t) = -\int_{t_1}^{t} f(x)g(T-x)dx \qquad (7)$$

$$I(t) = -f(\mu)\int_{\mu}^{t} g(T-x)dx - \int_{t_1}^{\mu} f(x)g(T-x)dx \qquad (8)$$

The holding cost of the inventory during $[0, t_1]$ is

$$H = \int_0^{\gamma}\left[\int_t^{\gamma}(c_h+\phi t)f(x)e^{-rt}dx + \int_{\gamma}^{t_1}(c_h+\phi t)f(x)e^{\alpha(x-\gamma)^{\beta}}e^{-rt}dx\right]dt$$

$$+(c_h+\phi t)\int_{\gamma}^{t_1} e^{-\alpha(t-\gamma)^{\beta}}\left[\int_t^{t_1} f(x)e^{-\alpha(x-\gamma)^{\beta}}dx\right]e^{-rt}\,dt \qquad (9)$$

**3.2 Present value of deterioration cost during [0, $t_1$] is**

$$D = c_d\int_{\gamma}^{t_1} f(t)e^{-\alpha(t-\gamma)^{\beta}}e^{-rt}dt - c_d\int_{\gamma}^{t_1} f(t)e^{-rt}dt \qquad (10)$$

**3.3 Present value of shortage cost during [$t_1$, $T$] is**

$$S = c_s\int_{t_1}^{\mu}\left[\int_{t_1}^{t} f(x)g(T-x)dx\right]e^{-rt}dt$$

$$+c_s\int_{\mu}^{T}\left[f(\mu)\int_{\mu}^{t} g(T-x)dx - \int_{t_1}^{\mu} f(x)g(T-x)dx\right]e^{-rt}dt \qquad (11)$$

**3.4 Present value of opportunity cost due to lost sale during [$t_1$, $T$] is**

$$O = c_o\int_{t_1}^{\mu}\left[1-g(T-t)\right]f(t)e^{-rt}dt + c_o f(\mu)\int_{\mu}^{T}\left[1-g(T-t)\right]e^{-rt}dt \qquad (12)$$

### 3.5 Present value of the total cost

$TC_1(t_1) = H + D + S + O$

$$= \int_0^{\gamma}\left[\int_t^{\gamma}(c_h+\phi t) f(x)e^{-rt}dx + \int_{\gamma}^{t_1}(c_h+\phi t) f(x)e^{\alpha(x-\gamma)^{\beta}}e^{-rt}dx\right]dt$$

$$+(c_h+\phi t)\int_{\gamma}^{t_1} e^{-\alpha(t-\gamma)^{\beta}}\left[\int_t^{t_1} f(x)e^{-\alpha(x-\gamma)^{\beta}}dx\right]e^{-rt}dt \qquad +$$

$$c_d\int_{\gamma}^{t_1} f(t)e^{-\alpha(t-\gamma)^{\beta}}e^{-rt}dt - c_d\int_{\gamma}^{t_1} f(t)e^{-rt}dt + c_s\int_{t_1}^{\mu}\left[\int_{t_1}^{t} f(x)g(T-x)dx\right]e^{-rt}dt$$

$$+c_s\int_{\mu}^{T}\left[f(\mu)\int_{\mu}^{t} g(T-x)dx + \int_{t_1}^{\mu} f(x)g(T-x)dx\right]e^{-rt}dt$$

$$+c_o\int_{t_1}^{\mu}\left[1-g(T-t)\right]f(t)e^{-rt}dt + c_o f(\mu)\int_{\mu}^{T}\left[1-g(T-t)\right]e^{-rt}dt$$

(13)

