Machine Repair Problem with Additional Repairman and Mixed Standbys

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Abstract

This paper is concerned with the analysis of a multi-component state dependent machine repair problem consisting of M operating units with two types of spares in the system to ensure the desired reliability for system. There is a provision of repair facility containing permanent as well as the additional repairman in order to provide repair to failed units one by one according to first come first served (FCFS) discipline. When all the spare units are exhausted then the system works in short mode; in this case a failed unit works with degraded rate due to stress. The life-time and repair time are both exponentially distributed. The recursive technique is used to find out the steady-state probabilities. Some system indices such as expected number of failed units in the system, expected number of cold as well as warm standby units, etc. are derived in terms of probabilities.

KEYWORDS: Machine repair problem, Permanent repairman, Additional repairman, Mixed Spares, Queue size distribution

1. INTRODUCTION

Machines are the integral part of any machining system. In recent years, the analysis of machine repair problem via queue-theoretic approach has played an important role to predict the system performance. The operating machines on failure can be replaced by the spare units if available. A standby machine is of the cold standby type if its failure rate is zero, whereas it is warm standby if its failure rate is nonzero and less than the failure rate of an operating machine. The provision of mixed standbys can be done to improve the grade of service in machining system.

Much work has been done in this direction from time to time since the inception of queueing theory. The M/M/C machine repair problem with cold and warm standbys has been discussed by **Toft and Boothroyd (1959)**. **Elsayed (1981)** considered two optimal repair policies for a machine repair problem with two modes of failure. **Wang and Sivazlian (1992)** studied the cost analysis of the M/M/R problem with spares under variable service rates. **Hsieh and Wang (1995)** discussed the reliability statistics of a repairable system with operating machines, spare machines and removable repair facility. **Jain (1997)** developed the diffusion approximation model for (m, M) machine repair problem with spares and state dependent rate. **Wang and Kuo (2000)** considered cost and probability analysis of service system with mixed standby components. The multi- component machinery system with spares and state dependent rates was studied by **Jain and Baghel (2001).**

Jain and Baghel (2003) analyzed a repairable system with spares, state dependent rates and additional repairmen. **Jain and Moses (2004)** developed the M/M/C interdependent machining system with mixed spares and controllable rates of failure and repair**. Jain *et al.* (2005)** discussed state dependent M/M/C/K/N machining system with mixed spares and removable repairmen. **Refael and Delia (2006)** developed a machining system with multiple warm standbys assuming operation and repair times under phase types distributions. **Jain *et al.* (2007)** considered machine repair model with cold standbys and maintenance float system. **Jain *et al.* (2008)** developed a performance modeling of state dependent system with spares and reneging. **Wahab *et al.* (2008)** studied a generic approach to measure the machine flexibility of manufacturing systems. **Eryilmaz (2009)** examined the reliability properties of consecutive k out of n systems of arbitrary dependent components. **Maheshwari et al. (2010)** analyzed machine repair problem with k-type warm spares, multiple vacations for repairmen and reneging. **Yue D, Yue W, Qi H (2012) discussed** performance analysis and optimization of a machine repair problem with warm spares and two heterogeneous repairmen. **Dong Yoh Yang and Ya-Dun Chang (2017)** developed a Sensitivity analysis of the machine repair problem with general repeated attempts. **Wang, K. H (2017)** studied the Profit analysis of the M/M/R machine repair problem with spares and server breakdowns.

A mixed standby machining system with additional repairman and state dependent rates is investigated in the present paper. For the solution purpose birth–death process is employed. The organization of the paper is as follows. In section 2, the model description and notations are provided. In section 3, we describe the balance equations at steady state. The queue size distribution is obtained in section 4. In section 5, some performance measures are established. Finally, the conclusion is drawn in section 6.

2. MODEL DESCRIPTION

Consider a machining system consisting of M operating units and two types of spare units with single service station. Whenever the operating unit fails, it is replaced by a spare unit if available. The failure of the unit in case when all spares have been used makes the system in degraded mode. The life time and repair time of the operating units as well as that of warm standbys are exponentially distributed. If any operating unit fails, it is sent for repair to the service station and a spare is put in place of failed unit if a spare is available. The failed units are repaired by the service station according to first come first served (FCFS) discipline. When the repairing of a failed unit is completed, it works as good as new one and joins to the standby group if M operating units are present in the system otherwise the repaired unit will immediately go into operating group. When there are failed machines more than the permanent repairmen the additional removal repairman turns on; the additional repairman turns off when the queue level of failed units again ceases to R.