**Case: II ($t_1 \geq \mu$)**

In this case the differential equations governing instantaneous state of $I(t)$ are given by:

| | | | |
|---|---|---|---|
| $I'(t) = -f(t)$ | with $I(\gamma_-) = I(\gamma_+)$ | $0 \leq t \leq \gamma$ | (14) |
| $I'(t) + \alpha\beta(t-\gamma)^{\beta-1}I(t) = -f(t)$ | with $I(\mu_-) = I(\mu_+)$ | $\gamma \leq t \leq \mu$ | (15) |
| $I'(t) + \alpha\beta(t-\gamma)^{\beta-1}I(t) = -f(\mu)$ | with $I(t_1) = 0$ | $\mu \leq t \leq t_1$ | (16) |
| *and* $I'(t) = -f(\mu)\, g(T-t)$ | | $\mu \leq t \leq T$ | (17) |

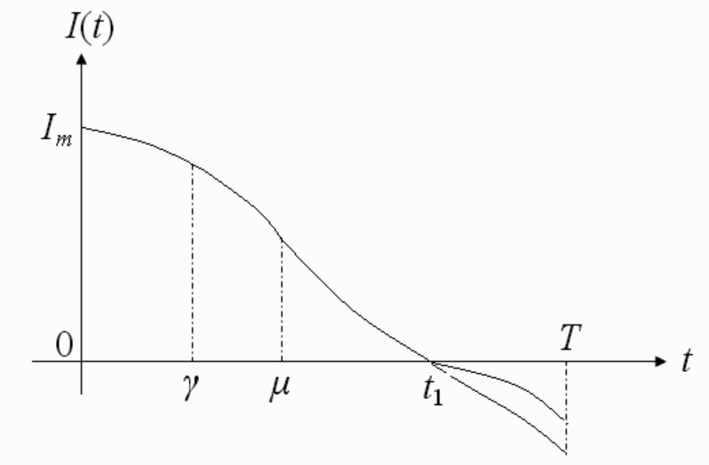


**Fig.2. Inventory level when $T_1 > \mu$**

Solutions of (14) – (17) are given by:

$$I(t)=\int_t^{\gamma} f(x)dx+\int_{\gamma}^{\mu} f(x)e^{\alpha(x-\gamma)^{\beta}}dx+f(\mu)\int_{\mu}^{t_1} e^{\alpha(x-\gamma)^{\beta}}dx \quad 0\le t\le \gamma \tag{18}$$

$$I(t)=e^{-\alpha(t-\gamma)^{\beta}}\left[\int_t^{\mu} f(x)e^{\alpha(x-\gamma)^{\beta}}dx+f(\mu)\int_{\mu}^{t_1} e^{\alpha(x-\gamma)^{\beta}}dx\right] \quad \gamma\le t\le \mu \tag{19}$$

$$I(t)=e^{-\alpha(t-\gamma)^{\beta}} f(\mu)\int_t^{t_1} e^{\alpha(x-\gamma)^{\beta}}dx \qquad \mu\le t\le t_1 \tag{20}$$

and

$$I(t)=-f(\mu)\int_{t_1}^{t} g(T-x)dx \qquad \mu\le t\le T \tag{21}$$

**3.6 Present value of holding cost of the inventory during [0, $t_1$] is**

$$\begin{aligned}H=(c_h+\phi t)\Bigg[&\int_0^{\gamma}\left[\int_t^{\gamma} f(x)dx+\int_{\gamma}^{\mu} f(x)e^{\alpha(x-\gamma)^{\beta}}dx+f(\mu)\int_{\mu}^{t_1} e^{\alpha(x-\gamma)^{\beta}}dx\right]e^{-rt}dt\\ &+\int_{\gamma}^{\mu} e^{-\alpha(t-\gamma)^{\beta}}\left[\int_t^{\mu} f(x)e^{\alpha(x-\gamma)^{\beta}}dx+f(\mu)\int_{\mu}^{t_1} e^{\alpha(x-\gamma)^{\beta}}dx\right]e^{-rt}dt\\ &+\int_{\mu}^{t_1} e^{-\alpha(t-\gamma)^{\beta}}\left[f(\mu)\int_t^{t_1} e^{\alpha(x-\gamma)^{\beta}}dx\right]e^{-rt}dt\Bigg]\end{aligned} \tag{22}$$

**3.7 Present value of deterioration cost during [0, $t_1$] is**

$$D=c_d\left[\int_{\gamma}^{\mu} f(t)\left(e^{\alpha(t-\gamma)^{\beta}}-1\right)e^{-rt}dt+f(\mu)\int_{\mu}^{t_1}\left(e^{\alpha(t-\gamma)^{\beta}}-1\right)e^{-rt}\,dt\right] \tag{23}$$

**3.8 Present value of shortage cost during [$t_1$, $T$] is**

$$S=c_s f(\mu)\int_{t_1}^{T}\left[\int_{t_1}^{t} g(T-x)dx\right]e^{-rt}dt \tag{24}$$

**3.9 Present value of opportunity cost due to lost sale during [$t_1$, $T$] is**

$$O=c_o f(\mu)\int_{t_1}^{T}[1-g(T-t)]e^{-rt}dt \tag{25}$$

**3.10 Present value of the total cost**

$TC_2(t_1) = H + D + S + O$

$$=(c_h+\phi t)\left[\int_0^{\gamma}\left[\int_t^{\gamma} f(x)dx+\int_{\gamma}^{\mu} f(x)e^{\alpha(x-\gamma)^{\beta}}dx+f(\mu)\int_{\mu}^{t_1} e^{\alpha(x-\gamma)^{\beta}}dx\right]e^{-rt}dt\right.$$

$$+\int_{\gamma}^{\mu} e^{-\alpha(t-\gamma)^{\beta}}\left[\int_{t}^{\mu} f(x)e^{\alpha(x-\gamma)^{\beta}}dx + f(\mu)\int_{\mu}^{t_1} e^{\alpha(x-\gamma)^{\beta}}dx\right]e^{-rt}dt$$

$$+\int_{\mu}^{t_1} e^{-\alpha(t-\gamma)^{\beta}}\left[f(\mu)\int_{t}^{t_1} e^{\alpha(x-\gamma)^{\beta}}dx\right]e^{-rt}dt\Bigg]$$

$$+c_d\left[\int_{\gamma}^{\mu} f(t)\left(e^{\alpha(t-\gamma)^{\beta}}-1\right)e^{-rt}dt + f(\mu)\int_{\mu}^{t_1}\left(e^{\alpha(t-\gamma)^{\beta}}-1\right)e^{-rt}dt\right]$$

$$+c_s f(\mu)\int_{t_1}^{T}\left[\int_{t_1}^{t} g(T-x)dx\right]e^{-rt}dt + c_o f(\mu)\int_{t_1}^{T}\left[1-g(T-t)\right]e^{-rt}dt \qquad (26)$$

The total present value of the system over [0, $T$] takes the form:

$$TC(t_1) = \begin{cases} TC_1(t_1) & \text{if } t_1 \le \mu \\ TC_2(t_1) & \text{if } t_1 > \mu \end{cases} \qquad (27)$$

It can be easily verified that $TC(t_1)$ is a continuous function at $t = \mu$. Our problem is to minimize the present value of the total cost function $TC(t_1)$ having its two branches.

Optimal replenishment policy:

We present the result which ensure the existence of a unique $t_1$ say $t_1^*$ which minimizes the total cost function $TC(t_1)$.

$$\frac{dTC_1(t_1)}{dt_1} = f(t_1)\,h(t_1) \qquad (28)$$

This implies that $h(t_1)$ is strictly increasing function of $t_1$, and we have $h(0) < 0$ and $h(T) > 0$, and by our assumption $f(t_1) > 0$, so the derivative $\frac{dTC_1(t_1)}{dt_1}$ vanishes at $t_1^*$, with $0 < t_1^* < T$, which is unique root of $h(t_1) = 0$, for this $t_1^*$ we have

$$\frac{d^2TC_1(t_1^*)}{dt_1^{*2}} = \frac{df(t_1^*)}{dt_1^*}h(t_1^*) + f(t_1^*)\frac{dh(t_1^*)}{dt_1^*} = f(t_1^*)\frac{dh(t_1^*)}{dt_1^*} > 0$$