The following notations have been used to describe the Markovian model under consideration:

M Number of operating units in the system.

S_1 (S_2) Number of cold (warm) spare units in the system.

K Total number of units in the system (i.e. $K=M+ S_1 + S_2$).

λ Failure rate of operating units when at least one spare unit is available.

α_2 Failure rate of warm spare units.

ε Mean dependence rate.

λ_d Degraded failure rate of operating units when all spare units are exhausted.

$\mu_1 (\mu_2)$ Repair rate of permanent (additional) repairman.

μ_f Faster repair rate of permanent repairmen when all cold standby units are used.

The failure rates of the failed units are given by

(i) For level 0

$$\lambda_n^0 = \begin{cases} M(\lambda-\varepsilon)+S_2(\alpha_2-\varepsilon), & 0 \le n < S_1 \\ M(\lambda-\varepsilon)+(S_1+S_2-n)(\alpha_2-\varepsilon), & S_1 \le n < S_1+S_2 \end{cases}$$

(ii) For level 1

$$\lambda_n^1 = \begin{cases} M(\lambda-\varepsilon)+(S_1+S_2-n)(\alpha_2-\varepsilon), & S_1 \le n < S_1+S_2 \\ (K-n)(\lambda_d-\varepsilon), & S_1+S_2 \le n < K \end{cases}$$

The repair rates are given by

(i) For level 0

$$\mu_n^0 = \begin{cases} (\mu_1-\varepsilon), & 0 < n \le S_1 \\ (\mu^{(2)}-2\varepsilon), & S_1 < n \le S_1+S_2 \end{cases}$$

where $\mu^{(2)} = \mu_1 + \mu_2$.

(ii) For level 1

$$\mu_n^1 = (\mu_f^{(2)}-2\varepsilon), \qquad S_1 \le n \le K$$

where $\mu_f^{(2)} = \mu_f + \mu_2$.

3. The Governing Balance Equations

The steady state probabilities are defined as follows:

$P_n(i)$ = The steady state probability of n failed units in the system at level i (i=0,1). Here $P_n(0)$ exists for $0 \le n \le S_1+S_2-1$ and $P_n(1)$ exists for $S_1+1 \le n \le K$.

Chapman Kolmogrov equations for steady state are constructed by using state transition flow rates (i.e. failure and repair rates) and are given below:

$$-\left[M(\lambda-\varepsilon)+S_2(\alpha_2-\varepsilon)\right]P_0(0)+(\mu_1-\varepsilon)P_1(0)=0 \quad \text{... (1)}$$

$$--\left[M(\lambda-\varepsilon)+S_2(\alpha_2-\varepsilon)+n(\mu_1-\varepsilon)\right]P_n(0)+\left[M(\lambda-\varepsilon)+S_2(\alpha_2-\varepsilon)\right]P_{n-1}(0)$$
$$+(\mu_1-\varepsilon)P_{n+1}(0)=0, \qquad 1 \le n \le S_1 \quad \text{... (2)}$$

$$-[M(\lambda-\varepsilon)+S_2(\alpha_2-\varepsilon)+(\mu_1-\varepsilon)]P_{S_1}(0)+[M(\lambda-\varepsilon)+S_2(\alpha_2-\varepsilon)]P_{S_1-1}(0)$$
$$+(\mu^{(2)}-2\varepsilon)P_{S_1+1}(0)+(\mu_f^{(2)}-2\varepsilon)P_{S_1+1}(1)=0 \quad \text{... (3)}$$

$$-[M(\lambda-\varepsilon)+(S_1+S_2-n)(\alpha_2-\varepsilon)+(\mu^{(2)}-2\varepsilon)]P_n(0)+[M(\lambda-\varepsilon)+(S_1+S_2-n+1)(\alpha_2-\varepsilon)]$$
$$\times P_{n-1}(0)+(\mu^{(2)}-2\varepsilon)P_{n+1}(0)=0, \;\; S_1<n<S_1+S_2-1 \quad \text{...(4)}$$