$t_1^*$ corresponds to the unconstraint global minimum of $TC_1(t_1)$, if $t_1^* \le \mu$, *i.e.* if $t_1^*$ is feasible, the optimal value of the order level is $I_m = I(0)$ and is given by

$$I_m^* = \int_0^{\gamma} f(t)dt + \int_{\gamma}^{t_1^*} f(t)\,e^{\alpha(t-\gamma)^{\beta}}dt \qquad (29)$$

And optimal order quantity $Q^*$ is given by

$$Q^* = I_m^* + \int_{t_1^*}^{\mu} f(t)g(T-t)dt + f(\mu)\int_{\mu}^{T} g(T-t)dt \qquad (30)$$

And the minimum cost $TC(t_1)$ is given by equation (13).

When $g(x) = 0$, we have $h(0)<0$ and $\frac{dh(t_1)}{dt_1}>0$,

If $h(T) > 0$, $t_1^*$ corresponds to the unconstraint global minimum of $TC_1(t_1)$, moreover if $t_1^* \leq \mu$, the optimal value of the order level and the optimal order quantity are given by equations (29) and (30) respectively.

If $h(T) < 0$, $TC_1(t_1)$ is strictly decreasing and attaining its minimum at $T$.

Now consider the branch $TC_2(t_1)$. Its first and second order derivatives are

$$\frac{dTC_2(t_1)}{dt_1} = f(\mu)\,h(t_1)\,, \qquad \frac{d^2TC_2(t_1)}{dt_1^{\,2}} = f(\mu)\frac{dh(t_1)}{dt_1} \tag{31}$$

$TC_2(t_1)$ is strictly increasing when $0 < g(x) \leq 1$, and as above $\frac{dh(t_1)}{dt_1}$ has its unique root $t_1^*$ with $0 <t_1^* <T$. This $t_1^*$ corresponds to the unconstraint global minimum of $TC_2(t_1)$, if $t_1^*$ is feasible *i.e.* if $t_1^* > \mu$, the optimal value of the order level is $S = I(0)$ and is given by

$$I_m^{\;*} = \int_0^{\gamma} f(t)dt + \int_{\gamma}^{\mu} f(t)\,e^{\alpha(t-\gamma)^{\beta}}dt + f(\mu)\int_{\mu}^{t_1^*} e^{\alpha(t-\gamma)^{\beta}}dt \tag{32}$$

And the optimal order quantity $Q^*$ is given by

$$Q^* = I_m^{\;*} + f(\mu)\int_{t_1^*}^{T} g(T-t)dt \tag{33}$$

And the minimum cost $TC(t_1)$ is given by equation (26).

The analysis done so far shows that the two functions $TC_1(t_1)$ and $TC_2(t_1)$ have the same (unique) unconstraint minimum point $t_1^* \in (0, T)$. From equation (28) we can further shows that the function $TC(t_1)$ is differentiable at the point $\mu$, so $\mu$ cannot be a corner point for $TC(t_1)$.

For the case $g(x) = 0$, $\frac{dh(t_1)}{dt_1}>0$ and $h(0) > 0$, again $h(T)$ may be less than or greater than zero. If $h(T) > 0$, $t_1^*$ corresponds to the unconstraint global minimum of $TC_2(t_1)$, moreover if $t_1^*$ is feasible *i.e.* if $t_1^* > \mu$, the optimal value of the order level and order quantity are given by equations (32) and (33) respectively. If $h(T) < 0$, $TC_1(t_1)$ is strictly decreasing and attaining its minimum at $T$.

**4. The Algorithm to find the Optimal Replenishment Policy:**

**For the case $0 < g(x) \leq 1$**

Step i: compute $t_1^*$.

Step ii: compare $t_1^*$ to $\mu$, if $t_1^* \le \mu$, the optimal order quantity and total cost function are given by equations (6.30) and (6.13) respectively. If $t_1^* > \mu$, the optimal order quantity and total cost function are given by equations (6.33) and (6.26) respectively.

**For the case $g(x) = 0$,**

Step i: compute $h(T)$.

Step ii: If $h(T) > 0$, go to step i of previous case.

If $h(T) \le 0$, $t_1^* = T$, and consequently

$$I_m^* = Q^* = \int_0^{\gamma} f(t)dt + \int_{\gamma}^{T} f(t)\, e^{\alpha(t-\gamma)^{\beta}} dt$$

$$i.e.\ Q^* = \int_0^{\gamma} f(t)dt + \int_{\gamma}^{\mu} f(t)\, e^{\alpha(t-\gamma)^{\beta}} dt + f(\mu)\int_{\mu}^{T} e^{\alpha(t-\gamma)^{\beta}} dt \tag{34}$$

## 5. Numerical Examples:

Example 1: Let us take $\alpha = 0.01$, $\beta = 2$, $\gamma = 0.3$ year, $\mu = 0.9$ year, $c_h$ = \$ 3 per unit per year, $c_d$ = \$ 5 per unit per year, $c_s$ = \$ 15 per unit per year, $c_o$ = \$ 20 per unit per year, and $R = 0.2$, $T = 1$ year, $f(t) = 3\,e^{4.5t}$, and $g(x) = e^{-0.2x}$, the optimal value of $t_1$ as $t_1^* = 0.8487 < \mu$. The optimal ordering quantity is $Q^* = 54.5342$, S*=27.3421, and the minimum cost is $TC$= 82.6875.

Example 2: This example is identical to example 1, except $\mu = 0.6$ year. The optimal value of $t_1$ is $t_1^* = 0.8278 > \mu$. The optimal ordering quantity is $Q^* = 26.3453$, S*=19.8394, and the minimum cost is $TC = 42.8934$.

## 6. Sensitivity Analysis:

Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 25% and 50%, and taking one parameter at a time, keeping the remaining parameters unchanged. Table is given on the next page.

**Table-1 (when $t1 \le \mu$)**
**Sensitive Analysis with Respect to Different to Parameters**

| Parameters | % Change | $t_1^*$ | $S^*$ | $Q^*$ | $TC_1^*$ |
|---|---|---|---|---|---|
| $\alpha$ | –50 | 0.8533 | 27.1585 | 54.2782 | 82.4356 |
| | –25 | 0.8521 | 27.2627 | 54.3536 | 82.5903 |
| | +25 | 0.8467 | 27.4462 | 54.7389 | 82.7824 |
| | +50 | 0.8412 | 27.4828 | 54.8102 | 82.8493 |
| $\beta$ | –50 | 0.8431 | 27.1039 | 54.7803 | 83.5291 |
| | –25 | 0.8466 | 27.2635 | 54.6772 | 82.9839 |
| | +25 | 0.8492 | 27.5482 | 54.2168 | 82.3548 |
| | +50 | 0.8501 | 27.6744 | 54.1673 | 81.7904 |
| $\gamma$ | –50 | 0.8410 | 27.0473 | 54.7836 | 81.0285 |
| | -25 | 0.8462 | 27.1348 | 54.5890 | 81.3342 |
| | +25 | 0.8490 | 27.3894 | 54.3784 | 82.7785 |
| | +50 | 0.8495 | 27.4203 | 54.2913 | 82.8954 |
| $c_h$ | –50 | 0.8523 | 29.6431 | 54.7190 | 78.6951 |
| | –25 | 0.8503 | 28.9204 | 54.6738 | 80.2607 |
| | +25 | 0.8367 | 25.7823 | 54.3847 | 86.8342 |
| | +50 | 0.8328 | 22.9038 | 54.2134 | 92.1460 |