$$[M(\lambda-\varepsilon)+2(\alpha_2-\varepsilon)]P_{S_1+S_2-2}(0)-[M(\lambda-\varepsilon)+(\alpha_2-\varepsilon)+(\mu^{(2)}-2\varepsilon)]P_{S_1+S_2-1}(0)=0 \quad \text{...(5)}$$

$$-[M(\lambda-\varepsilon)+(S_1-1)(\alpha_2-\varepsilon)+(\mu_f^{(2)}-2\varepsilon)]P_{S_1+1}(1)+(\mu_f^{(2)}-2\varepsilon)P_{S_1+2}(1)=0 \quad \text{...(6)}$$

$$-[M(\lambda-\varepsilon)+(S_1+S_2-n)(\alpha_2-\varepsilon)+(\mu_f^{(2)}-2\varepsilon)]P_n(1)$$
$$+[M(\lambda-\varepsilon)+(S_1+S_2-n+1)(\alpha_2-\varepsilon)]P_{n-1}(1)+(\mu_f^{(2)}-2\varepsilon)P_{n+1}(1)=0, \;\; S_1+1<n<S_1+S_2 \quad \text{... (7)}$$

$$-[M(\lambda_d-\varepsilon)+(\mu_f^{(2)}-2\varepsilon)]P_{S_1+S_2}(1)+[M(\lambda-\varepsilon)+(\alpha_2-\varepsilon)]P_{S_1+S_2-1}(1)$$
$$+(\mu_f^{(2)}-2\varepsilon)P_{S_1+S_2+1}(1)+[M(\lambda-\varepsilon)+(\alpha_2-\varepsilon)]P_{S_1+S_2-1}(0)=0 \quad \text{... (8)}$$

$$-[(K-n)(\lambda_d-\varepsilon)+(\mu_f^{(2)}-2\varepsilon)]P_{n-1}(1)+[(K-n+1)(\lambda_d-\varepsilon)]P_{n-1}(1)$$
$$+(\mu_f^{(2)}-2\varepsilon)P_{n+1}(1)=0, \;\; S_1+S_2+1 \le n < K \quad \text{... (9)}$$

4. Queue Size Distribution

Using equations (1) and (2), we have

$$P_n(0) = A^n P_0(0), \qquad 0 < n \le S_1 \qquad ...(10)$$

where $A = \dfrac{[M(\lambda-\varepsilon)+S_2(\alpha_2-\varepsilon)]}{(\mu_1-\varepsilon)}$.

From equations (3) and (4), to find $P_n(0)$ for $S_1+1 \le n \le S_1+S_2-1$, we employ recursive method, and obtain

$$P_n(0) = \prod_{i=1}^{n-S_1-1} K_i \beta P_0(0) - \left[1 + \sum_{j=1}^{n-S_1-1} \prod_{i=j}^{n-S_1-1} K_i\right]\left(\frac{\mu_f^{(2)}-2\varepsilon}{\mu^{(2)}-2\varepsilon}\right) P_{s_1+1}(1),$$

$$S_1+1 \le n \le S_1+S_2-1 \qquad ...(11)$$

where $K_i = \dfrac{[M(\lambda-\varepsilon)+(S_2-i)(\alpha_2-\varepsilon)]}{(\mu^{(2)}-2\varepsilon)}$ and $\beta = \dfrac{[M(\lambda-\varepsilon)+S_2(\alpha_2-\varepsilon)]^{S_1+1}}{(\mu_1-\varepsilon)^{S_1}(\mu^{(2)}-2\varepsilon)}$.

To obtain $P_{S_1+1}(1)$ in terms of $P_0(0)$, using equation (5), we get

$$P_{S_1+1}(1) = \frac{\prod_{i=1}^{S_2-1} K_i \beta P_0(0)}{\left[1 + \sum_{j=1}^{S_2-1}\prod_{i=j}^{S_2-1} K_i\right]\left(\frac{\mu_f^{(2)}-2\varepsilon}{\mu^{(2)}-2\varepsilon}\right)} \qquad ...(12)$$

Combined equations (11) and (12) yield

$$P_n(0) = \left[\prod_{i=1}^{n-S_1-1} K_i - \frac{\left(1+\sum_{j=1}^{n-S_1-1}\prod_{i=j}^{n-S_1-1} K_i\right)\left(\prod_{i=1}^{S_2-1} K_i\right)}{\left(1+\sum_{j=1}^{S_2-1}\prod_{i=j}^{S_2-1} K_i\right)}\right]\beta P_0(0), \quad S_1+1 \le n \le S_1+S_2-1$$