**Table-2 (When $t_1 > \mu$)**

| Parameters | % Change | $t_1^*$ | $S^*$ | $Q^*$ | $TC_2^*$ |
|---|---|---|---|---|---|
| $\alpha$ | –50 | 0.8423 | 20.3227 | 24.8532 | 40.1727 |
| | –25 | 0.8378 | 19.9371 | 25.4574 | 41.3236 |
| | +25 | 0.8234 | 18.5289 | 26.5248 | 43.1334 |
| | +50 | 0.8146 | 17.8654 | 27.0453 | 44.3282 |
| $\beta$ | –50 | 0.8210 | 17.0964 | 27.3109 | 43.1389 |
| | –25 | 0.8234 | 18.5427 | 26.4942 | 42.9323 |
| | +25 | 0.8299 | 19.9542 | 25.5735 | 42.2427 |
| | +50 | 0.8389 | 20.2564 | 24.0852 | 41.3894 |
| $\gamma$ | –50 | 0.8034 | 18.4797 | 27.5166 | 43.3562 |
| | –25 | 0.8190 | 19.0459 | 26.8347 | 43.1082 |
| | +25 | 0.8378 | 19.9075 | 25.8943 | 42.1728 |
| | +50 | 0.8482 | 20.1043 | 24.2809 | 41.5458 |
| $c_h$ | –50 | 0.8554 | 20.5432 | 27.1275 | 40.7364 |
| | –25 | 0.8411 | 19.9076 | 26.7344 | 41.2782 |
| | +25 | 0.8155 | 18.5763 | 25.2289 | 43.5945 |
| | +50 | 0.7959 | 17.5780 | 24.1833 | 45.7829 |

From table 1 and table 2, we can conclude:

1. The percentage change in the optimal total cost is almost equal for both positive and negative changes of all the parameters except $\beta$, $\gamma$, $c_h$ and $c_o$.
2. It is seen that the model is more sensitive for a negative change than an equal positive change in either of the parameters $\beta$, $\gamma$, $c_h$ and $c_d$.
3. The optimal cost increases and decreases with the increase and decrease in the parameters $\alpha$, $c_h$, $c_d$, $c_s$ and $c_o$. But this trend is reversed for parameters $\beta$, $\gamma$ and $R$.
4. The proposed model is low sensitive for parameters $\alpha$, $\beta$, $\gamma$ and $c_d$, moderately sensitive to changes in the parameters $c_o$ and $R$. It is highly sensitive to changes in the parameters $c_h$ and $c_s$.

**7. Conclusion:**

This paper deals an order level inventory model for deteriorating items with ramp type demand rate. The items that incur a gradual loss in quality or quantity over time while in inventory are usually called deteriorating items. It was assumed that the goods in the inventory deteriorate at a three parameter Weibull distribution rate. The nature of demand of seasonal and fashionable products is increasing-steady-decreasing and becomes asymptotic. For seasonal products like clothes, air conditions etc, demand of these items are very high at the starting of the season and become steady after certain time. The demand pattern assumed here is found to occur not only for all types of seasonal products but also for fashion apparel, computer chips of advanced computers, spare parts, etc. The procedure presented here may be applied to many practical situations. Retailers in supermarket face this type of problem to deal with highly perishable seasonal products. The model is fairly general as the demand rate which is a ramp type function of time. The shortages were allowed and partially backlogged in the model. The effects of inflation and time-value of money were incorporated into the model. Furthermore, numerical examples are given to illustrate the solution procedure for the mathematical model developed. Finally, they also implement sensitivity analysis of the optimal solution with respect to major parameters. Numerical results are shown with the help of graph.

The proposed model can be extended in several ways, for instance we may consider finite planning horizon. Also we can extend the deterministic demand function to stochastic demand patterns. Furthermore, we can generalize the model to allow permissible delay in payments.

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