...(13)

Using equations (6) to (9), we obtain

$$P_n(1) = \begin{cases} \left[1+\sum_{i=1}^{n-S_1-1}\prod_{l=i}^{n-S_1-1}\delta_l\right] P_{S_1+1}(1), & S_1+1 \le n \le S_1+S_2 \\ \left(\prod_{i=0}^{n-S_1-S_2-1}\frac{(M-i)(\lambda_d-\varepsilon)}{(\mu_f^{(2)}-2\varepsilon)}\right)\left(1+\sum_{i=1}^{S_2-1}\prod_{l=i}^{S_2-1}\delta_l\right) P_{S_1+1}(1), & S_1+S_2 < n \le K \end{cases}$$

...(14)

where $\delta_\ell = \dfrac{[M(\lambda-\varepsilon)+(S_2-\ell)(\alpha_2-\varepsilon)]}{(\mu_f^{(2)}-2\varepsilon)}$.

To find $P_0(0)$, we use the normalizing condition given by

$$\sum_{n=0}^{S_1+S_2} P_n(0) + \sum_{n=S_1+1}^{K} P_n(1) = 1 \qquad ...(15)$$

Now, we obtain $P_0(0)$ as

$$[P_n(0)]^{-1} = \sum_{n=0}^{S_1} A^n + \beta \sum_{n=S_1+1}^{S_1+S_2-1} \left[\prod_{i=1}^{n-S_1-1} K_i - \frac{\left(1+\sum_{j=1}^{n-S_1-1} \prod_{i=j}^{n-S_1-1} K_i\right)\left(\prod_{i=1}^{S_2-1} K_i\right)}{\left(1+\sum_{j=1}^{S_2-1}\prod_{i=j}^{S_2-1} K_i\right)} \right]$$

$$+\frac{[\prod_{i=1}^{S_2-1} K_i]\beta P_0(0)}{\left(1+\sum_{j=1}^{S_2-1}\prod_{i=j}^{S_2-1} K_i\right)\left(\frac{\mu_f^{(2)}-2\varepsilon}{\mu^{(2)}-2\varepsilon}\right)} \left[\sum_{n=S_1+1}^{S_1+S_2} \left(1+\sum_{i=1}^{n-S_1-1}\prod_{l=i}^{n-S_1-1} \delta_l\right) + \left(1+\sum_{i=1}^{S_2-1}\prod_{l=i}^{S_2-1}\delta_l\right) \times \sum_{n=S_1+S_2+1}^{K} \prod_{i=0}^{n-S_1-S_2-1} \frac{(M-i)(\lambda_d-\varepsilon)}{(\mu_f^{(2)}-2\varepsilon)} \right]$$

...(16)

5. SOME PERFORMANCE MEASURES

Performance measures characterizing the machining system can be obtained using queue size distribution as follows:

- Expected number of failed units in the system is

$$L = \sum_{n=0}^{S_1+S_2} nP_n(0) + \sum_{n=S_1+1}^{K} nP_n(1) \quad ...(17)$$

- Expected number of cold standby units is given by

$$E(S_1) = \sum_{n=0}^{S_1} (S_1 - n)P_n(0) \quad ...(18)$$

- Expected number of warm standby units is obtained as

$$E(S_2) = S_2 \sum_{n=0}^{S_1} P_n(0) + \sum_{n=S_1+1}^{S_1+S_2} (S_1+S_2-n)P_n(0) + \sum_{n=S_1+1}^{S_1+S_2} (S_1+S_2-n)P_n(1) \quad ...(19)$$

6. Conclusions

In this paper we have developed the M/M/R machine repair model with two types of spares. A repair facility consisting of R permanent and one additional removable repairman to restore failed units is taken into consideration. The steady state queue size distribution is obtained which is further employed to establish various performance indices. The provision of spare part support and additional repairman can improve the system efficiency and availability to the desired extent.

The model developed has many applications in computer, communication, manufacturing and production systems wherein an optimal combination of spares and repairmen is desired for smooth functioning of the system at minimum cost.

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